Solutions

for *Conceptual Physics*, *Light and Matter*, *Mechanics*, *Simple Nature*, *Fields and Circuits*, and *Modern Physics*

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1 Light and Matter

0-1 (a) If your calculator has parentheses, you have to use them around the whole denominator. On a calculator without parentheses, you'd add up the denominator, put the sum in memory, and then do the division. The answer is 0.4951584.
(b) $\$3.5 \times 10^5$ equals \$350,000, the price of a big house, and $\$3.5^5$ equals \$525.21, the price of a big TV.

0-2 (a) 8 cm; (b) 1.33 m; (c) doesn't make sense; (d) 60 miles/hour

0-3 $23.4\text{ m} + 2 \times 0.294\text{ m} = 23.988\text{ m}$, which should be rounded to 24.0 m (3 significant figures)

0-4

$$\begin{aligned}&3.0 \times 10^8\text{ m/s}\\ &\times\left(\frac{100\text{ cm}}{1\text{ m}}\right)\left(\frac{1\text{ inch}}{2.54\text{ cm}}\right)\left(\frac{1\text{ foot}}{12\text{ inches}}\right)\\ &\times\left(\frac{1\text{ yard}}{3\text{ feet}}\right)\left(\frac{1\text{ furlong}}{220\text{ yards}}\right)\\ &\times\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right)\left(\frac{24\text{ hr}}{1\text{ day}}\right)\left(\frac{14\text{ days}}{1\text{ forthnight}}\right)\\ &= 1.8 \times 10^{12}\text{ furlongs/fortnight}\end{aligned}$$

0-5 (a) 10^4 μg; (b) 10^{10} μg; (c) 10^{10} μg; (d) 10^{11} μg; (e) 1 μg

0-7 (a)

$$\begin{aligned}\frac{0.5\text{ mg}}{0.2\text{ ng/hamburger}} &= \frac{0.5 \times 10^{-3}\text{ g}}{0.2 \times 10^{-9}\text{ g}}\text{ hamburgers}\\ &= 2\text{ million hamburgers!}\end{aligned}$$

(Note how the fake units of "hamburgers" help to reassure us that dividing the first number by the second one gives the right result.)
(b) (0.5 mg)/(300 ng/serving)=2000 servings

0-9 The range of uncertainty amounts to more than a factor of 2. That means that each of the two figures has been quoted with some completely meaningless sig figs. A better way to write the numbers would be 6 days and 14 days.

0-10 The weight is given in U.S. units with not much more than one sig fig of precision, but the metric conversion is given with three sig figs. It's silly to try to give tenths of a gram here; there must be an order of magnitude more variation than that from one bag to the next, since one pretzel is probably several grams.

0-11 Both r and h have units of meters. The product rh has units of m^2, and taking its square root gives units of meters again. That makes sense, because the result is supposed to be a distance.

0-13 The estimate of $4,480$ calories is stated with three significant figures, which is absurd. Individuals often differ by a factor of two in their food consumption, and in any case this is a rough estimate for a species that has been extinct for tens of thousands of years.

1-1

$$(1\text{ ft}^3) \times (12\text{ in/1 ft})^3 = 1728\text{ in}^3$$

1-2 Volume scales like the third power of length, so the dog has 8 times more brain cells.

1-3 Solving the proportionality $A \propto L^2$ for L, we have $L \propto \sqrt{A}$. Converting this into the language of ratios gives $L_{LA}/L_{SF} = \sqrt{A_{LA}/A_{SF}} = 1.2$.

1-4 Area and volume are different units. You can't compare 10 in^2 to 1 in^3. 10 in^2 is not "greater than" 1 in^3. (The dog's nose membrane is crumpled up like a crumpled piece of paper.)

1-5 I don't even know the exact width of a football field, but let's guess the dimensions are 100 yd × 30 yd, which is close enough to 100 m × 30 m, for a total area of 3000 m^2. If we assume each blade of grass takes up on the order of 1 mm^2, then the number of blades equals $3000\text{ m}^2/1\text{ mm}^2 = 3 \times 10^9\text{ mm}^2/1\text{ mm}^2 = 3 \times 10^9$. Call it 10^9 blades.

1-6 Let's say the chip is 1 cm × 1 cm. It contains about 10^8 bits (64×10^6 bytes$\times8$ bits/byte), so the area of each circuit is about 10^{-8} cm^2.

Assuming the circuits are little squares, their dimensions would be about 10^{-4} cm on a side, since $10^{-4}\text{ cm}\times 10^{-4}\text{ cm} = 10^{-8}\text{ cm}^2$.

1-7 The world's population is about 5 billion, or call it 10^{10}. Suppose each person gets an apartment that's 5 m x 5 m x 2 m, for a volume of 50 m^3, or round off to 100 m^3. The total volume of the building needs to be about 10^{12} m^3. Its area at the base is 10^8 m^2, so its height needs to be $10^{12}\text{ m}^3/10^8\text{ m}^2 = 10^4\text{ m} = 10$ km.

1-8 Suppose one bongo burger is 1/4 lb, which is about 0.1 kg. If the usable meat on a cow amounts to 100 kg, then one cow makes 1000 burgers. Clearly the cow has to eat much more over the course of its life than its own adult weight. (After all, you could not raise a human to adulthood on 100-200 lb of food.) Let's say 10000 kg of feed are needed to raise a several-hundred-kg cow. Then 10000 kg of feed makes 1000 burgers, or 10 kg of feed makes 1 burger. That means $\sim 10^{11}$ kg of feed were needed to make 10^{10} burgers.

1-9 Apply two of the general rules for making order-of-magnitude estimates: don't try to estimate volume directly, and approximate complicated shapes as simple ones. Suppose a person is sort of a rectangle, 2 m tall, 0.5 m wide, and 0.2 m from front to back. The volume would then be $0.2\text{ m}^3 = 2\times 10^5\text{ cm}^3$, which we can round off to 10^5 cm^3.

1-13 (a) You can get two A5 sheets by cutting an A4 in half, so the area of an A5 is half that of an A4. The two types are the same shape, so the area and linear dimensions are related by $A \propto L^2$, or $L \propto \sqrt{A}$. Cutting the area in half means reducing all the linear dimensions by a factor of $1/\sqrt{2}$.

(b) Let the dimensions of the B0 box, in units of meters, be a, b, and c ($a > b > c$). If you can pack two B1 boxes inside a B0 box, two of the dimensions of a B1 must be the same as the two smallest dimensions of the B0, b and c, while the third is $a/2$. Since $L \propto V^{1/3}$, the dimensions of the B1 must be smaller than those of the B0 by a factor of $1/2^{1/3}$. If the series is to continue with B2, B3, …, the three sides of a particular box must be related by this same factor:

$$\begin{aligned} b &= \frac{a}{2^{1/3}} \\ c &= \frac{b}{2^{1/3}} = \frac{a}{2^{2/3}} \end{aligned}$$

Since the volume is one cubic meter, we have

$$\begin{aligned} 1\text{ m}^3 &= abc \\ &= (a)\left(\frac{a}{2^{1/3}}\right)\left(\frac{a}{2^{2/3}}\right) \\ &= \frac{a^3}{2} \\ a &= 2^{1/3}\text{ m} \\ b &= 1\text{ m} \\ c &= 1/2^{1/3}\text{ m} \end{aligned}$$

1-14 We never try to estimate mass or volume directly; we estimate linear dimensions first. Let's say the hair has a diameter of 0.1 mm, and a length of 1 cm. There's no real point in trying to compute its volume as a cylinder, since this is just an estimate. Let's just consider it to be a long, skinny box, with dimensions 0.1 mm × 0.1 mm × 1 cm. We'll assume it has the same density as water, 1 g/cm^3, so let's get all the dimensions in terms of centimeters before computing the volume and mass: $0.01\text{ cm}\times 0.01\text{ cm}\times 1\text{ cm} = 10^{-4}\text{ cm}^3$. Using the density, we get a mass of $10^{-4}\text{ g} = 10^{-7}$ kg.

1-15 First let's estimate the total volume of the Earth's atmosphere. We know that by going to the top of a mountain that's a few miles high, we can get above most of the atmosphere, so let's say the atmosphere is roughly 10 km, or 10^4 m thick. The diameter of the earth is about 12000 km, or about 10^7 m, so its surface area is on the order of 10^{14} m^2, and multiplying by the thickness of the atmosphere gives about 10^{18} m^3. A human breath is probably on the order of a liter, or 10^{-3} m^3, so Caesar's last breath has been diluted by a factor of something like $10^{18}/10^{-3} \sim 10^{21}$.

Now the question is whether your breath has a lot more than, or a lot less than, 10^{21} atoms in it. If you've taken chemistry, you know that a mole is something like 10^{24} atoms, and a mole of air would only be on the order of 10 grams. That means your breath has a lot more than 10^{21} atoms in it, so it's probably true that it contains many of the atoms from Caesar's last breath. If you haven't taken chemistry, you might not have as good a feel for this, but you could simply look up the density of air and the mass of an oxygen molecule on the internet or at the library.

(One implicit assumption that could be wrong here is that air stays in the atmosphere indefinitely. Some air could spend a lot of its time in plants, or dissolved in the oceans or in rocks.)

1-16 Area scales like the square of the linear dimensions, so Earth's total surface area is four times that of Mars. Since only 30 percent of Earth's area is land, the ratio is 1.2 to 1 (Earth to Mars).

1-18 Area scales like the square of the linear dimensions, so 0.35 of the linear dimensions corresponds to $0.35^2 = 0.11$, or 11% of the area.

1-22 (a) Living things typically have about the same density as water, $1\text{g/cm}^3 = 1000\ \text{kg/m}^3$. The linear dimensions are about 1 μm, so the volume is about $10^{-18}\ \text{m}^3$, giving a mass of 10^{-15} kg.
(b) Estimating the mass of the feces to be 1 kg (some of us are more full of it than others), we get about 10^{15} bacteria in the body. This is two orders of magnitude greater than the number of human cells.
(c) Since the human cells are much less numerous, but take up much more space, they must be many orders of magnitude bigger than bacterial cells.

1-23 (a) Converting the relation $n \sim A^{1/3}$ into the language of ratios, we have $n_1/n_2 = (A_1/A_2)^{1/3}$. Plugging in the numbers, the result is $n_1 \approx 14$, which not in bad agreement with the real-world figure of 22. (b) Suppose that the exponent was equal to 1. Then if we took two adjacent tracts of land with equal area and counted up all the taxa, we would find twice as many taxa in the combined area than in either of its halves. In other words, no taxon would be found in both areas. This is of course unrealistic. Almost certainly there will be taxa that sprawl across the border.

1-24 Humans and ants are both mostly water, so the ratio of their body masses is probably about the same as the ratio of their volumes. We therefore have $m \propto V \propto L^3$, or $L \propto m^{1/3}$. Since the ratio of the body masses is $m_1/m_2 \sim 7 \times 10^6$, the ratio of the thicknesses is $L_1/L_2 \sim (m_1/m_2)^{1/3} \sim 200$.

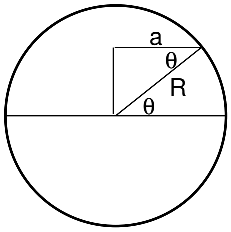
1-25 In the language of proportionalities, we have $L \propto t$, and $V \propto L^3$, so $t \propto V^{1/3}$. Translating this into a statement about ratios, we have $t_2/t_1 = (V_2/V_1)^{1/3} = 100^{1/3} = 4.64$. The signals have already been propagating outward for $t_1 =$ 90 years, so to fill 100 times the volume, they would have to propagate for (90 years)(4.64), or 418 years. Adding this to 1920, we get 2338 AD.

1-27 The relationship between volume and linear dimensions is $V \propto L^3$. We're given information about V and asked to find out about L, so we solve this for $L \propto V^{1/3}$. Converting this into a statement about ratios, we have $L_1/L_2 = (V_1/V_2)^{1/3}$. The result is that the balls could have a diameter as big as 4.48 cm.

1-28 A thousand metric tons is 10^9 g of plutonium. To estimate the volume of the earth's oceans, let r be the earth's radius and d the average depth of the ocean. Looking up r, estimating d to be 1 km, and taking the earth to be mostly ocean, we get a volume of $4\pi r^2 d$, or about $10^9\ \text{km}^3 = 10^{24}\ \text{cm}^3$. The concentration comes out to be $\sim 10^{-15}\ \text{g/cm}^3$, which is fifty times lower than the government standard. This method of disposal would therefore have no measurable human health effects (and probably none on any part of the biosphere).

2-1 (a) $v_{av} = 90\ \text{m}/14\ \text{s} = 6.4\ \text{m/s}$
(b) The tangent line seems to have the greatest slope at the very end. Drawing the tangent line, we get a slope of about 40 m/2 s=20 m/s.

2-2 (a) Let a be the radius of the circle made by the point on the earth's surface. $a = R\cos\theta$, and $L = 2\pi a$, so $L = 2\pi R\cos\theta$.
(b) Plugging in $\theta = 34°$ and $R = 6.4 \times 10^6$ m gives $L = 3.3 \times 10^7$ m, so the speed is $L/24\ \text{hr} = 1.4 \times 10^6\ \text{m/hr} = 890\ \text{mi/hr}$.



2-3 The definition of velocity is the slope of the tangent line. (Note that $v = \Delta y/\Delta t$ is *not* the definition, and isn't even true here, since the graph isn't a line.) Applying the definition directly, we could make a graph, draw in a tangent line, and find its slope.

As a shortcut, we can realize that most of the graph would be irrelevant. To draw a tangent line at $t = 7.0$ s, we only need the part of the graph very close to that time. As an even more extreme shortcut, we don't even need to make a graph at all. Under a microscope, the part of the curve near $t = 7.0$ s looks like a straight line, so all we really need to do is take two points that are very close to that time, and use them to compute the velocity.

Plugging in $t = 7.0000$ s gives $y = 9588.3508$ m, and $t = 7.0001$ s gives $y = 9588.3470$ m. The velocity is approximately (9588.3470 m − 9588.3508 m)/0.0001 s = −38 m/s. (Note that this is a situation where we have to keep lots of extra significant figures until the end, or we'd get zero for our answer because both y's would round off to the same number.)

2-6 $v = \mathrm{d}x/\,\mathrm{d}t = 10 - 3t^2 = -17$ m/s.

2-8 We're only going to be able to make a rough estimate, due to perspective and the changing speed of the leaves. The angle between the leaves appears to be about 180°, 90°, and 0° in the three successive frames, so each leaf travels about the same distance in the two time intervals. The second time interval, 44 minutes, is much shorter than the first, 96 minutes, so the greater speed seems to be during the second one. The distance traveled by the tip of one of the leaves looks like about 2 cm from one image to the next, so the top speed is something like

$$\begin{aligned}\frac{2\text{ cm}}{44\text{ min}} &= \frac{0.02\text{ m}}{2600\text{ s}}\\ &\approx 8\times10^{-6}\text{ m/s}\qquad,\end{aligned}$$

rounding off to one significant figure because of the many approximations involved.

2-9 (a) Let the subscripts A, E, and G refer to the arrow, Eowyn, and the ground. Let the positive x axis point forward.

$$\begin{aligned}v_{EG} &= 11\text{ m/s}\\ v_{AE} &= -25\text{ m/s}\end{aligned}$$

(b)

$$\begin{aligned}v_{AG} &= v_{AE} + v_{EG}\\ &= -14\text{ m/s}\end{aligned}$$

The answer makes sense: the arrow flies backward, at less than normal speed.

2-11 (a) Since b is added to t^2, b must have units of s^2. We want x to come out in units of meters, so a must have units of $\mathrm{m}\cdot\mathrm{s}^2$ (meters *times* seconds squared, not meters divided by seconds squared as in acceleration).

(b) From the given information, we can find her speed $v(t)$ by differentiation, and we can then find the maximum by the usual technique of setting the derivative equal to zero. That means taking two derivatives in a row: a second derivative. The second derivative of the position is the acceleration, so what it boils down to is finding the time when her acceleration is zero. (Note that the a in the original equation doesn't stand for acceleration.)

$$\begin{aligned}\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} &= \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[a\left(b+t^2\right)^{-1}\right]\\ &= a\frac{\mathrm{d}}{\mathrm{d}t}\left[-\left(b+t^2\right)^{-2}(2t)\right]\\ &= a\left[2\left(b+t^2\right)^{-3}(4t^2)-\left(b+t^2\right)^{-2}(2)\right]\\ &= 0\end{aligned}$$

$$
\begin{aligned}
8\left(b+t^2\right)^{-3}\left(t^2\right) &= 2\left(b+t^2\right)^{-2} \\
8t^2 &= 2\left(b+t^2\right) \\
6t^2 &= 2b \\
t &= \pm\sqrt{b/3}
\end{aligned}
$$

Plugging $t = \pm\sqrt{b/3}$ into the first derivative, we get $|v| = (3^{3/2}/8)ab^{-3/2}$.

(c) The $3^{3/2}/8$ is unitless, so the units come from the a and b factors: $\mathrm{m}\cdot\mathrm{s}^2\cdot(\mathrm{s}^2)^{-3/2} = \mathrm{m/s}$.

2-12 Take a frame defined by an observer in another car who is in the lane next to yours, who is initially moving at the same speed as you, and who continues at the speed even while you slow down. In that observer's frame, you started out with zero speed and then gained some (backward) speed when you stepped on the brakes.

2-13

$$\frac{3000\ \text{km}}{1\ \text{month}} \times \frac{1\ \text{month}}{30\ \text{days}} \times \frac{1\ \text{day}}{24\ \text{hours}} \approx 4\ \text{km/hr}$$

That's a pretty good average speed to keep up, against water resistance, for that long. It probably doesn't go in a straight line, either, so this is presumably an underestimate of how fast it typically moves. If you ever get a chance to see them at feeding time at the Monterey Bay Aquarium, don't miss it — they're impressive sprinters.

2-14 (a) The input to the exponential function has to be unitless, so b must have units of s^{-1} (inverse seconds). The output of the exponential is also unitless, so if the right-hand side is to have units of meters, c must have units of m/s.

(b) Differentiating, we find that the velocity is

$$v = c(1-bt)e^{-bt} \qquad .$$

We want to maximize this as a function of time, which we can do by taking its derivative and setting it equal to zero:

$$0 = bc(bt-2)e^{-bt}$$

The maximum occurs at $t = 2/b$, and the velocity at this time ia $-ce^{-2}$, so the maximum speed is the absolute value of this, ce^{-2}. (Higher speeds occur at earlier times, but we're only finding the maximum speed as it comes back to the closed position.)

(c) We already know that c has units of m/s, so this does have the right units to be a velocity.

2-15 Draining the can completely isn't optimal, because then its center of mass will be that of the empty can, at its own center. The center of mass is a kind of average. Any beer below the center will lower the average relative to the empty can, while any beer above it will raise it. You should drink half the beer before setting it down.

2-16 (a) Split the isoceles triangle into two right triangles, and let the long sides of one of the right triangles be L_p and L, where the subscript p refers to the runner who is passing. Applying the definition of the cosine gives $L/L_P = \cos\theta$. Since the runners cover these distances in equal times, we have $v_p = L_P/t$ and $v = L/t$, so that $v_p = (L_P/L)v = v/\cos\theta$. Therefore $\Delta v = v_p - v = (1/\cos\theta - 1)v$.

(b) The cosine is unitless, so the entire factor $(1/\cos\theta - 1)$ in parentheses is also unitless. The velocity v has units of m/s, so multiplying by the quantity in parentheses still gives m/s. This makes sense, because the thing we're calculating, on the left side of the equals sign, is supposed to be a velocity, with units of m/s.

(c) Since the cosine of 0 is 1, the result in the case of $\theta = 0$ is $\Delta v = 0$. This makes sense. If there is a hundred-mile straightaway available in which to pass, the extra speed required is going to be nearly zero.

(d) Plugging in numbers gives $\Delta v = 1.1 \times 10^{-3}$ m/s. This is not just small (as expected from part c) but insanely small — in fact too small to matter — basically because L_p does not differ from L by as much as we would intuitively expect. It appears that in reality, the extra speed required by the passing runner is not the speed required to run the extra distance, but simply the same extra speed that would have been required if the runners had been in parallel lanes. That is, our geometrical model turned out to include the factor that was unimportant, and ignore the one that would really be important.

2-17 The speed of light is constant, so the equation $v = \Delta x/\Delta t$ holds. The distance traveled by the ray of light is $\Delta x = 2L$. The spacing between teeth is $1/n$ of a revolution, so during the time Δt the wheel moves through $1/2n$ of a revolution. Setting up a ratio between a whole revolution and this small part of a revolution, we have

$$\frac{\Delta t}{\text{time for one revolution}} = \frac{1}{2n}$$

The time for one revolution is $1/f$, so $\Delta t = 1/2nf$. Substituting into the equation for the velocity, we have $c = \Delta x/\Delta t = (2L)/(1/2nf) = 4Lnf$.

(b) The factor of $4n$ on the right is unitless. The units of Lf are $(\text{m})(\text{s}^{-1}) = \text{m/s}$, which are the units we expect for a velocity.

(c) All three numbers are being multiplied on the right, so an increase in any one of these corresponds to an *increase* in c. Since c is big, Fizeau wants all three numbers to be big: a long distance to the mirror, a large number of teeth, and a fast rotation of the wheel.

(d) The result is 3.13×10^8 m/s.

2-18 (a) The circumference of a circle is 2π times the radius, so we have $v = 2\pi R/T$.

(b) The factor of 2π is unitless. The radius R has units of meters, and T has units of seconds. Therefore the right-hand side has units of m/s, which makes sense, because we're supposed to be calculating a velocity.

(c) When T is very large, we'd be dividing by a very large number in our expression from part a. Dividing by a very large number gives a very small result. This makes sense, because if the earth stopped rotating, the velocity should equal zero.

(d) Plugging in numbers gives 460 m/s. This is faster than the speed of sound, over a thousand miles an hour! (I rounded the result to 2 sig figs, since a really precise answer would have to take into account other factors, such as how far Nairobi is from the equator, how much the earth deviates from being a perfect sphere, and the distinction between a solar day and a sideral day.)

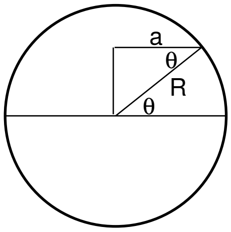
2-19 (a) Let a be the radius of the circle made by the point on the earth's surface. $a = R\cos\theta$, and $L = 2\pi a$, so $L = 2\pi R\cos\theta$.

(b) Dividing by T gives $v = L/T = 2\pi R\cos\theta/T$.

(c) The factor of $2\pi\cos\theta$ is unitless. The radius R has units of meters, and T has units of seconds. Therefore the right-hand side has units of m/s, which makes sense, because we're supposed to be calculating a velocity.

(d) At the north pole, $\theta = 90°$ and $\cos\theta = 0$, so $v = 0$, which makes sense. At the equator, $\theta = 0$ and $\cos\theta = 1$, so we recover the result of the previous problem.

(b) Plugging in $\theta = 34°$ and $R = 6.4 \times 10^6$ m gives $v = 380$ m/s.



2-20 (a) Let the length of the race be 2ℓ. The time required for the first half is ℓ/u, and for the second ℓ/v, so the total time is $\ell/u + \ell/v$. Dividing the total distance by the total time gives an over-all speed of $2/(1/u + 1/v)$.

(b) The units of $1/u$ are seconds per meter, and so are the units of $1/v$. Since they have the same units, it makes sense to add them. The final result has units that are the inverse of these, meters per second, and that makes sense, because what we're trying to calculate is a velocity.

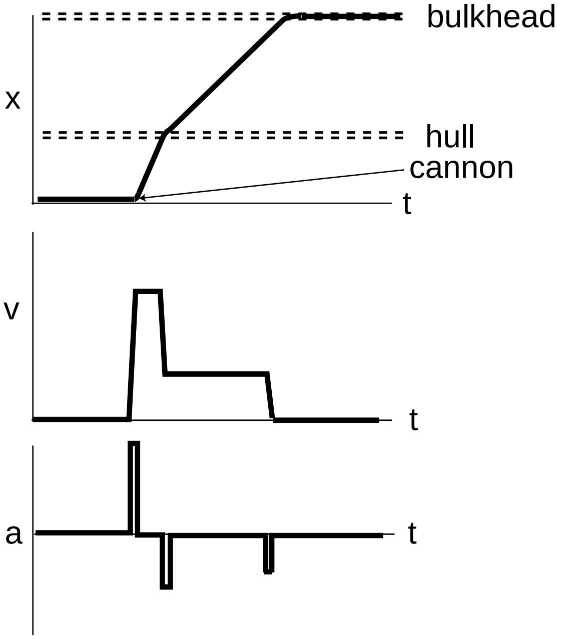
(c) When $u = v$, we have $2/(1/u + 1/u) = u$, which makes sense.

(d) Mathematically, making u bigger makes $1/u$ smaller, but $1/u$ is in the denominator, so the final result gets bigger. This makes sense physically: if you do the first half of the race faster, your over-all speed is higher. The analysis for v is identical.

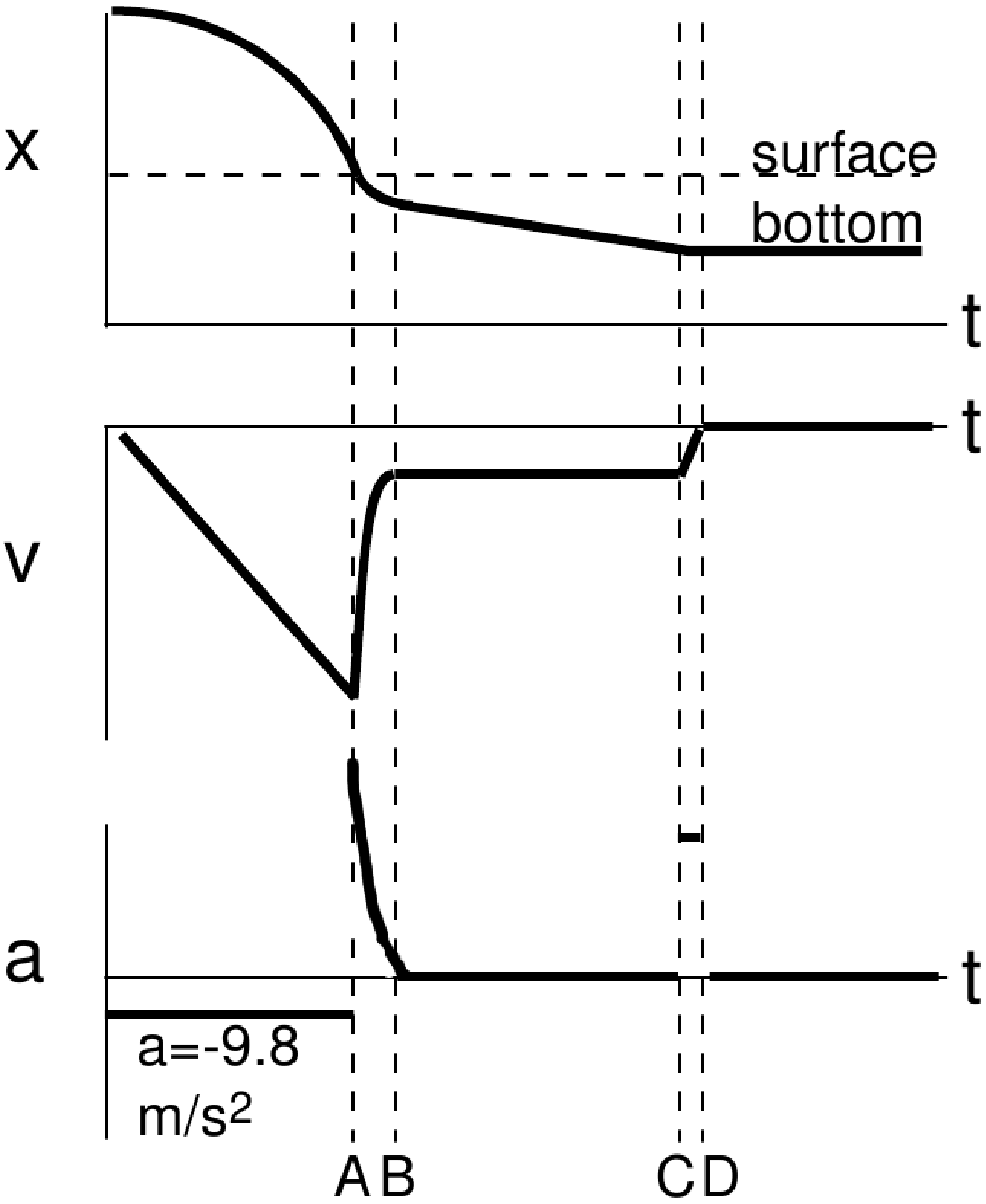
3-1 (a) When its velocity is negative it's coming back toward the hive, so at $t = 8.7$ s it is as far from the hive as it will ever get. (b) Between $t = 0$ and $t = 8.7$ s, the area under the curve is about 22 squares. Each square is 1 s wide and 1 m/s tall, so each one represents 1 m. The greatest distance from the hive is 22 m. (c) The area from $t = 8.7$ s to the end is under the axis, so it counts as negative. It's about 2 squares, so at the end the bee is about 20 m from the hive.

3-2 I picked coordinates with the positive x axis pointing up. Air resistance is negligible for a dense, heavy object like a rock dropped from a small height like this. In other words, it would take a much longer time for the rock to get up to speeds where air resistance was important, and eventually reach its terminal velocity in air. So while it's in the air, it has essentially a constant acceleration equal to $-g = -9.8\text{ m/s}^2$. Resistance from the water, on the other hand, is very strong, and the rock's terminal velocity in water is going to be very low – that's why you see rocks sink fairly slowly down to the bottom of a pool of water. So the rock is going to slow down when it enters the water because of the water's strong upward force on it (time interval AB), and it will then reach its terminal velocity, which it will have until it hits the bottom (BC). When it does hit the bottom, its velocity has to change quite a bit in a short amount of time (CD), so it has to have a large acceleration. A common difficulty in this type of reasoning involves plus and minus signs. As an example, considering the interval from the start to time A, the easiest way to get the sign is to reason that the slope of a tangent line on the $x - t$ graph is negative (sloping down as you go to the right on the graph), so the velocity must be negative during the same time interval. Similarly, the negative slope of the $v - t$ graph during that interval means that the acceleration must be negative.

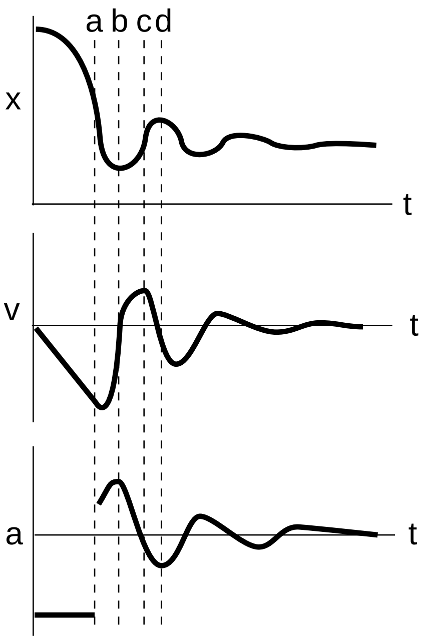
3-3 See the diagram. (Mine is tiny because I did it with computer drawing software; yours should be much bigger.) We're dealing with the horizontal motion only, so there is no acceleration while it's in the air. During those periods of zero acceleration, its speed has to be constant, and its $x - t$ graph has to be a line. Of course it's going to slow down from passing through the hull. It's at rest at the beginning (in the cannon, not yet fired) and at the end (lodged in the bulkhead), so there its x has to be constant and it has to have $v = 0$ and $a = 0$.



3-4 From the start to the point (a) where the cord becomes taut, the only force acting on the person is her weight, so her acceleration is constant (-9.8 m/s^2), her velocity is decreasing linearly with time, and her $x - t$ graph looks like a parabola. After that, the $x - t$ graph shows her bobbing up and down. At a, the slope of her $x - t$ graph is large and negative, so her v is large and negative. At b and d, the slope of her $x - t$ graph is zero, so her v is zero. At c, $x - t$ has large positive slope, so v is large and positive. Similarly, the accleration follows the slope of the $v - t$ graph, e.g. the slope of her $v - t$ graph is large and positive at b, so her acceleration is large and positive there.



Problem 3-2.



3-5 The speed is always increasing, because without friction there are only forces trying to speed it up, no forces trying to slow it down. A ball rolling without friction across a flat surface would neither speed up or slow down, i.e. would have zero acceleration, so the acceleration is greatest when the slope is greatest. The acceleration is increasing at first, then constant, then decreasing to a final acceleration of zero as it rolls away to the right at constant speed.

3-6 No. The acceleration is greatest at the ends, where $v = 0$. Just because the velocity is passing through zero on a graph doesn't mean the acceleration is zero. The velocity is changing rapidly (large Δv), even though $v = 0$.

3-7 Zero. $\Delta v = 0$, because the velocity didn't change.

3-8 The units don't work out: $\text{m/s}^2 \times \text{s} = \text{m/s}$, not m.

3-9 $s = \frac{1}{2}at^2$, and $a = g = 10\ \text{m/s}^2$, so $s = 45$ m, or call it 40 m (1 sig fig).

3-10 (a) The trip consists of four parts of equal duration. The duration, $T/4$, of each part is related to half the distance, d, to Proxima Centauri, $d/2 = \frac{1}{2}a(T/4)^2$, so $T = 4\sqrt{d/a}$.

(b) Plugging in to the formula from (a), we get 2.6×10^8 s = 7 years.

(c) The maximum speed is $(a)(T/4) = 6 \times 10^8$ m/s. This is more than twice the speed of light, which is not actually possible.

3-11 We have $v^2 = 2a\Delta x$, and the factor of $2a$ is the same in both cases, so we can rewrite this as a proportionality $v^2 \propto \Delta x$. Solving for v gives $v \propto \sqrt{\Delta x}$, and converting this into the language of ratios results in $v_1/v_2 = \sqrt{\Delta x_1/\Delta x_2} = \sqrt{2}$.

3-12 They both have accelerations equal to g. Let's say that the bullet leaves the gun with a velocity of 100 m/s, and for convenience, let's just say g is approximately 10 m/s^2. Then the rock's speed at one-second intervals goes like 0, 10, 20, ... m/s, while the bullet goes 100, 110, 120, ... m/s.

3-13 (a) b: meters, c: meters/s (because you multiply it by t to get something with units of meters), k: units of s (because it doesn't make sense to take $e^{-t/k}$ unless the units of $-t/k$ cancel out)

(b) $v = \mathrm{d}y/\mathrm{d}t = \mathrm{d}(-ct)/\mathrm{d}t + \mathrm{d}(-cke^{-t/k})/\mathrm{d}t = -c + ce^{-t/k}$

(c) For large values of t, $e^{-t/k}$ is nearly zero, so v approaches c. Thus, c can be interpreted as the maximum velocity she will ever reach (known as her terminal velocity).

(d) Differentiating again, we get $a = \mathrm{d}v/\mathrm{d}t = \mathrm{d}(-c + ce^{-t/k})/\mathrm{d}t = -(c/k)e^{-t/k}$.

(e) For large t, the exponential becomes very small, so her acceleration is nearly zero.

3-23 The area under the curve gives the change in velocity. If I just estimate fractions of a box by eye, I get about 36 boxes under the curve, but it's possible to do better than that, because you're given an exact geometric description of the curve. The area under the curve from 0 s to 4 s fits perfectly on top of the part from 5 s to 9 s, forming a complete rectangle with a width of 4 boxes and a height of 7 boxes, an area of 28 boxes. The part from 4 s to 5 s consists of a triangle with an area of 1.5 boxes sitting on top of a 1×2 rectangle, for a total area of 3.5 boxes, and similarly from 9 s to 10 s we have an area of 6 boxes. The total area is exactly

28.0+3.5+6.0 = 37.5 boxes Each box has a width of 1 s, and a height of 2 m/s^2, so its area is 2 m/s. The change in velocity is 37.5×2 m/s = 75.0m/s, so the chipmunk's final velocity is -3.1 m/s + 75.0 m/s = 71.9 m/s. The number of significant figures on the final result reflects the fact that its accuracy is limited by the accuracy of the initial velocity, which is given to a tenth of a meter per second.

3-24 The calculation of $\Delta v = a\Delta t$ is correct, because the equation it's based on, $a = \Delta v/\Delta t$, is valid for constant acceleration. The calculation of $\Delta x = v\Delta t$ is wrong, because the equation it's based on, $v = \Delta x/\Delta t$, is only valid for constant velocity. We know the velocity is increasing steadily, not staying constant. The speed of 10 m/s is only correct for the very end of the time period, not the whole time.

3-25 They're not equivalent. The slope of the tangent line will only be the same as $\Delta v/\Delta t$ if the $v - t$ graph is linear, i.e., if the acceleration is constant. For instance, suppose I drive my car for one minute at 50 km/hr, and then over the course of the second minute I speed up from 50 to 60 km/hr. I could use the $\Delta v/\Delta t$ definition to calculate my acceleration using $\Delta v = 10$ km/hr and $\Delta t = 2$ minutes, but that would give me a misleading result. My actual acceleration (slope of the tangent line) was zero for the first minute, and during the second minute, my acceleration was twice as much as $\Delta v/\Delta t$.

3-26 For example, suppose you accelerate away from a stop sign to 10 km/hr, then 20 km/hr, then 30 km/hr. The equation is telling you that the distance you cover increases faster than your speed. The distance from the stop sign to where your speedometer reads 20 km/hr is four times the distance from the stop sign to where it read 10 km/hr, and likewise the total distance from the stop sign to where it reads 30 km/hr is nine times the distance at 10 km/hr. This makes sense, because later in the motion, you're covering ground more quickly.

3-27 (a) The variables involved are distance, acceleration, and time, so we want the constant-acceleration equation that relates them, which is $\Delta x = \frac{1}{2}g\Delta t^2$. The balls are two separate objects, so we need to have two separate equations of this form.

$$\begin{aligned}
\Delta x_1 &= \frac{1}{2}g\Delta t_1^2 \\
\Delta x_2 &= \frac{1}{2}g\Delta t_2^2 \\
\Delta t_1 &= \sqrt{\frac{2\Delta x_1}{g}} \\
\Delta t_2 &= \sqrt{\frac{2\Delta x_2}{g}} \\
T &= \Delta t_2 - \Delta t_1 \\
&= \sqrt{\frac{2\Delta x_2}{g}} - \sqrt{\frac{2\Delta x_1}{g}} \\
g &= \left(\frac{\sqrt{2\Delta x_2} - \sqrt{2\Delta x_1}}{T}\right)^2
\end{aligned}$$

(b) The units inside the parentheses are m$^{1/2}$/s, so squaring this quantity, we have m/s^2.

(c) When $\Delta x_1 = 0$, T is the same as Δt_2, and we have simply $g = (\sqrt{2\Delta x_2}/\Delta t_2)^2$, which is what we would have gotten if we had just solved $\Delta x_2 = \frac{1}{2}g\Delta t_2^2$ for g. This case reassures us that the result is correct, but it is useless in practical terms: we have to make Δx_1 great enough so that the ball will make a sound we can record.

(d) Mathematically, we get zero over zero, which is undefined. Physically, we cannot measure all the quantities in the equation with perfect precision, so we will not get exactly zero on the top or on the bottom; the result will be not an error on our calculator but rather a wildly unreliable answer! Nature is giving us a hint that the most accurate results will be obtained when the two heights are as different as possible, and will get less and less reliable as the heights are made closer to one another.

3-28 (a)

$$v_f^2 = v_\text{o}^2 + 2a\Delta x$$

The initial velocity v_o, is the velocity of the spinning earth's surface, which at the equator is $(2\pi r)/(1 \text{ day}) \approx 500$ m/s. This is an order of magnitude less than the orbital velocity given in this problem. Since v_o is much less than v_f, v_o^2 is even more negligible compared to v_f^2, and we're justified in ignoring it for the purposes of this approximate calculation, whose accuracy is really limited by the assumed acceleration.

$$\begin{aligned} v_f^2 &= 2a\Delta x \\ \Delta x &= \frac{v_f^2}{2a} \\ &= 1\times 10^6\ \text{m} \\ &= 1000\ \text{km} \end{aligned}$$

(b) It's not very practical to build a railgun that would stretch halfway across a continent. The railgun idea is definitely not going to be practical for crewed launches, but it might work for lofting payloads that could handle more violent accelerations, such as raw materials. For instance, a payload that could handle $300g$ of acceleration would only need a 10-kilometer railgun.

3-29 We're given the distance it takes the flea to slow back down to a stop, $\Delta x = 30$ cm, so it's natural to use the equation $v^2 = 2a\Delta x$, which tells us the distance required in order to slow down from v to rest, or speed up from rest to v. The flea first accelerates rapidly, then, once its feet have left the ground, it begins decelerating. The acceleration probably isn't constant, but this is just an estimate anyway, so we'll assume it is. We then have $2g\Delta x = 2ah$, where a is the acceleration as it's taking off, and h is the distance its center of mass travels upward while its legs are straightening. The result is $a = g\Delta x/h$. Estimating $h = 0.1$ cm, we find $a \sim 300g$.

3-30 We're given distance and acceleration, and we want to find time. Of the constant acceleration equations, the one that has the right variables in it is $\Delta x = v_\text{o}\Delta t + \frac{1}{2}a\Delta t^2$. The initial velocity is zero, and solving for Δt, we have $\Delta t = \sqrt{2\Delta x/a}$. Converting the given distance to SI (meter-kilogram-second) units, we have 4000 mi $= 6.4\times 10^6$ m, which results in $\Delta t = 1100$ s, or about 20 minutes (1 sig fig).

3-31 Of the time t he spent aloft, he spent $t/2$ rising and $t/2$ falling. The acceleration is $g = (9.8\ \text{m/s}^2)/6$. $\Delta x = (1/2)g(t/2)^2 = 0.43$ m.

3-32 (a) We want to use a constant-acceleration equation to relate distance and time. We don't know the velocity, and we don't want to know it. The equation that has the right variables in it is $\ell = (1/2)at^2$ (using ℓ instead of x and taking $v_o = 0$). The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $a/2$, giving $\ell \propto t^2$. Solving for t gives $t \propto \ell^{1/2}$. Change this into a statement about ratios:

$$\frac{t_d}{t_h} = \left(\frac{\ell_d}{\ell_h}\right)^{1/2} = \sqrt{4} = 2 \qquad ,$$

where "d" refers to the dinosaur and "h" to a human.

(b) The distance each animal moves in a quarter stride is proportional to ℓ, so $v \propto \ell/t \propto \ell/\ell^{1/2} = \ell^{1/2}$. In terms of ratios,

$$\frac{v_d}{v_h} = \left(\frac{\ell_d}{\ell_h}\right)^{1/2} = \sqrt{4} = 2 \qquad .$$

In other words, the dinosaur's stride is longer by a factor of four, but it loses half of that advantage because its steps are slower by a factor of two.

3-33 We want to use a constant-acceleration equation to relate acceleration and time for a fixed distance. We don't know the velocity, and we don't want to know it. The equation that has the right variables in it is $x = (1/2)at^2$ (setting $v_o = 0$). Solving for the acceleration gives $a = 2x/t^2$. The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $2x$, giving $a \propto t^{-2}$. Change this into a statement about ratios:

$$\frac{a_H}{a_r} = \left(\frac{t_H}{t_r}\right)^{-2} = (0.47)^{-2} = 4.5 \qquad ,$$

where "H" refers to Hatton and "r" to the reporter.

3-34 We want to use a constant-acceleration equation to relate acceleration, speed, and distance. We don't know the time, and we don't want to know it. The equation that has the right variables in it is $v_f^2 = v_o^2 + 2a\Delta x$. The final velocity is zero. Solving for the acceleration gives $a = -v_o^2/2\Delta x$. The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $-1/2$, giving $a \propto v_o^2\Delta x^{-1}$. Change this into a statement about ratios:

$$\frac{a_F}{a_S} = \left(\frac{v_F}{v_S}\right)^2 \left(\frac{\Delta x_F}{\Delta x_S}\right)^{-1} = (4.8)^2(1.9)^{-1} = 12$$

where "F" refers to the F-14 and "S" to the Sopwith.

3-35 We want to use a constant-acceleration equation to relate acceleration and distance for a fixed final speed. We don't know the time, and we don't want to know it. The equation that has the right variables in it is $v_f^2 = 2a\Delta x$ (setting $v_o = 0$). Solving for the acceleration gives $a = v_f^2/2\Delta x$. The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $v_f^2/2$, giving $a \propto \Delta x^{-1}$. Change this into a statement about ratios:

$$\frac{a_H}{a_m} = \left(\frac{\Delta x_H}{\Delta x_m}\right)^{-1} = (1.62\,\mathrm{m}/1.75\,\mathrm{m})^{-1} = 1.08$$

where "H" refers to Harrell and "m" to a pitcher of the nominal minimum height.

3-36 We want to use a constant-acceleration equation to relate acceleration and speed for a fixed distance. We don't know the time, and we don't want to know it. The equation that has the right variables in it is $v_f^2 = v_o^2 + 2a\Delta x$. The final velocity is zero. Solving for the initial speed gives $v_o = (-2a\Delta x)^{1/2}$. The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $\sqrt{-2\Delta x}$, giving $v_o \propto a^{1/2}$. Change this into a statement about ratios:

$$\frac{v_C}{v_D} = \left(\frac{a_C}{a_D}\right)^{1/2} = (1.09)^{1/2} = 1.04 \qquad ,$$

where "C" refers to the Chevy cop car and "D" to the Dodge.

3-37 (a) Each rectangle has a width of 1 s and a height of 1 m/s, so it represents a distance of 1 m. There are 14 rectangles between the curve and the t axis, so the change in position is 14 m. (b) The square below the axis counts as a negative change in position, so the result is 7 m.

3-38 The constant-acceleration acceleration that has the right variables in it is $v_f^2 = v_0^2 + 2a\Delta x$. The final velocity is zero, so we have $a = -v_0^2/2\Delta x$. If we wanted to worry about the sign, then we would have to pick a coordinate so that we could define a sign for Δx, and then we would get some sign for the acceleration, expressed in those coordinates. However, we're only trying to find the magnitude of the deceleration, so ignoring signs, the result is 1.5 m/s^2.

3-39 The constant-acceleration equation with the right variables in it is $\Delta x = v_0\Delta t + (1/2)a\Delta t^2$. The initial velocity is zero. Solving for the acceleration gives $a = 2\Delta x/\Delta t^2 = 16.3$ m/s^2.

3-40

$$, \qquad \begin{aligned} H &= v_0T + \frac{1}{2}gT^2 \\ v_0 &= H/T - \frac{1}{2}gT \end{aligned}$$

4-1 No, we can only conclude that if there are forces acting on it, their sum is zero.

4-2 The mass is increased by a factor of 1.5, so the acceleration is reduced by a factor of 1.5, to 2 m/s^2.

4-3 (a) We are supposed to find an equation for mass based on information about forces and accelerations. Newton's second law, $a = F_{total}/m$, is what relates these three quantities. There are two forces acting on the elevator: a downward force from gravity and an upward force from the cable. Let's use positive numbers for upward forces and negative numbers for downward ones. The force of gravity will then be $-mg$, and the maximum force the cable can exert will be $+T$, giving $F_{total} = -mg + T$. Substituting this into Newton's second law gives

$$a = \frac{-mg + T}{m} \quad .$$

Solving for m gives

$$m = \frac{T}{a + g} \quad .$$

(b) If $a = 0$, then the equation becomes $m = T/g$, i.e. $T = mg$. Interpretation: if the elevator is not accelerating, then the maximum mass is the mass that results in a gravitational force as great as the maximum tension in the cable. This is intuitively obvious if the elevator is at rest, but perhaps a little surprising for the case where the elevator is, say, moving upward at constant speed. Most people would not expect that the tension in the cable would be the same whether the car was at rest or moving upward at constant speed.

If the car is accelerating downward at 9.8 m/s^2, that would be $a = -g$, since we are using positive for up and negative for down. Plugging this in to the equation gives $m = \infty$ (division by zero). Interpretation: this is the case where the elevator is simply dropping like a rock. The cable does not need to exert any force in this situation, and so there is no limit on the mass of the car.

4-4 (a) Newton's second law says $F_{air} - F_{weight} = ma$, or $F_{air} - mg = ma$. Since the forces are constant, this will be motion with constant acceleration, so $v = at = (F_{air} - mg)t/m$.
(b) 7.8 m/s

4-5 (a) $F_W = mg$, so reducing g by 0.01 m/s^2 reduces his weight by

$$72 \text{ kg} \times 0.01 \text{ m/s}^2 = 0.72 \text{ N}$$

His mass stays the same.

(b) For motion with constant acceleration, $v_f^2 = v_i^2 + 2ax$. At the top of his jump, his vertical velocity is zero, so $v_f = 0$, and $0 = v_i^2 + 2ax$. His acceleration equals $-g$. Solving for x, the height to which he can jump is, $x = v_i^2/2g$. Since the height to which he can jump is inversely proportional to g, a 0.1% decrease in g will result in a 0.1% increase in height, or 2 mm.

4-6 $F = ma = kt$, so $a = (k/m)t$. Integrating once gives its velocity, $v = (k/2m)t^2 + v_o$. The constant of integration, v_o, equals zero, because the initial speed is zero, so $v = (k/2m)t^2$. Integrating again gives $x = (k/6m)t^3 + x_o$.

4-8 Their claim doesn't make sense. The branch isn't accelerating, so evidently the total force on it is zero. The two sides of the shears must be making equal forces on the two sides of the branch, in opposite directions. This has to be true regardless of the type of shears.

4-9 (a)

$$\begin{aligned} v_f^2 &= v_\text{o}^2 + 2a\Delta x \\ 0 &= v^2 + 2ad \\ a &= -\frac{v^2}{2d} \\ F &= ma \\ &= -\frac{mv^2}{2d} \end{aligned}$$

(b) The quantity $-\frac{mv^2}{2d}$ has units of

$$\frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{m}} = \text{kg}\cdot\text{m/s}^2 \quad ,$$

which are the same as the units of force (mass multiplied by acceleration).

(c) If m gets bigger, F gets bigger. That makes sense, because more force would be required in order to stop a more massive object in the same distance. If v increases, so does F, and that also makes sense. Dividing by a bigger number gives a smaller result, so if d gets bigger, F gets smaller. That also makes sense, because a smaller force would take more distance to stop a moving object.

(d) The only thing to be careful about here is that if we want an answer in SI (meter-kilogram-second) units, we have to make sure to convert all the raw data to SI before plugging in:

$$\begin{aligned} F &= -\frac{mv^2}{2d} \\ &= -\frac{\left(6.7\times10^{-27}\ \text{kg}\right)\left(2.0\times10^{7}\ \text{m/s}\right)^2}{2\left(7.1\times10^{-4}\ \text{m}\right))} \\ &= -1.9\times10^{-9}\ \text{N} \end{aligned}$$

4-11 Let's use a coordinate system in which positive x is down. Then the downward force of gravity on Sally is positive, and the blaster's upward force on Sally is negative. (a)

$$\begin{aligned} a &= \frac{F_{total}}{m} \\ &= \frac{mg - F_B}{m} \\ &= g - F_B/m \\ h &= \frac{1}{2}at^2 \\ t &= \sqrt{\frac{2h}{a}} \\ &= \sqrt{\frac{2h}{g - F_B/m}} \end{aligned}$$

(b) The units of g are m/s^2. F_B has units of newtons, and dividing newtons by kilograms gives m/s^2 (as in Newton's second law). In the denominator, we're subtracting m/s^2 from m/s^2. That's good, because you can't subtract things that have different units. Dividing h, with units of m, by m/s^2 gives units of s^2, and taking the square root of that gives units of seconds, which matches up correctly with the units of the variable t on the left side.

(c) For large values of F_B, the quantity inside the square root becomes negative, and the result for the time becomes an imaginary number. The point where this bogus behavior sets in is the point at which the denominator goes from positive to negative, i.e., when the denominator is zero: $g - F_B/m = 0$, or $F_B = mg$. In other words, if F_B is greater than the force of gravity, she'll fly up, and will never even hit the ground.

4-12 (a) Friction would be parallel to the ceiling, but all the motion is vertical. (b) If the rice were to stay stuck onto the ceiling of the cup, it would have to accelerate along with the cup. When you're banging on something like this the acceleration can be huge, perhaps thousands of meters per second squared. This is much more than anything gravity could supply. (c) The cup is the only object touching a grain of rice that could be exerting a force on it, and Newton's second law would then require that the this force equal ma. As the cup decelerates, we therefore need a large upward force from the cup on the rice if the rice is to decelerate along with it. This would be some kind of sticky force, and there's a limit on how strong such a force can be. If it isn't strong enough, then the grain's acceleration can't match that of the cup, and it will come loose.

4-13 For motion with constant acceleration, starting from rest, we have $v^2 = 2a\Delta x$, or $v \propto a^{1/2}$. Newton's second law gives $a \propto m^{-1}$, so $v \propto m^{-1/2}$. The result is

$$\frac{v_2}{v_1} = \left(\frac{340\ \text{kg} + 60\ \text{kg}}{340\ \text{kg}}\right)^{-1/2} = 0.92$$

4-14 For motion with constant acceleration, starting from rest, we have $\Delta x = (1/2)at^2$. For a fixed Δx, this becomes $t \propto a^{-1/2}$. Newton's second law gives $a \propto F$, so $t \propto F^{-1/2}$. The result is

$$\frac{t_1}{t_2} = \left(\frac{F_1}{F_2}\right)^{-1/2} = (1/1.9)^{-1/2} = 1.4$$

4-16 (a) We have $v^2 = 2aL$ for the average acceleration, so the average force is $F = ma = mv^2/2L$.

(b) The units on the right-hand side are kg $\cdot$ (m/s)^2/m = kg$\cdot$m/s^2, which is the same as newtons.

(c) Mathematically, increasing m increases F. That makes sense, because more force would be required in order to accelerate a more massive bullet. The dependence on v also makes sense, because accelerating the bullet up to a higher speed should take more force. Raising L in our answer would lower F, and that makes sense, because if we have a greater distance over which to accelerate the bullet, we can do it more gradually.

4-17 (a) Let, e.g., F_{23} mean 2's force on 3. By Newton's first law, the total force on block 3 is zero, so $F_{23} - M_3 g = 0$, so $F_{23} = M_3 g$. Newton's third law then gives $F_{32} = -F_{23} = -M_3 g$.

(b) Applying Newton's first law to block 2, we have $F_{12} + F_{32} - M_2 g = 0$, so $F_{12} = (M_2 + M_3)g$. This makes sense because 1 is supporting the entire weight of 2 and 3.

5-1 Newton's third law says their forces on each other are equal. Since $a = F/m$, the less massive old lady has a greater acceleration.

5-2 When you drop an object, the earth does accelerate up to meet it, but the acceleration is very small because the earth's mass is so big.

5-3 Yes, they are equal and opposite, because otherwise the total force on you would not be zero, and you would accelerate vertically. No, Newton's third law does not apply, because there are three objects involved, not just two: the earth, your feet, and the floor. Newton's third law says that two objects' forces on each other are equal and opposite, so it never applies to two forces both acting on the same object.

5-4

5-5

5-6

5-7

5-8

5-9 (a) The two static frictional forces must together cancel out the elevator's weight $F_W = Mg$, so

$$F_s = \frac{Mg}{2}$$

The maximum amount of static friction is given by $F_s < \mu_s F_N$, so

$$F_N > \frac{Mg}{2\mu_s}$$

(b) If friction is weaker, we'd need to press more firmly. The equation demonstrates this behavior, since dividing by a smaller value of μ_s will give a greater result.

5-10 (a) See table on last page.

(b) Let up be positive, and let a be the (positive) acceleration of mass m. Newton's second law applied to mass m gives

$$a = \frac{T - mg}{m} \qquad ,$$

and for mass M,

$$-a = \frac{T - Mg}{M} \qquad .$$

We have two equations in two unknowns, T and a. To solve them, we need to eliminate one variable. Let's eliminate a.

$$\begin{aligned} \frac{T - mg}{m} &= -\frac{T - Mg}{M} \\ \frac{T}{m} - g &= -\frac{T}{M} + g \\ \left(\frac{1}{m} + \frac{1}{M}\right) T &= 2g \\ (M + m)T &= 2gMm \\ T &= \frac{2gMm}{M + m} \end{aligned}$$

Now we can find a.

$$\begin{aligned} a &= \frac{T - mg}{m} \\ &= \frac{T}{m} - g \\ &= \frac{2gM}{M + m} - g \\ &= \left(\frac{2M}{M + m} - 1\right) g \\ &= \left(\frac{2M}{M + m} - \frac{M + m}{M + m}\right) g \\ &= \left(\frac{M - m}{M + m}\right) g \end{aligned}$$

(c) No. If the masses are equal, the acceleration will be zero, not the velocity. The equal masses could be left at rest, and would stay at rest, regardless of their relative heights. They

Problem ??:

force acting on magnet			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
magnetism	up	car	magnetism	down	magnet
gravity	down	planet earth	gravity	up	magnet
normal	down	car	normal	up	magnet

Problem ??:
(a) A monkey climbing a palm tree.

force acting on monkey			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	right	tree	normal	left	monkey (right hand)
normal	left	tree	normal	right	monkey (left hand)
gravity	down	planet Earth	gravity	up	monkey
static friction	up	tree	static friction	down	monkey (right hand)
static friction	up	tree	static friction	down	monkey (left hand)

Problem ??:
(b) A piece of tape stuck on the ceiling.

force acting on tape			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	tape
sticky	up	ceiling	sticky	down	tape

Problem ??:

force acting on rower			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	↑	seat	normal	↓	rower
gravity	↓	Earth	gravity	↑	rower
normal	→	oar	normal	←	rower
static friction	←	seat	static friction	→	rower
normal	↖	footrest	normal	↘	rower

Problem ??:

force acting on farmer			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
static friction	up	cow	static friction	down	farmer
static friction	up	wall	static friction	down	farmer
gravity	down	planet earth	gravity	up	farmer
normal	left	cow	normal	right	farmer
normal	right	wall	normal	left	farmer

Problem ??:

(The "lift" force below is really not so different from a normal force.)

force acting on plane			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal?	east	air	normal?	west	plane (propeller)
friction	west	air	friction	east	plane
lift	up	air	lift	down	plane (wing)
gravity	down	earth	gravity	up	plane

Problem ??:

force acting on mass m			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	up	string	normal	down	mass m
gravity	down	planet Earth	gravity	up	mass m

could also be left in motion, and would stay in motion, without speeding up or slowing down.
(d) (found above)
(e) Mathematically, the expression for T vanishes when either mass is zero. Physically this makes sense, because if either mass is not present, the rope will go slack.

5-11 (a) (b) (c) Let's choose a coordinate system in which positive is forward. The forces acting on the tugboat give

$$a = \frac{F_f - T}{m}$$

and the ones acting on the ship result in

$$a = \frac{T}{m} \qquad .$$

Note that we use the same symbol a in both cases: they're tied together, so their accelerations must be the same. To solve for T, we need to eliminate a, which we can do by setting the two equations equal:

$$\frac{F - T}{m} = \frac{T}{M}$$

Bringing all the T stuff to one side,

$$\frac{F}{m} = \left(\frac{1}{m} + \frac{1}{M}\right) T \qquad ,$$

so

$$\begin{aligned} T &= \frac{F}{m(\frac{1}{m} + \frac{1}{M})} \\ &= \frac{F}{1 + m/M} \\ &= \frac{M}{M+m} F \qquad . \end{aligned}$$

(d) For $M = 0$, the tension is zero, which makes sense. For $M = \infty$, we have $M/(M+m) = 1$, since the bottom becomes essentially the same as M, so $T = F$. The interpretation is that the tugboat is so light that essentially none of the force from the water on its propellers is required in order to accelerate it.

5-12 If kinetic friction were stronger than static friction, then we could have, for example, a situation in which 4 N was the force necessary to break static friction and make an object slide across a horizontal surface, but 6 N was necessary in order to produce a force greater than kinetic friction in order to produce an acceleration. This wouldn't make sense, because what would happen if you applied 5 N? It would break static friction, but then there would be no acceleration because the force would be weaker than kinetic friction. The way you would actually measure a static frictional force experimentally would be to measure how much force you had to apply in order to get the object to accelerate, so there's no way you could have determined the 4 N force for static friction in the first place.

5-13 Let the tension in the rope connected to the right-hand weight be T. By arguments similar to the ones in the example in section 5.6, we know that the tension in the rope connected to the left-hand weight is $2T$. The accelerations of the two weights are also related. For every centimeter that the weight on the left moves up, the pulley in the middle will move one centimeter to the right. This causes one centimeter of rope to come around the middle pulley, lengthening the part the goes over the right-hand pulley and down to the weight. The right-hand weight therefore drops two centimeters: one centimeter because an extra centimeter of rope is being fed around the middle pulley, and another centimeter because the middle pulley is moving a centimeter to the right. Therefore the acceleration of the right-hand weight is twice as much as the acceleration of the left-hand one, and is in the opposite direction, $a_2 = -2a_1$. Applying Newton's second law to each weight (with positive being up), we have the following set of three equations in three unknowns:

Problem ??:
Part a

force acting on tugboat			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
fluid fr.	forward	water	fluid fr.	backward	tugboat
tension	back	ship	tension	forward	tugboat

Problem ??:
Part b

force acting on ship			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
tension	forward	tugboat	tension	back	ship

$$\begin{aligned} a_2 &= -2a_1 \\ a_1 &= \frac{2T-mg}{m} \\ a_2 &= \frac{T-mg}{m} \end{aligned}$$

Substituting the first equation into the third one reduces this to two equations in two unknowns:

$$\begin{aligned} a_1 &= 2T/m - g \\ -2a_1 &= T/m - g \end{aligned}$$

Now we solve the second equation for $T/m = -2a_1 + g$ and substitute into the first one, which makes it one equation in one unknown:

$$a_1 = 2(-2a_1 + g) - g$$

Solving this, we find $a_1 = g/5$. The positive sign indicates that this mass (the one on the left) goes up.

5-15 The parallel case is trivial: $k = k_1 + k_2$ by the same reasoning as before. The series case requires more delicacy, because the strings don't get elongated by the same amount. For simplicity of notation, let $x_\text{o} = 0$ for both springs. Since their forces on each other are equal by Newton's third law, the elongations are related by $F = k_1x_1 = k_2x_2$. The equivalent spring constant is $F/(x_1 + x_2) = k_1x_1/(x_1 + x_2) = k_1x_1/(x_1 + x_1k_1/k_2) = k_1k_2/(k_1 + k_2)$. Another way of writing this is $1/(1/k_1 + 1/k_2)$.

5-17 (a) From problem 16 we have $k_{whole} = (A/L)E$. The simplest way to work this is to imagine a fiber consisting of only two atoms. Then we have $A = b^2$ and $L = b$, and $k = k_{whole}$, resulting in $k = bE$. (b) The result is on the order of 1 N/m. (c) The stretching of the bond is $x = F/k = mg/k$. A typical mass of an atom would be on the order of 10^{-26} kg, as you can find, for example, by looking up the masses of subatomic particles in the back of the book. (Another method would be to multiply the atomic volume b^3 by a typical density of solid matter.) The resulting extension of the length of the bond is on the order of 10^{-25} m, which is negligible, even compared to b. In other words, individual molecules won't be distorted significantly by gravitational stresses.

5-19 (a) See table on last page.

(b) See table on last page.

Problem ??:
Part a:

force acting on Ginny			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	Ginny
static friction	left	ropes	static friction	right	Ginny
normal	up	sled	normal	down	Ginny
static friction	right	sled	static friction	left	Ginny

Problem ??:
Part b:

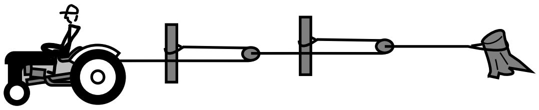
force acting on sled			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	sled
normal	down	Ginny	normal	up	sled
static friction	left	Ginny	static friction	right	sled
normal	up	snow	normal	down	sled
friction	right	snow	friction	left	sled

(c) The vertical forces on Ginny cancel out, so the normal force between Ginny's pants and the sled has magnitude $N_2 = Mg$. Similarly, the normal force between the sled and the snow is $N_1 = (M+m)g$, since the snow supports the total mass of both Ginny and the sled. In the situation where she can just barely accomplish what she wants to do, the initial acceleration is vanishingly small; to find the limiting case, we set it to zero. That means there must be zero total horizontal force on her, and zero total horizontal force on the sled. There are two frictional forces on the sled, F_2 from Giny and F_1 from the snow. If they're canceling, their magnitudes must be equal, $|F_2| = |F_1|$. If the sled just barely starts slipping over the snow before Ginny's pants start slipping over the sled, then both these forces are at their maximum value for static friction,

$$\begin{aligned}\mu_2 N_2 &= \mu_1 N_1\\ \mu_2 Mg &= \mu_1(M+m)g\\ \mu_2 &= \mu_1(1+m/M)\\ M &= \frac{m}{\mu_2/\mu_1 - 1}\end{aligned}$$

(d) For $\mu_2 = \mu_1$, M becomes infinite. For smaller values of μ_2, we have a negative value for M, which is also impossible. In other words, for $\mu_2 \le \mu_1$, it wouldn't matter if Ginny carried a backpack full of lead bricks; she'd still slip off the sled.

5-21 The simplest thing to do is to hook up two of the pulley setups in a row, so that the second one doubles the force output from the first one.



5-22 The scale reads too high when the backpack is on the way down and decelerating, and also when it's on the way up and accelerating. It reads too low when the backpack is on the way up and decelerating, and also when it's on the way down and accelerating. In other words, when I'm making an upward force on the backpack that's greater than the force required to support its weight, Newton's third law says the backpack must be making a correspondingly large force downward on me, which registers on the scale. Conversely, when I make an unusually small upward force on the pack, it makes an unusually small downward force on me.

5-23

$$\begin{aligned} v &= \sqrt{2aL} \\ &= \sqrt{2\frac{F_k}{m}L} \\ &= \sqrt{2\frac{\mu_k F_N}{m}L} \\ &= \sqrt{2\frac{\mu_k mg}{m}L} \\ &= \sqrt{2\mu_k gL} \end{aligned}$$

Note that the cop doesn't even need to know anything about the car other than the coefficient of kinetic friction between its tires and the asphalt μ_k, and μ_k probably doesn't vary all that much from one car to another.

5-24 The acceleration a in Newton's second law is the acceleration of the center of mass. The atoms at the point of contact between the ball and the bat move together and have equal accelerations, but that doesn't mean the centers of mass have equal accelerations. The ball and the bat both start squashing on impact.

5-27 (a) See table.

(b) The vertical forces cancel, so $F_N = mg$. Picking a coordinate system in which positive is forward, the horizontal force of friction produces an acceleration $a = F_k/m = -\mu_k F_N/m = -\mu_k g$. Applying the constant-acceleration equation $v_f^2 = v_i^2 + 2ax$, we find

$$x = \frac{v_i^2}{2\mu_k g} \qquad .$$

(c) The coefficient of friction is unitless, because it's defined as the ratio of two forces. The units of v_i^2 are m^2/s^2, and dividing by m/s^2 gives meters.

(d) Mathematically, a higher v_i gives a higher result, and this makes sense physically, because you can go farther if you start out going faster. A greater value of μ_k gives a smaller final result, and this also makes sense; increasing friction (e.g., by roughening the ice) would make the skater stop sooner. Increasing g would also decrease the final result; this is because a stronger gravitational field would press the skater more firmly against the ice, increasing F_N, and thereby indirectly increasing friction.

(e) The result is 2.4 km.

(f) This is clearly unrealistic. Probably the biggest reason for the unrealistic result is that we've neglected air resistance. Air resistance increases rapidly with speed, so that could be a reasonable approximation at lower speeds, but at these speeds (about 50% faster than an olympic sprinter), it's probably a poor approximation. When you see pictures of olympic-level speed skaters, they're always wearing special suits that are designed to cut down on air resistance; this shows that air resistance is a very important effect at the speeds they get up to.

5-28 (a) See the tables for an analysis of the forces involved.

Applying Newton's second law to the climber of mass M, and letting the positive direction be downward, we have

$$Mg - T = Ma \qquad .$$

The climber of mass m doesn't sink into the snow or fly up in the air, so

$$F_N - mg = 0 \qquad .$$

Newton's second law for m, with positive to the right, gives

$$T - \mu_k F_N = ma \qquad .$$

These are three equations in three unknowns, a, T, and F_N. Eliminating $F_N = mg$ reduces it

Problem ??:

force acting on skater			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	Earth	gravity	up	skater
normal	up	ice	normal	down	skater
kin. fr.	back	ice	kin. fr.	forward	skater

Problem ??:

force acting on climber of mass m			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	climber
kinetic friction	left	snow	kinetic friction	right	climber
normal	up	snow	normal	down	climber

Problem ??:

force acting on climber of mass M			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	climber
normal	up	rope	normal	down	climber

two a system of two equations in two unknowns:

$$\begin{aligned} Mg - T &= Ma \\ T - \mu_k mg &= ma \end{aligned}$$

Next we solve the first equation for $T = M(g-a)$ and substitute into the other equation, giving one equation with one unknown:

$$M(g-a) - \mu_k mg = ma$$

Solving this for a, we find

$$a = \frac{M - \mu_k m}{M + m} g \qquad .$$

(b) The definition of μ_k is $F_k = \mu_k F_N$, so μ_k is unitless. That means that it makes sense to do the subtraction $M - \mu_k m$, since both terms have units of kg. The quotient $(M - \mu_k m)/(M + m)$ is unitless, and multiplying it by g gives units of acceleration, which checks out.

(c) Mathematically, increasing g increases a. That makes sense because more gravity should make them accelerate faster. Mathematically, increasing μ_k decreases a, and this checks out because increasing friction should keep them from accelerating as fast. Increasing m decreases the numerator and increases the denominator, and both of these effects will reduce a. This makes sense because adding extra weight to m will help both by increasing friction and by increasing the inertia of the whole system, which is measured by $M + m$. M is the hardest variable to analyze mathematically. Increasing M increases both the numerator and the denominator, so there are two countervailing effects, and it's not obvious which is the greater. But since the numerator is smaller than the denominator, increasing M by a certain amount will cause a greater percent change in the numerator, so the net effect will be to increase a. This makes sense physically, because if M was miniature French poodle, we wouldn't expect the acceleration to be as fast.

5-30

5-31 (a)

(b) The car's acceleration is zero, so the leftward and rightward forces acting on the car must be canceling. They are equal in strength.

(c) Although the car is moving now, its acceleration is still zero, so the leftward and rightward forces are still equal in strength.

(d) The man's leftward force on the car must now be less than the mud's rightward force.

5-32 (a) Let the mass of the block be M. The downward force Mg of gravity on it is canceled by the upward normal force from the ledge, so the strength of the normal force is $F_N = Mg$. The maximum force of static friction is $\mu_s F_N = \mu_s Mg = \mu_s \rho V g$. Since we assume there is no friction between the rope and the lip of the cliff, this is the maximum tension that can be sustained in the rope. Equating this to the climber's body weight gives $\mu_s = m/\rho V$.

(b) The units are mass over mass, which cancels out. That makes sense, because the definition of μ_s implies that it is unitless.

(c) If m goes up, our equation says that μ_s also goes up. That makes sense, because a heavier climber would need more friction. If ρ or V gets bigger, the result gets smaller. That also makes sense, because increasing either of these would make the block heavier, which would allow it to hold without such a high coefficient of friction. (d) The result is 0.04, which is smaller by an order of magnitude than the reference value for granite on granite. This is a large margin of safety.

5-33 (a) We have the following three equations to start off with:

$$\begin{aligned} v^2 &= 2ax \\ F &= \mu_k mg \\ a &= F/m \end{aligned}$$

Eliminating F and a gives

$$\mu_k = \frac{v^2}{2gx} = 0.037.$$

5-34 (a) See tables.

(b) Let N_1 be the normal force between the ground and block 1, and N_2 the normal force between the two blocks. Then because block 2's acceleration is zero, the total force on it is zero, and $T - \mu_k N_2 = 0$, so $T = \mu_k M_2 g$.

Problem ??:

force acting on climber			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	up	rope	normal	down	climber
gravity	down	planet earth	gravity	up	climber

Problem ??:

force acting on car			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet earth	gravity	up	car
normal	up	mud	normal	down	car
normal	left	man	normal	right	car
static friction	right	mud	static friction	left	car

Problem ??:

force acting on block 1			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	↓	Earth	gravity	↑	block 1
normal	↑	ground	normal	↓	block 1
kin. fr.	←	ground	kin. fr.	→	block 1
normal	↓	block 2	normal	↑	block 1
kin. fr.	←	block 2	kin. fr.	→	block 1
not given	→	not given	not given	←	block 1

Problem ??:

force acting on block 2			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	↓	Earth	gravity	↑	block 2
normal	↑	block 1	normal	↓	block 2
kin. fr.	→	block 1	kin. fr.	←	block 2
normal?	←	rope	normal?	→	block 2

(c) Newton's second law gives

$$\begin{aligned} M_1 a &= F - \mu_k N_1 - \mu_k N_2 \\ &= F - \mu_k (M_1 + M_2) g - \mu_k M_2 g \\ &= F - \mu_k (M_1 + 2M_2) g \\ a &= \frac{F - \mu_k (M_1 + 2M_2) g}{M_1} \end{aligned}$$

5-35 Because the pulleys are frictionless, the rope that the person is grabbing has equal tension F throughout. Therefore the pulley immediately above the load has an upward force $2F$ acting on it from the two strands of rope above it. This tells us that the system has a mechanical advantage of 2, and the person can lift a weight $2F$, which corresponds to a mass $2F/g$.

5-36 (a) Let the positive x axis be to the right, so that the forces are F_1 and $-F_2$. If we consider the pair of blocks as a single object, then applying Newton's second law to that object gives $a = (F_1 - F_2)/(m_1 + m_2)$. Applying the second law to m_2 gives $a = (f - F_2)/m_2$. These are two equations in the two unknowns a and f. Eliminating a and solving for f gives

$$f = \frac{m_2 F_1 + m_1 F_2}{m_1 + m_2}.$$

This is a sort of weighted average of the two forces, but with the weights swapped, the weight of force 1 being mass 2 and vice versa.

(b) In the symmetric case, with $F = F_1 = F_2$, the result becomes $f = 2mF/2m = F$, which makes sense, because each block feels forces that cancel.

(c) Setting $f = 0$ gives $F_1/m_1 = -F_2/m_2$. This makes sense. One of the forces has to be in the opposite direction compared to the original diagram, so that each block feels a force in the same direction. These forces have to be in proportion to the masses, so that each mass can respond independently to the external force acting on it with the same acceleration.

6-1 (a) For its motion from the hand to the top of its arc, it has a constant acceleration equal to g, so $v^2 = 2gy$, and $y = v^2/2g$. (b) Only the vertical component, $v_y = v \sin\theta$, of the initial velocity has any effect on its vertical motion, so the equation becomes $y = v^2 \sin^2\theta/2g$.

6-2 In Miss Lettuce's frame of reference, the float is at rest and the sidewalk is moving. The bouquet is initially at rest, so when she drops it, it falls straight down and lands at her feet.

In her fans' frame, the float is moving and the sidewalk is at rest. The bouquet is initially moving to the right, and even after she drops it, it retains that horizontal motion, because there is no horizontal force that would change it. They see a parabolic arc going to the right.

6-3 (a) The horizontal and vertical motions are independent, so they hit simultaneously. (b) While she's in the air, the only force on her is gravity. There is no horizontal force on acting on her, so her horizontal acceleration is zero, meaning she keeps a constant speed. Her speed on impact is the same as when she went over the edge, 30 km/hr. (c) The same as when he went over the edge, 40 km/hr. (d) They have the same vertical velocities, but different horizontal velocities, so Bill's speed is greater.

6-4 (a) The horizontal and vertical motions are independent, so we can write down separate equations for them:

$x = v_x t$ [motion with constant velocity]

$y = y_o - \frac{1}{2} g t^2$ [motion with constant acceleration]

We're not given information about time, and we don't want to find it either, so we need to eliminate it. Solving the first equation for t and substituting into the second equation gives

$$y = y_o - \frac{1}{2} g \left(\frac{x}{v_x}\right)^2 .$$

We're interested in $x = L$, and the variable d corresponds to $y_o - y$, so we have

$$d = \frac{1}{2} g \left(\frac{L}{v_x}\right)^2 .$$

(b) Convert everything to mks to start with. 73.3 mi/hr=32.8 m/s, 60.0 ft=18.3 m. Plugging in, we get $d = 1.53$ m=5.00 ft. I calculated this using the two-sig-fig value $g = 9.8\ \mathrm{m/s^2}$, so that ends up being the limiting factor for accuracy, and the result should be rounded off to two sig figs, $d = 5.0$ ft.

6-6 Since v and g are fixed, we can ignore the constant factor of $2v^2/g$ and just maximize the expression $\sin\theta\cos\theta$.

The maxima and minima of a differentiable function occur where it has a derivative equal to zero, so we are looking for places where

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\sin\theta\cos\theta) = 0 \qquad .$$

Using the product rule, we find

$$\sin^2\theta - \cos^2\theta = 0 \qquad ,$$

which means $\sin^2\theta = \cos^2\theta$, or $\sin\theta = \pm\cos\theta$. Since we're interested in angles between 0 and 90°, both the sine and cosine will be positive, so $\sin\theta = \cos\theta$, which occurs at $\theta = 45°$.

6-7 The acceleration is mainly vertical. Each car's v_y starts out zero, increases as the bump's slope gets steeper, reaches a maximum, and then decreases back to zero as the car reaches the top of the bump. Riding back down the bump is similar.

The equation $a = \Delta v/\Delta t$ isn't strictly true here, since a isn't constant. However, we can still use it to get a comparison between the two cars. We then have $a_P/a_M = (\Delta v_P/\Delta v_M)(\Delta t_M/\Delta t_P)$. The first factor is clearly equal to 37/25, but so is the second one, since the Maserati's lower speed causes it to take more time to go through the same motion. Therefore $a_P/a_M = (37/25)^2 = 2.2$.

6-8 (a) For the vertical motion, we have $h = (1/2)gt^2$, which gives the air time $t = \sqrt{2h/g}$. You need to travel a horizontal distance $h\tan\theta$ in that time, so $v = h\tan\theta/t = \sqrt{gh/2}\tan\theta$.

(b) The units are $\sqrt{\mathrm{m^2/s^2}} = \mathrm{m/s}$, which makes sense.

(c) A higher g means you have to run faster, which makes sense physically. A higher h also means you have to run faster, which makes sense, e.g., you can't jump off of the top of a mountain and land at the base, but you could easily jump from a small scale model of the mountain. A bigger θ requires a higher speed, which makes sense. In the case of $\theta = 0$, we have $v = 0$, which makes sense because you can just step off the cliff. For $\theta = 90°$, the velocity becomes infinite, and that also makes sense: the "cliff" is now just flat ground, and you can't jump off of Kansas.

(d) Plugging in numbers gives 1.8 m/s.

6-9 (a) The vertical and horizontal motions are independent of each other. Since the balls reach the same height, their vertical motion is the same, so the initial y velocity of the green ball is also v_0.

(b) The times are the same, for the same reasons as in part a.

(c) The x velocity is $v_x = \sqrt{(2v_0)^2 - v_0^2} = \sqrt{3}v_0$. The time in the air is $2v_0/g$. Multiplying these gives the range, $2\sqrt{3}v_0^2/g$, and since $v_0^2 = 2gH$, this range can be expressed in terms of H as $4\sqrt{3}H$.

6-11 The problem is stated for a cylinder, but the minimum velocity will be one that is possible only in the case where the bullet flies in the plane of the figure, so we can reduce the problem to two dimensions.

The relevant physical conditions are the radius r of the cylinder and the strength g of the gravitational field. Based on units, the solution must be of the form $\sqrt{gr}$ multiplied by a unitless constant. Therefore we can simplify the math by taking $g = 1$ and $r = 1$, and waiting until the end of the problem to put the factor of $\sqrt{gr}$ back in.

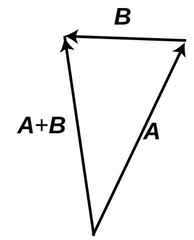
There are two qualitatively different ways in which the bullet could fail to clear the cylinder. It could spend a finite time in the air and then hit, or it could simply fail to lift off. If we only consider the second condition, then there are several methods that allow a very short (even one-line) solution, but they involve tricks (such as approximating the bullet's motion as circular motion) or fancy math (such as matching the curvatures of the circle and parabola by setting the second-order terms of their Taylor series equal). We will instead take an approach that uses straightforward algebra and no tricks, and that addresses both possible modes of failure.

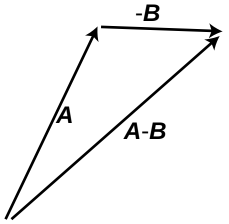
Let v be the initial speed of the bullet, and choose Cartesian coordinates with the usual orientation and the origin at the center of the circle. Then its motion is given by $x = vt$, $y = 1 - (1/2)t^2$, and eliminating t gives $y = 1 - (1/2)(x/v)^2$, the equation of the bullet's parabolic trajectory. For big enough values of v, this parabola will only intersect the circle at $x = 0$. For too-small values, it will intersect it in at least one other place. By symmetry, any such intersections will occur in pairs on the left and the right, but we consider only $x > 0$. If the bullet achieves lift-off but then hits the circle, then there should be two intersections for $x > 0$, one where the bullet enters the circle and one where it exits again at the bottom. If it never achieves lift-off, then there should be only one, where it exits.

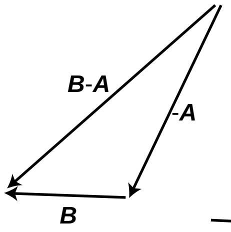
To locate and count these intersections, we equate the equations of the circle and the parabola, $1-(1/2)(x/v)^2 = \pm\sqrt{1-x^2}$. The only positive solution is $x = 2v\sqrt{1-v^2}$, which exists if the square root is real. Since there is never more than one positive solution, we have shown rigorously that the only way in which we can fail is if the bullet never lifts off.

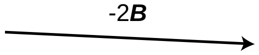
The solution fails to exist if $v > 1$, so the minimum speed is 1. Reinserting the factors that contain units, we have $v = \sqrt{gr}$ for the minimum speed.

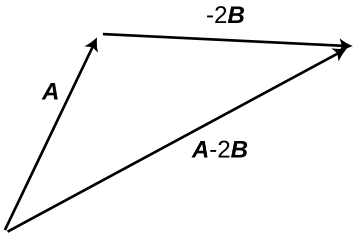
7-1











7-2 (a) Let's use a coordinate system with positive x being east and positive y being north. In these coordinates, the vector from Bangkok to Phnom Penh has

$$\Delta x = 470 \text{ km} \qquad \Delta y = -250 \text{ km}$$

and the one from Phnom Penh to Hanoi has

$$\Delta x = 60 \text{ km} \qquad \Delta y = 1030 \text{ km} \qquad .$$

(b) The components of the vector from Bangkok to Hanoi equal the sum of the components of the two vectors above,

$$\Delta x = 530 \text{ km} \qquad \Delta y = 780 \text{ km} \qquad .$$

7-3 Since the two given angles are both measured counterclockwise from east, we can just use a coordinate system with positive x being east and positive y north, and the positive and negative signs of the components will come out right if we just use cosines for x components and sines for y components. We find the components of the total Δr vector by adding the components of the two individual vectors:

$$\begin{aligned}\Delta x &= \Delta x_1 + \Delta x_2\\ &= (35\text{ km})\cos\ 25° + (22\text{ km})\cos 230°\\ &= 17.6\text{ km}\end{aligned}$$

(temporarily keeping 1 extra sig fig).

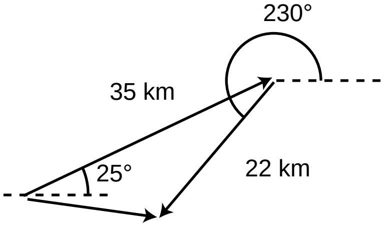
$$\begin{aligned}\Delta y &= \Delta y_1 + \Delta y_2\\ &= (35\text{ km})\sin 25° + (22\text{ km})\sin 230°\\ &= -2.1\text{ km}\end{aligned}$$

(temporarily keeping 1 extra sig fig).

The magnitude of the total Δr vector is 18 km, and the direction is

$$\tan^{-1}(-2.1\text{ km}/17.6\text{ km}) = -7° \text{ or } 173° \qquad .$$

Checking against the diagram, the correct arctangent is $-7°$.



7-4 This problem has to be done analytically, not graphically, because it requires such high precision. The two given angles are both measured counterclockwise from the right, which is convenient; for angles defined this way, we can always find x components using a cosine, and y components using a sine, and the plus and minus signs will be come out automatically. The first vector's components are

$$\begin{aligned}\Delta x_1 &= (35.24\text{ mm})\cos 217.3°\\ &= -28.03\text{ mm}\\ \Delta y_1 &= (35.24\text{ mm})\sin 217.3°\\ &= -21.36\text{ mm} \qquad ,\end{aligned}$$

and the second one's are

$$\begin{aligned}\Delta x_2 &= (48.01\text{ mm})\cos 11.7°\\ &= 47.01\text{ mm}\\ \Delta y_2 &= (48.01\text{ mm})\sin 11.7°\\ &= 9.74\text{ mm} \qquad .\end{aligned}$$

Checking these against the figure, their sizes and signs look reasonable. Adding these would give the vector from the top hole to the right-hand hole, but she wants to go the opposite way, so the components we want are

$$\begin{aligned}\Delta x_3 &= -(\Delta x_1 + \Delta x_2)\\ &= -18.98\text{ mm}\end{aligned}$$

and

$$\begin{aligned}\Delta y_3 &= -(\Delta y_1 + \Delta y_2)\\ &= 11.62\text{ mm} \qquad .\end{aligned}$$

Again, the signs and sizes look about right. The distance she has to move is given by the Pythagorean theorem,

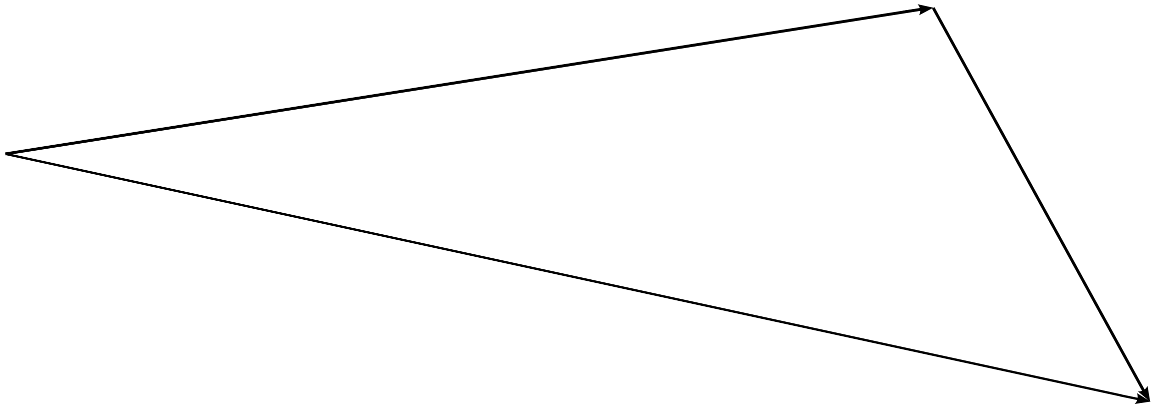
$$\begin{aligned}|\Delta \mathbf{r}_3| &= \sqrt{\Delta x_3^2 + \Delta y_3^2}\\ &= 22.25\text{ mm} \qquad .\end{aligned}$$

The direction is

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\Delta y_3}{\Delta x_3}\right)\\ &= -31.4° \text{ or } 148.5°\end{aligned}$$

(measured counterclockwise from the right). We can tell from looking at the figure that the second arctangent is the correct one. (I've given the final answers with four sig figs of accuracy, although you could argue that it should be three, since the least accurate piece of given data is the 11.7° angle. In reality, however, that angle isn't any less accurate than the 217.3° one, i.e., changing either one by a tenth of a degree will have about the same effect on the final result.)

7-5 To prove that this operation is not rotationally invariant, it suffices to find one example



Problem 8-1.

in which converting the inputs **A** and B into a different, rotated coordinate system causes the output **C** to be a completely different vector, not just the same vector represented in a different system of coordinates. Almost any example will show this, unless we really manage to shoot ourselves in the foot with an unusual choice like $\mathbf{A}=0$, $B=0$. As an example, let's take $B=1$, and $\mathbf{A}=\hat{\mathbf{x}}+\hat{\mathbf{y}}-\hat{\mathbf{z}}$ in the original coordinate system, resulting in $\mathbf{C}=2\hat{\mathbf{x}}+2\hat{\mathbf{y}}$. Now rotate the coordinate system by 180 degrees about the z axis. In this new coordinate system, $\mathbf{A}=-\hat{\mathbf{x}}-\hat{\mathbf{y}}-\hat{\mathbf{z}}$, but the scalar B stays the same, giving $\mathbf{C}=0$. This new **C** is clearly not just the same vector expressed in a differently oriented coordinate system, since its magnitude has changed from a nonzero value to zero.

8-1 See the figure. I chose a scale of 0.5 cm ↔ 10^{-7} m/s. The vector sum is 14.3 cm in length, which converts to 2.9×10^{-7} m/s. The direction, measured with a protractor, is about 12° south of east.

8-2 Yes, it would be slowing down.

8-3 The original velocity vector had $v_x=21.1$ m/s, $v_y=0$. The later one has $v_x=(21.1\ \text{m/s})\times\cos 7^\circ=20.94$ m/s and $v_y=(21.1\ \text{m/s})\times\sin 7^\circ=2.57$ m/s. The change in the velocity vector is

$$\Delta v_x=-0.16\ \text{m/s}\qquad\text{and}$$
$$\Delta v_y=2.57\ \text{m/s}\qquad,$$

so the acceleration vector has components

$$a_x=\Delta v_x/\Delta t=-0.16\ \text{m/s}^2\qquad\text{and}$$
$$a_y=\Delta v_y/\Delta t=2.57\ \text{m/s}^2\qquad,$$

giving a magnitude of

$$\sqrt{(0.16\ \text{m/s}^2)^2+(2.57\ \text{m/s}^2)^2}=2.6\ \text{m/s}^2\qquad.$$

Since the x component is negative and the y component is positive, it has to be pointing northwest, and the angle measured counterclockwise from east is $\tan^{-1}(2.57/-0.16)=92^\circ$. (Your calculator probably gives you the other arctangent, -88°, which is not the appropriate one.)

8-4 (a) There are three forces acting on the part of the rope right under the person's feet: the two tensions and the normal force of the person's feet on the rope, which equals the person's weight. The vector sum of the three forces must be zero, i.e. their x components must add up to zero and so must their y components. The x components will just naturally cancel out by symmetry, so that's not helpful information. The y components of the two tension forces are both

equal to each other, so double the y component of the tension in one side must equal the person's weight. The y component of the tension in one side is $T\sin\theta$, where θ is the angle of the rope above horizontal. Therefore, we must have $Mg = 2T\sin\theta$. The length of each half of the tightrope is given by the Pythagorean theorem, $\sqrt{h^2 + (L/2)^2}$, and using trig, $\sin\theta = h/\sqrt{h^2+(L/2)^2} = 1/\sqrt{1+(L/2h)^2}$. Solving for T, $T = Mg/(2\sin\theta) = \frac{1}{2}Mg\sqrt{1+(L/2h)^2}$.
(b) Examining the equation $T = Mg/(2\sin\theta) = \frac{1}{2}Mg\sqrt{1+(L/2h)^2}$, if h was zero, $L/2h$ would be infinite, so the tension would have to be infinite. Physically, the rope would break at some nonzero value of h.

8-5 The main reason that this problem is difficult is that you have to realize there are two separate cases. Pressing the block and the wall together can result in either an upward or a downward frictional force on the block. If you're applying almost enough force to slide the block up the wall, then the frictional force is downward, but if you're applying just barely enough to keep it from sliding down, then the frictional force is upward. Because the vertical component of the frictional force has an opposite signs in the two cases, it is necessary to do a separate version of the algebra in each case.

Case A: upward friction on the block

Let the positive x axis be to the right and the positive y axis up. Then both the x and y components of the force on the block must equal zero. Let $\mathbf{F}_N$ be the normal force of the wall on the block. The maximum force of static friction is given by $|\mathbf{F}_{s,max}| = \mu_s|\mathbf{F}_N|$, and it is the maximum in which we're interested, since we're looking for the largest possible value of $|\mathbf{F}_H|$.

zero total force in the x direction:

$$|\mathbf{F}_H|\cos\theta - |\mathbf{F}_N| = 0$$

zero total force in the y direction:

$$|\mathbf{F}_H|\sin\theta - mg + \mu_s|\mathbf{F}_N| = 0$$

We solve the first equation for $|\mathbf{F}_N| = |\mathbf{F}_H|\cos\theta$, then substitute into the second equation and solve to find

$$|\mathbf{F}_H| = \frac{mg}{\sin\theta + \mu_s\cos\theta}$$

Case B: downward friction on the block

The solution of this part is exactly analogous, except that the total force in the y direction is $|\mathbf{F}_H|\sin\theta - mg - \mu_s|\mathbf{F}_N| = 0$, with a minus sign in front of the frictional force. The result is

$$|\mathbf{F}_H| = \frac{mg}{\sin\theta - \mu_s\cos\theta}$$

In the case of $\theta = 90°$, both the maximum and the minimum are mg, which makes sense. The maximum force blows up to infinity for $\theta = \tan^{-1}\mu_s$. With this angle, no amount of force will make the block slip, because the greater normal force leads to a greater frictional force, which is great enough to counteract the greater vertical force from the hand. For angles smaller than this, the equation for the maximum force gives a negative result, which is not physically meaningful because it was supposed to represent the magnitude of the force.

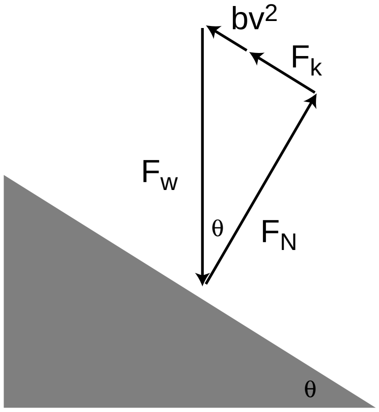
8-6 (a) There are four forces acting on the skier, a normal force F_N from the snow, a gravity force F_W, a kinetic friction force $F_k = \mu_k F_N$, and an air friction force bv^2. If the skier is to approach a constant velocity, the total force on her must become nearly zero. Laying the vectors tip-to-tail results in the right triangle shown in the figure. At first the bv^2 force will not be big enough to close the gap in the triagle, and the skier will be accelerating, but eventually as her velocity grows and the bv^2 force gets stronger the total force will approach zero and she will stop accelerating. Trigonometry gives

$$bv^2 + F_k = F_W\sin\theta \qquad \text{and}$$
$$F_N = F_W\cos\theta \qquad ,$$

and using these two equations plus $F_W = mg$ and $F_k = \mu_k F_N$ we find

$$v = \sqrt{\frac{mg}{b}\left(\sin\theta - \mu_k\cos\theta\right)} \qquad .$$

(b) For θ less than $\tan^{-1}\mu_k$, we'd be taking the square root of a negative number. These are angles so shallow that the skier would slow to a halt.



8-7 (a) The units of the first term are just the units of b, so b must have units of meters. The units of c have to be m/s, so that the seconds will cancel and give meters. Similarly d must have units of m/s^2, i.e., acceleration.

(b)

$$\begin{aligned}\mathbf{v} &= \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\\ &= c\hat{\mathbf{y}} + 2dt\hat{\mathbf{z}}\\ \mathbf{a} &= \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\\ &= 2d\hat{\mathbf{z}}\end{aligned}$$

(c) The only force acting on the bullet is gravity, so its acceleration $2d$ equals g, and $\hat{\mathbf{z}}$ points down. The bullet's velocity at $t = 0$ is $c\hat{\mathbf{y}}$, so c is its initial velocity, and $\hat{\mathbf{y}}$ points to the west. If the coordinate system is right-handed, then $\hat{\mathbf{x}}$ must point south, and b is simply the bullet's location in the north-south direction, measured from some arbitrary origin.

8-12 (a) $v_f = \Delta x/\Delta t = w/t_b$.

(b) We know its final velocity, and the distance it traveled, and we want to find its acceleration. The constant acceleration equation that has the right variables in it is $v_f^2 = v_o^2 + 2a\Delta x$, which in our case translates into $v_f^2 = 2ax$. Solving for the unknown and expressing it in terms of the things we're actually able to measure directly, we have

$$\begin{aligned}a &= \frac{v_f^2}{2x}\\ &= \frac{w^2}{2xt_b^2}\end{aligned}$$

(c) See table on last page.

(d) This is easiest if we adopt a coordinate system where x is along the track, and y is perpendicular to the track. The cart only has an acceleration along the x axis, and the only force acting on the cart that has an x component is gravity.

$$\begin{aligned}a_x &= \frac{F_x}{m}\\ &= \frac{F\sin\theta}{m}\\ &= \frac{mg\sin\theta}{m}\\ &= g\sin\theta\end{aligned}$$

You might wonder how I knew it was a sine rather than a cosine. Well, the sine of zero is zero, and I know that the acceleration is zero if the angle of the track is zero.

8-13 In each case, the y component, T_y, of the tension has to be enough to support half the boy's weight. The y component of the tension is going to be given by either the sine or the cosine, and we can see that it must be the cosine because that gives the right answer for the first positoin, $T_y = T\cos\theta = T$. Since T_y is always the same, the tension must be proportional to $1/\cos\theta$. The tensions are in the ratios of 1 to $1/\cos 45°$ to $1/\cos 60°$, or 1 to 1.41 to 2.

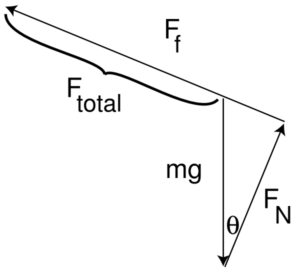
8-14 (a) See table on last page.

Problem ??:

force acting on cart			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	earth	gravity	up	cart
normal	up and right	track	normal	down and left	cart

Problem ??:

force acting on car			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	car
static friction	uphill	road	static friction	downhill	car
normal	up and forward	road	normal	down and backward	car



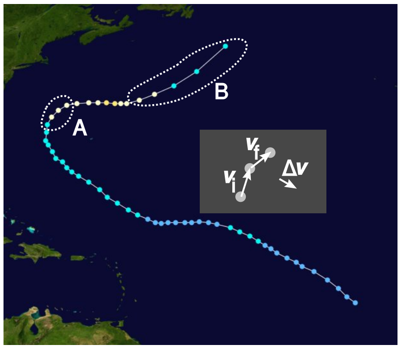
(b) From the figure, we see that the total force is equal to the force of static friction, minus the length of the short leg of the right triangle, $F_s - mg\sin\theta$. The maximum possible deceleration will be obtained if static friction has its maximum value $F_s = \mu_s F_N = \mu_s mg\cos\theta$. The result for the total force is $\mu_s mg\cos\theta - mg\sin\theta = mg(\mu_s\cos\theta - \sin\theta)$. By Newton's second law, the maximum deceleration is the total force divided by the mass, or $g(\mu_s\cos\theta - \sin\theta)$.

(c) Increasing the car's mass increases its inertia, which makes it harder to stop, but it also increases the gravitational force on it, which causes a greater normal force, and therefore a greater maximum value for the static friction force.

(d) At $\theta = 0$, we find that the maximum deceleration of a car on a flat road is $\mu_s g$. For $\theta < 0$, the $\sin\theta$ is negative, which means the result for the deceleration is *greater* than on a flat road. That makes sense: it's easier to make a panic stop when you're going uphill.

(e) As θ increases, the $\cos\theta$ term gets smaller and the $\sin\theta$ term gets bigger. At some angle, the result for the deceleration will become zero. This is the steepest slope the car can be on without slipping. For angles greater than this, static friction must give way to kinetic friction, and our result from part b becomes incorrect, since we derived it assuming static friction.

8-15 The velocity vector points in the direction of motion. There are two time periods during which the direction of motion is northeast:



During period A, around Aug. 15, the path is curving to the right. The $\Delta\mathbf{r}$ vectors connecting the points in dot-to-dot fashion can be thought of as synonyms for the $\mathbf{v}$ vectors, since $\mathbf{v} \approx \Delta\mathbf{r}/\Delta t$, and Δt is just a constant (6 hours). As shown in the inset figure, vector subtraction of these **v** vectors gives a $\Delta\mathbf{v}$ that points south-east, so the acceleration is to the southeast.

During period B, a similar vector subtraction gives an acceleration to the northwest, which is not what is required by the question.

8-16 (a) Let the positive x axis be to the right and positive y up. The x components of the forces cancel by symmetry, so they don't give us any useful information. The y components of the three forces are $S\cos\theta/2$, $S\cos\theta/2$, and $-L$. Setting the sum of these equal to zero, we find $S = L/(2\cos\theta/2)$. (b) For $\theta = 0$, we find $S = L/2$, which makes sense physically because two forces S are cooperating perfectly to cancel L. (c) For $\theta = 180°$, we have $S = \infty$, which is not physically possible, but does make sense. If we were to grab the two anchor lines in our hands and pull them to the right and left away from each other, it would take infinite force to make $\theta = 180°$ exactly. (c) This occurs when $2\cos\theta/2 = 1$, i.e., $\theta = 2\cos^{-1}(1/2) = 120°$.

8-17 (a) Most of the forces on the box are either parallel to or perpendicular to the slope, so it's convenient to pick coordinate axes aligned with the slope. Let positive x be uphill, and positive y perpendicular to that (up and to the left). Four forces act on the box: a normal force F_N in the positive y direction, a kinetic frictional force F_k in the $-x$ direction, a force from the rope with magnitude T in the $+x$ direction, and a downward force of gravity with magnitude Mg. The gravitational force is the only one with both an x and a y component; it has x component $-Mg\sin\theta$ and y component $Mg\cos\theta$.

Since the box doesn't hop off of or sink into the slope, the total force on the box in the y direction must be zero:

$$0 = F_N - Mg\cos\theta$$

To predict the box's acceleration, we apply Newton's second law in the x direction:

$$a = (T - Mg\sin\theta - \mu_k F_N)/M$$

These are two equations in the three unknowns F_N, a, and T. Since we don't have enough equations to solve for the unknowns, we apply Newton's second law to the person:

$$a = (mg - T)/m \qquad ,$$

where we make use of the facts that the tension in a rope is unaffected by going around a frictionless pulley and that the person's acceleration must match that of the box, since they're tied together. Solving these equations for a results in

$$a = (m - M\sin\theta - \mu_k M\cos\theta)g/(M + m) \qquad .$$

(b) All the terms in the factor $m - M\sin\theta - \mu_k M\cos\theta$ have units of kilograms, and these cancel with the units of the $M + m$, giving nothing but acceleration units from the factor of g.

(c) When $M = 0$, the rope should be slack and the person should simply free-fall with acceleration g. This is indeed what happens when we plug in $M = 0$.

When $\theta = -90°$, the slope is rotated clockwise so that it becomes a vertical wall above the person's head. There is no normal force between the box and the slope, so there is no friction. The person and the box should both simply free-fall with acceleration g. Plugging in $\theta = -90°$ does produce this result.

8-18

$F_{F1,x}$	small, $-$	$-F_{F1}\sin\theta$
$F_{F1,y}$	large, $+$	$F_{F1}\cos\theta$
$F_{F2,x}$	small, $-$	$-F_{F2}\sin\theta$
$F_{F2,y}$	large, $+$	$F_{F2}\cos\theta$
$F_{N1,x}$	large, $-$	$-F_{N1}\cos\theta$
$F_{N1,y}$	small, $-$	$-F_{N1}\sin\theta$
$F_{N2,x}$	large, $+$	$F_{N2}\cos\theta$
$F_{N2,y}$	small, $+$	$F_{N2}\sin\theta$
$F_{W,x}$		0
$F_{W,y}$		$-F_W$

8-19 (a) Let positive x be to the right and positive y up. Zero total force acts on the carabiner in the y direction,

$$-L + 2T\cos(\theta/2) = 0 \qquad .$$

Zero total force also acts on the left-hand anchor, so

$$S_y - T\cos(\theta/2) = 0 \qquad \text{and}$$
$$S_x + T + T\sin(\theta/2) = 0 \qquad .$$

Algebra leads to the result

$$S = L\frac{\sqrt{(1+\sin(\theta/2))/2}}{\cos(\theta/2)} \qquad .$$

(b) When $\theta = 0$, we have $S = L/\sqrt{2}$, which makes sense because in this situation $T = L/2$, and each anchor experiences two forces equal in magnitude to T, acting at right angles. This is worse than the correct setup, in which $\theta = 0$ gives $S = L/2$.

8-20 (a) The tension at the center describes a force that each half of the wire exerts on the other half. Therefore we should analyze the forces on one half of the wire, say the left half.

(Here "cohesive" is just a name I made up for the force that holds the wire together and keeps it from breaking.)

Let the magnitude normal force be F_N, and let the tension in the middle of the wire be T_m. By Newton's first law, the total force acting on the left half of the wire must be zero. Let's do analytic addition, which means adding components. Let the positive x axis be to the right, and the positive y axis up. Then adding the horizontal components gives

$$T_m - F_N\cos\theta = 0 \qquad ,$$

and adding the vertical ones gives

$$F_N\sin\theta - \frac{1}{2}mg = 0 \qquad .$$

The result is $T_m = mg/2\tan\theta$.

(b) The tension at the ends equals F_N. Solving the original two equations for this unknown, we have $F_N = mg/2\sin\theta$. Since $\sin\theta$ is always less than $\tan\theta$ for angles between 0 and 90°, we find $F_N > T_m$. The tension at the ends is greater than in the middle. This is easy to verify intuitively by imagining the case of $\theta \approx 90°$.

8-21 (a) Let positive x be to the right, and positive y up. The horizontal motion is at constant velocity $u\cos\alpha$, so

$$x = ut\cos\alpha.$$

The vertical motion has initial velocity $u\sin\theta$ and acceleration $-g$, so

$$y = ut\sin\alpha - \frac{1}{2}gt^2.$$

Setting $x = \ell$ in the first equation and solving for t gives $t = \ell/u\cos\alpha$. Now we set $y = h$ in the second equation and substitute for t, which gives

$$h = \ell\tan\alpha - \frac{1}{2}g(\ell/u\cos\alpha)^2.$$

Solving this for u results in

$$u = \sqrt{\frac{gl}{2\cos^2\alpha(\tan\alpha - h/\ell)}}.$$

(b) The ratio h/ℓ is unitless, so it makes sense to subtract it from $\tan\alpha$. The units of the quantity inside the square root come solely from the factor gl, which has units of m^2/s^2. Taking the square root gives m/s, which makes sense.

(c) Physically, if gravity is stronger we'll need to roll the ball faster to get it up to the hole. Mathematically, g is on top, so increasing it increases u.

(d) Physically, if the height is increased we'll need to increase the speed. Mathematically, h/ℓ is subtracted on the bottom, so increasing h reduces the bottom, which increases the result.

(e) If $\alpha = 90°$, then u blows up to infinity. This makes sense, because if the ball is launched straight up, then it doesn't matter how fast we roll it, it will never cover the horizontal distance h.

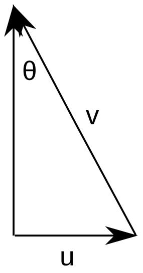
(f) When $\tan\alpha = h/\ell$, the ramp is pointed straight at the hole. If we were playing the game in zero gravity, this would be just fine, but when there is gravity, the path of the ball can't be straight, so there is no way it can get to the hole when it's aimed this way. Mathematically, our result for u blows up under these conditions, which makes sense.

(g) Plugging in gives $u = 4.1$ m/s.

8-22 The addition of the velocity vectors looks like this:

Problem ??:

force acting on left half of wire			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet earth	gravity	up	left half
cohesive	right	right half	cohesive	left	left half
normal	up and left	hook	normal	down and right	left half



(a) We have $\theta = \sin^{-1}(u/v)$.

(b) The velocity of the plane relative to the ground is the left side of the triangle, $w = \sqrt{v^2 - u^2}$, so the time to get to the city is $t = D/w = D/\sqrt{v^2-u^2}$.

(c) The units are $\text{m}/\sqrt{(\text{m}^2/\text{s}^2)}$, which does turn out to be seconds.

(d) If $u = v$ we get an infinite result. This makes sense, because in this situation the plane can only stand still by flying directly into the wind.

8-23 Let the positive x axis be to the right, and positive y up.

(a) The block isn't popping up into the air or sinking into the surface, so the y forces cancel:

$$F_N - Mg - F\sin\theta = 0$$
$$F_N = Mg + F\sin\theta$$

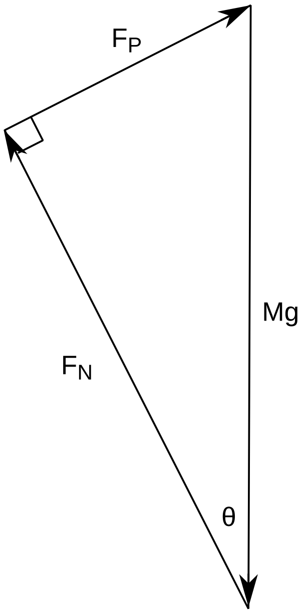
(b) The case where it's just about to slip is the one where it's in equilibrium horizontally and the force of static friction is at its maximum value $\mu_s F_N$, so that $F\cos\theta - \mu_s F_N = 0$. Substituting the answer from part a for F_N and solving for F, we find

$$F = \frac{Mg}{\cos\theta/\mu_s - \sin\theta}.$$

(c) This is the same as part b, but with μ_k.

(d) The answer to part b misbehaves starting at the critical angle for which the denominator is zero, which is $\tan^{-1}(1/\mu_k)$. Beyond this angle, the result for F is negative, which is unphysical.

8-24 (a) Three forces act on the mass: a gravitational force Mg, a normal force from the ramp, and a force from the rope that equals the reading F_P on the force probe. As shown in the figure, these three forces, when assembled tip-to-tail for vector addition, make a right triangle with Mg on the hypotenuse and F_P on the leg opposite to the angle θ. The result is that $F_P = Mg\sin\theta$.



(b) The expression found in part a gives 0 when $\theta = 0$ and Mg when $\theta = 90°$. Both of these make sense.

8-26 The horizontal motion is at the constant velocity v_0, while the vertical motion is with constant acceleration g. The vertical component of the impact velocity is given by $v_y^2 = 2gh$. Given the x and y components of the vector, we can use the inverse tangent function to find its direction. The result is

$$\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{v_0}\right)$$

8-27 (a) Let positive x be to the right and positive y up. The block is acted on by four forces: the two external forces of magnitude F, along with a normal force and a frictional force from the surface. If the block is to remain at rest, then the forces in the x direction must cancel, $F - F\cos\theta - \mu_s F_N = 0$, and similarly in the y direction, $F_N - F\sin\theta = 0$. Eliminating F_N and solving for the coefficient of friction results in

$$\mu_s = \frac{1-\cos\theta}{\sin\theta}.$$

(b) At $\theta = 0$ we expect that the block should be able to remain at rest without friction, since the horizontal forces cancel by symmetry. Our expression for μ_s is an indeterminate form when $\theta = 0$, but application of l'Hôpital's rule gives

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta} = \lim\frac{\sin\theta}{\cos\theta} = 0,$$

as required.

9-1 (a) Only an inward force, i.e., a force perpendicular to the surface of contact, can have an effect on whether the batter separates from the beater. Friction is always parallel to the surface of contact.

(b) See table on last page.

(c) An inward force is required to keep the batter from following Newton's first law and flying off straight, on a tangent to the circle. The amount of force required is given by Newton's second law, $F = ma = mv^2/r$. If the speed increases, so does amount of inward sticky force needed to continue deflecting the batter off of its desired straight-line path. There's a maximum amount of force that the stickum can supply, beyond which it becomes unstuck.

9-2 $(\mathrm{m/s})^2/\mathrm{m} = \mathrm{m}^2/\mathrm{s}^2/\mathrm{m} = \mathrm{m/s}^2$

9-3 If neither the seat nor the seatbelt is applying a force to the pilot, then the only force on him is the earth's gravity, i.e. his weight. When the only force on an object is gravity, its acceleration is g. The speed needs to be such that the acceleration equals g. The acceleration is v^2/r, so $v^2/r = g$, or $v = \sqrt{gr} = 99$ m/s.

9-4 (a) $\theta = vt/r$, so $\mathbf{r} = r(\cos(vt/r))\hat{\mathbf{x}} + r(\sin(vt/r))\hat{\mathbf{y}}$.

(b) We need to differentiate twice with respect to t, using the chain rule and the fact that $\mathrm{d}(\sin x)/\,\mathrm{d}x = \cos x$ and $\mathrm{d}(\cos x)/\,\mathrm{d}x = -\sin x$. The first differentiation gives a velocity vector

$$\begin{aligned}\mathbf{v} &= \left(\frac{v}{r}\right) r\left(-\sin\frac{vt}{r}\right)\hat{\mathbf{x}} + \left(\frac{v}{r}\right) r\left(\cos\frac{vt}{r}\right)\hat{\mathbf{y}}\\ &= v\left(-\sin\frac{vt}{r}\right)\hat{\mathbf{x}} + v\left(\cos\frac{vt}{r}\right)\hat{\mathbf{y}}.\end{aligned}$$

The second differentiation gives

$$\begin{aligned}\mathbf{a} &= \left(\frac{v}{r}\right) v\left(-\cos\frac{vt}{r}\right)\hat{\mathbf{x}} + \left(\frac{v}{r}\right) v\left(-\sin\frac{vt}{r}\right)\hat{\mathbf{y}}\\ &= \frac{v^2}{r}\left[-\cos\frac{vt}{r}\hat{\mathbf{x}} - \sin\frac{vt}{r}\hat{\mathbf{y}}\right] \quad .\end{aligned}$$

(c) The magnitude of the vector in square brackets equals one, since $\cos^2 + \sin^2 = 1$. Multiplying by the scalar v^2/r therefore gives a vector with magnitude v^2/r.

9-6 (a)

$$\begin{aligned}a &= F/m\\ v^2/r &= F_N/m\\ F_s &= mg\\ \mu_s F_N &\geq mg\\ \mu_s mv^2/r &\geq mg\\ v &\geq \sqrt{gr/\mu_s}\end{aligned}$$

(b) The velocity is inversely proportional to the square root of the coefficient of static friction, so if her friction is three times weaker, she has to wait until the speed is greater by a factor of $\sqrt{3} = 1.7$.

Problem ??:
Part b:

force acting on tape			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
sticky	left	beater	sticky	right	tape

9-8 The "radius" concept is a bit fuzzy here, because the train has a finite width to it. Although the inside of the outer rail is where the force has to be applied when the train is starting to tip, its radius from the center of the curve isn't quite the same as the radius of the circle traveled by the train's center of mass. Then again, maybe the train's center of mass would be above the outer rail if it was just on the verge of flipping over. Anyhow, ignoring these niceties, we assume that there is a maximum acceleration, which is the same in both cases. Converting $a = v^2/r$ into a proportionality by throwing out the factor of a, we have $v^2 \propto r$, or $v \propto \sqrt{r}$. Rewriting this in terms of ratios gives $v_1/v_2 = \sqrt{r_1/r_2}$. Since we're doing a ratio, it doesn't matter whether we do diameter or radius. The result is $(0.95\text{ m/s})\sqrt{662\text{ mm}/1067\text{ mm}} = 0.75$ m/s. We know we set up the ratio the right way, since it makes sense that the speed comes out slower on the tighter curve.

9-9 (a)

equal and opposite force

force acting on ball involved in Newton's 3rd law

type of object exerting object

force direction the force type direction exerting it

tension in and up string tension down and out ball

gravity down earth gravity up ball

(b)

Newton's second law, horizontal component:

$v^2/r = F_T \sin\theta/m \qquad (1)$

Newton's second law, vertical component:

$0 = (F_T \cos\theta - mg)/m \qquad (2)$

geometry:

$r = L \sin\theta \qquad (3)$

$v = 2\pi r/P \qquad (4)$

The variable v is not given and not desired, so we eliminate it using equation (4):

$$4\pi^2 mr/P^2 = F_T \sin\theta (5)$$

We next use equation (3) to eliminate r:

$$4\pi^2 mL/P^2 \sin\theta = F_T \sin\theta \qquad (6)$$

$$4\pi^2 mL/P^2 = F_T \qquad , \qquad (7)$$

which has the unintended side-effect of making θ disappear. The equation is still useful, however, because it tells us F_T in terms of variables we know. We can use it to eliminate F_T in equation (2):

$$0 = 4\pi^2 L/P^2 \cos\theta - g \qquad , \qquad (8)$$

which can be solved for θ:

$$\theta = \cos^{-1}\left(\frac{gP^2}{4\pi^2 L}\right) (9)$$

(c) We can't take the arccosine of a number that's greater than one, so there would be no solution for sufficiently large values of P. Physically, if P approached infinity, the ball would be standing still, in which case θ would equal zero. In the case where the angle is zero, step (6) → (7) is invalid, since we can't divide both sides of the equation by zero.

9-10 (a) The force on the hamster is $k(L-b)$, where L is the length of the spring (which is the same as the radius of the circle). Newton's second law gives $v^2/L = k(L-b)/m$. We can eliminate $v = 2\pi L/T$, and straightforward algebra then gives

$$L = \frac{b}{1-\frac{4\pi^2 m}{kT^2}}$$

(b) For a certain value of T, the denominator becomes zero. If we were to speed up the motor gradually, L would grow without bound as we approached this value of T. Physically, the spring would have to break. The reason for this surprising behavior will become clearer after you learn about the phenomenon of resonance.

9-11 (a) Let the tension in each of the top bars be T, and in each bottom one U. The vertical components of the forces on the mass M must cancel, so

$$2U\cos\theta = Mg \qquad ,$$

and similarly for the vertical forces on each mass m,

$$(T-U)\cos\theta = mg \qquad .$$

The horizontal force of the bars on each mass m must satisfy Newton's second law,

$$(T+U)\sin\theta = \frac{4\pi^2 mL\sin\theta}{P^2} \qquad .$$

We now have three equations in the three unknowns U, T, and θ. We use the first equation to eliminate U, which reduces this to two equations in two unknowns:

$$T + \frac{Mg}{2\cos\theta} = \frac{4\pi^2 mL}{P^2}$$
$$T\cos\theta - \frac{Mg}{2} = mg$$

Solving the second equation for $T = (mg + Mg/2)/\cos\theta$, substituting into the first equation, and solving for θ, we find the result claimed for θ.

(b) Low speeds correspond to large values of the period, P. At low enough speeds, the equation derived in part a ends up being the arccosine of a number greater than one, i.e., there is no solution of that form. What happens at these speeds is the same things that happens at zero speed: everything hangs straight down. In this solution, the radius of the circle traveled by each mass m is zero. (The mathematical loophole here was that we canceled a $\sin\theta$ on both sides of one of the equations, but you can't divide both sides of an equation by zero.) By making a nonzero, we make such a zero-radius solution impossible, because even when the engine isn't spinning, each mass m is already at some distance from the axis.

9-12 Applying Newton's second law and $a = v^2/r$ to each block independently, we have

$$v_1^2/L_1 = (T_1 - T_2)/m_1$$
$$v_2^2/(L_1+L_2) = T_2/m_2$$

We can eliminate $v_1 = 2\pi L_1/P$ and $v_2 = 2\pi(L_1+L_2)/P$, and straightforward algebra then yields

$$T_1 = \left(\frac{4\pi^2}{P^2}\right)[m_1L_1 + m_2(L_1+L_2)]$$
$$T_2 = \left(\frac{4\pi^2}{P^2}\right)m_2(L_1+L_2)$$

9-13 The apparent paradox arises if you assume, incorrectly, that v and T are the same even though r is different. You can't change r while leaving v and T the same. A should have said that *for objects moving at the same speed*, the smaller circle gives a greater acceleration. B should have said that *for objects that take the same amount of time per cycle*, the smaller circle gives a smaller acceleration.

9-14 The force is $F = ma = m\omega^2 r = m(2\pi T)^2 r = 1.3\times10^{47}$ newtons. This is an absurd amount of force, and it clearly isn't being exerted by any physical object. There isn't actually any force, and Sirius isn't actually accelerating. It just appears to accelerate in this noninertial frame of reference.

9-15 (a) Let positive x be inward and positive y up. Then $a_x = F_x/m$ gives $v^2/r = F_N/m$, and $a_y = F_y/m$ results in $0 = \mu_s F_N - mg$. Solving for r, we find $r = \mu_s v^2/g$. (b) $(\mathrm{m/s})^2/(\mathrm{m/s}^2) = \mathrm{m}$. (c) Mathematically, increasing v increases r; this makes sense because it should be harder to do if r is bigger, so you should have to run faster. Increasing g decreases r, and that makes sense because, e.g., if g were 0, you could do this with an unlimited r. Increasing μ_s increases r; this makes sense because, e.g., in the absence of friction, it should be impossible to do the stunt, and the maximum r should be 0, i.e., it shouldn't be possible for any positive r. (d) The result is 50 meters, which is counterintuitively huge.

9-16 (a) Newton's second law applied to the driver's body gives

$$a = \frac{F_{\text{total}}}{m}.$$

Their acceleration is

$$a = \frac{v^2}{R}.$$

The total force acting on the driver's body is

$$\begin{aligned} F_{\text{total}} &= mg - N \\ &= \frac{2}{3}mg. \end{aligned}$$

Eliminating a and F_{total} gives

$$v = \sqrt{\frac{2}{3}gR}.$$

(b)

$$\frac{\mathrm{m}}{\mathrm{s}} = \sqrt{\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)\cdot\mathrm{m}}$$

(c) If g is bigger, v gets bigger. That makes sense, because if gravity is stronger, the car has to be going faster to get the same feeling of partial lift-off. If R is bigger, v gets bigger. That also makes sense, because a very large R indicates a nearly flat road, and you can't get any lift-off on a flat road.

9-17 (a) Newton's second law in the vertical direction:

$$T\sin\theta - Mg = 0.$$

Newton's second law in the horizontal direction:

$$T\cos\theta = Mv^2/R.$$

Eliminating T and solving for v gives

$$v = \sqrt{\frac{gR}{\tan\theta}}$$

(b)

$$\frac{\mathrm{m}}{\mathrm{s}} = \sqrt{\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)\cdot\mathrm{m}}$$

(c) If g is bigger, v gets bigger. That makes sense, because if gravity is stronger, Tommy has to whirl the brick faster to get it to rise as high. If R is bigger, v gets bigger. That also makes sense, because intuitively this would be very hard to do with a long rope. If θ gets bigger, v gets smaller. That makes sense, because the faster he whirls the brick, the higher it will rise. As $\theta \to 0$, $v \to \infty$, so Tommy can never get the brick to spin in a perfectly horizontal plane.

9-18 Let r be the distance of the heart from the axis, and let f be the rate of rotation in revolutions per second, $f = 296/360\ \mathrm{s}^{-1}$. The time for one rotation is $1/f$. The speed at which the heart is circling the axis is (circumference)/(time) $= 2\pi r f$. We then have $a/g = v^2/r$, or

$$\frac{a}{g} = \frac{4\pi^2 f^2 r}{g} = 1.8.$$

10-1 (a) Since he's on the earth's surface, either $F = mg$ or $F = GMm/r^2$ will work. $mg = 590$ N.
(b) Same as in part a (Newton's third law).
(c) $F = Gm_{Laurie}m_{Roy}/r^2 = 1.2 \times 10^{-7}$ N.
(d) Her distance from the sun is essentially the same as the center-to-center distance between the sun and the earth, 1.5×10^{11} m, so $F = Gm_{sun}m_{Laurie}/r^2 = 0.38$ N.

10-2 The relevant data are in the back of the book. Shortcut: for parts a, d and e, it's accurate enough to assume the distance between the moon and the sun equals the distance from the earth to the sun.

(a) sun on moon: -4.35×10^{20} N;
(b) earth on moon: 1.98×10^{20} N;
(c) sun on earth: -3.52×10^{22} N;
(d) total force on sun: $|\text{part a}|+|\text{part c}| = 3.56\times 10^{22}$ N;
(e) total force on moon: $a + |b| = -2.37 \times 10^{20}$ N;
(f) total force on earth: $-|b|-|c| = -3.54\times 10^{22}$ N

10-3 The earth's force on the moon points up and to the right, and has magnitude 1.99×10^{20} N, as calculated in problem #2. The sun's distance from the moon is slightly different than in problem #2, but not enough to cause a significant change in the force, which points to the left, and has magnitude 4.36×10^{20} N.

For vector addition, we need to find the x and y components. Let x be to the right, and let y be up.

$$\begin{aligned}
F_{sun,x} &= -4.36 \times 10^{20}\ \text{N} \\
F_{earth,x} &= (1.99 \times 10^{20}\ \text{N}) \cos 45^\circ \\
F_{total,x} &= -2.95 \times 10^{20}\ \text{N} \\
F_{sun,y} &= 0 \\
F_{earth,y} &= (1.99 \times 10^{20}\ \text{N}) \sin 45^\circ \\
F_{total,y} &= 1.41 \times 10^{20}\ \text{N}
\end{aligned}$$

The magnitude of the total force is $\sqrt{F^2_{total,x} + F^2_{total,y}} = 3.27 \times 10^{20}$ N.

10-4 First we take Newton's law of gravity, $F = Gm_1m_2/r^2$, and throw out the constant factor of Gm_1m_2, turning it into the proportionality $F \propto 1/r^2$, or $r \propto 1/\sqrt{F}$. Changing this into a statement about ratios, we have $r_1/r_2 = \sqrt{F_2/F_1} = 10$. At the earth's surface, r equals the earth's radius. To make it ten times bigger, we have to put the rocket nine earth radii above the surface.

10-5 Kepler's law of periods says $T \propto R^{3/2}$, and solving for R gives $R \propto T^{2/3}$. Converting this into a statement about ratios gives $R_1/R_2 = (T_1/T_2)^{2/3} = 2.9$, or 3 if we round off to one sig fig.

10-6 Not if they are well above the planet's atmosphere, where there is no air friction. They do not need any force other than the planet's gravity to keep them in orbit, any more than our moon needs a power plant to keep it going around the earth. As Kepler figured out, the only possible orbits are circles, parabolas, ellipses and hyperbolas. No spirals! If, however, they were low enough to get some air friction, it would be possible for their orbit to decay in a spiral.

The people who wrote these plots were probably reasoning based on a kind of intuitive Aristotelianism: the incorrect belief that things "naturally" slow down if there is no forward force on them.

In the special case where the orbit is a circle (not an ellipse), we can also relate this to what we learned in chapter 9 about circular motion. Only an inward force is required in order to cause circular motion, and in this situation the inward force is being supplied by the planet's gravity. If the ship is initially moving at the right speed to be in a circular orbit, there is no reason why it would slow down, and "the right speed" means the speed at which the planet's gravity is just the right amount of inward force to make the ship go in a circle.

10-7 (a) Let M be the mass of the whole planet, and m the mass of some object on the surface. Then

$$\begin{aligned}
g &= F/m - v^2/r \\
&= (GMm/r^2)/m - (2\pi r/T)^2/r \\
&= GM/r^2 - 4\pi^2 r/T^2 \qquad .
\end{aligned}$$

The mass, m, of the object has canceled out, which is good because otherwise we would not be able to solve the problem. Eliminating $M = \rho V = 4/3\pi r^3\rho$, we get

$$g = \frac{4}{3}\pi G r\rho - \frac{4\pi^2 r}{T^2} \qquad .$$

(b) Plugging in, the second term is 3.4×10^{-2} m/s^2. We know that the first term is 9.8 m/s^2, so the fractional change is 0.35%.

(c) $g = 0 = (4/3)\pi G r\rho - 4\pi^2 r/T^2$, so $T = \sqrt{\frac{3\pi}{G\rho}}$,

which does not depend on r.
(d) Plug in.
(e) Solving for ρ, $\rho = 3\pi/GT^2 = 1.4 \times 10^{11}$ kg/m^3 for $T = 1$ s.
(f) Plugging $\rho = 10^{17}$ kg/m^3 in to the equation in part c, we get $T \approx 1$ ms. If they went faster, they'd fly apart.

10-8 Your path to Mars is half of an elliptical orbit. Kepler's third law says that that the time required for an elliptical orbit is proportional to its long axis to the power 3/2. It is convenient to work the problem in units of astronomical units (a.u.) and years, where 1 a.u. is the average distance from the earth to the sun. The long axis of our ellipse equals the sum of the radii of the earth's and Mars' orbits, which is 1.00 a.u.+1.53 a.u.=2.53 a.u. Comparing with the earth's orbit, which takes one year, we have

$$\frac{\text{time for half of your orbit}}{\text{time for half the earth's orbit}} = \left(\frac{\text{long axis of your orbit}}{\text{diameter of earth's orbit}}\right)^{3/2}$$

The quantities in the denominators are 0.5 years and 2.0 a.u., and solving for the unknown we find that the outward leg of the trip takes 0.70 years.

10-9 (a) At the center of the earth, your weight is zero by symmetry, since all the outward forces cancel out. That makes one suspect weight would decrease steadily with depth. More quantitatively, the shell theorem says that the part of the earth at radii greater than yours exerts no net force on you, while the part at smaller radii exerts a force as if it was concentrated at the earth's center. If you assume constant density for the earth, then the amount of the earth's mass pulling on you is proportional to r^3, where r is your distance from the earth's center. Your weight is proportional to this mass divided by r^2, so under the assumption of constant density, your weight varies as r^3/r^2, i.e. it is proportional to r.
(b) Since this observation contradicts the logical conslusion of part a, based on the assumption of constant density, the assumption must have been wrong. The earth's density must be greater as you go deeper.

10-13 (a) Differentiating twice, we find

$$v = kpt^{p-1}$$
$$a = kp(p-1)t^{p-2}$$

This acceleration is given by Newton's second law:

$$a = F/m$$
$$= GMr^{-2}$$

To relate the two expressions for the acceleration, we need to get them both in terms of the same variable, say r:

$$t = (r/k)^{1/p}$$
$$a = kp(p-1)(r/k)^{\frac{p-2}{p}}$$
$$= GMr^{-2}$$

If the powers of r are to be the same, then $(p-2)/p = -2$, or $p = 2/3$. Setting the constants out in front equal to each other, we find $k = (9GM/2)^{1/3}$.

(b) It approaches zero. If you shoot the bullet straight up at exactly escape velocity, then it slows down more and more, but never quite stops and falls back down.

(c) This is a little tricky, because we have an equation for v, but it's in terms of time. We want the initial velocity, i.e., the velocity at the earth's surface, so we need to find the time t_o at which the bullet's distance from the center of the earth is r_o, the same as the earth's radius. The initial time is *not* zero! We already have the relation between t and r, so with a little algebra we find

$$t_o = (r_o/k)^{3/2}$$
$$v_o = \frac{2k^{3/2}}{3r_o^{1/2}}$$
$$= 1.1 \times 10^4 \text{ m/s} \qquad .$$

10-14 Let M be the mass of the mysterious object, and m the mass of one of the stars orbiting around it. The relevant equations are Newton's second law,

$$a = \frac{F}{m} \qquad ,$$

Newton's law of gravity,

$$F = \frac{GMm}{r^2} \qquad ,$$

and the circular-motion equation

$$a = \frac{v^2}{r} \qquad .$$

The acceleration a isn't given, and we don't want to know it either, so let's eliminate it by setting the first and third equations equal to each other. We're now down to two equations:

$$\begin{aligned} \frac{F}{m} &= \frac{v^2}{r} \\ F &= \frac{GMm}{r^2} \end{aligned}$$

The force is also something we don't know and don't care about, so we'll get rid of it by substituting the second equation into the first:

$$\begin{aligned} \frac{GMm/r^2}{m} &= \frac{v^2}{r} \\ \frac{GM}{r^2} &= \frac{v^2}{r} \\ M &= \frac{v^2 r}{G} \\ &\sim 10^{36}\ \text{kg} \qquad , \end{aligned}$$

which is something like a million times the mass of the sun!

10-18 (a) Yes. The two masses are treated in a symmetric manner in the equation. It doesn't matter which one you label 1 and which one you label 2. (b) The force on the greater mass would be greater, but not greater in proportion to its mass. For example, if the more massive object had four times the mass, the force on it would only be double. According to $a = F/m$, a heavier object would therefore fall with a smaller acceleration. (c) The raindrop's mass is less by a factor of 10000, but the force on it would be smaller by a factor of only 100, so its acceleration would be 100 times greater, i.e. 1000 m/s^2. Raindrops falling from any significant height would be as deadly as bullets! (d) Both halves would suddenly start accelerating more rapidly, and the lighter one would pull away from the heavier one. (e) Instead of raising both masses to the 1/2 power, we'd have to raise them to some power greater than 1, e.g. make the force proportional to the product of the cubes of the masses.

10-20 Looking up the masses of the earth and moon in the back of the book, we find that the earth is 81.2 times more massive than the moon. The cancellation point must therefore lie much closer to the moon than to the earth. Not 81.2 times closer, though, because gravity depends on the square of the distance. The distance to the moon only has to be $\sqrt{81.2} = 9.01$ times smaller than the distance to the earth, i.e., almost exactly 9/10 of the way from the earth to the moon.

10-21 (a)

$$\begin{aligned} \frac{g_U}{g_E} &= \frac{m_U/r_U^2}{m_E/r_E^2} \\ &= \left(\frac{m_U}{m_E}\right)\left(\frac{r_E}{r_U}\right)^2 \\ &= 0.91 \end{aligned}$$

(b) It's surprising that Uranus's gravitational field is weaker than Earth's, since Uranus is a giant planet. However, "giant" means two things: large in size and large in mass. The large size means that if you were at the visible surface of Uranus (the cloud-tops), you'd be very far away from most of its mass. The exact result of this competition between mass and distance depends on how dense the planet it. Uranus is far less dense than Earth.

10-22 It's tempting to say that the gravity on the space station is zero, since we see the astronauts floating around on TV. But that can't be true, because Newton's law of gravity implies $g \propto 1/r^2$, which could only be zero if r were infinite. (By the shell theorem, this holds as long as we're on or above the surface of the Earth.) The astronauts just seem weightless because they're orbiting the earth in the same orbit as the space station that surrounds them, so they move along with it. Let r be the radius of the Earth, which, by the shell theorem, is also the distance that appears in Newton's law of gravity for people living on the Earth's surface. Let primed variables refer to the space station, so that in particular r' is its distance from the center of the earth.

$$\begin{aligned} g' &= g\left(\frac{r}{r'}\right)^2 \\ &= g\left(\frac{r}{r+h}\right)^2 \\ &= g\left(\frac{1}{1+h/r}\right)^2 \qquad , \end{aligned}$$

where h is the space station's height above the earth's surface. Since h/r is small, the quantity in parentheses won't be too different from one, and g' won't be very different from g. Looking up r, we find g' to be about 8.8 m/s^2.

10-23 Let the positive x axis be toward the sun, and the positive y axis be toward the top of the figure. The sun's force then has a positive x component and a zero y component. Using Newton's law of gravity, we find

$$\begin{aligned} F_{s,x} &= 0.096\ \text{N} \\ F_{s,y} &= 0 \qquad . \end{aligned}$$

For Jupiter's force,

$$\begin{aligned} F_{J,x} &= (GMm/r^2)\cos 95.2529^\circ = -1.015\ \text{N} \\ F_{J,y} &= (GMm/r^2)\sin 95.2529^\circ = 11.044\ \text{N} \end{aligned}$$

The sun's force is about a thousand times smaller than Jupiter's. That makes sense, because although the sun's mass is greater than Jupiter's, the sun is also hundreds of times farther away, and the $1/r^2$ in Newton's law of gravity means that it depends very strongly on distance. As a further check, we also observe that Jupiter's force has a small negative x component and a large positive y; this makes sense geometrically.

The total force has components

$$\begin{aligned} F_x &= F_{s,x} + F_{J,x} = -0.919\ \text{N} \\ F_y &= F_{s,y} + F_{J,y} = 11.044\ \text{N} \end{aligned}$$

Its magnitude is 11.082 N, which is just slightly less than the 11.091 N magnitude of Jupiter's force alone. It makes sense that it's less. If the two forces had been in the same direction, the magnitude of the total would have been more than the magnitude of the individual forces. If they'd been perpendicular, the total would have been given by the Pythagorean theorem applied to the individual forces, and it would have been very slightly greater than Jupiter's force. In reality, the angle between the two forces is not 90 degrees, it's somewhat more than 90 degrees. This causes some cancellation in the x components.

10-24 (a) Combining Newton's law of gravity $F = GMm/r^2$ with Newton's second law $a = F/m$ gives $a = GM/r^2$, and if the motion is to be circular, then this has to equal v^2/r, so

$$\frac{v^2}{r} = \frac{GM}{r^2} \qquad .$$

We then have $v = \sqrt{GM/r}$, and since the volume of a sphere is $(4/3)\pi r^3$, we have $M = (4/3)\pi r^3 \rho$ and

$$v = r\sqrt{(4/3)\pi G\rho} \qquad .$$

(b) Solving for r and plugging in numbers gives about 3 km.

10-25 As the probe enters the interior of B, the shell theorem says that the probe feels zero force from B, but feels the same force from A that it would have felt if A's mass had been concentrated at its center. From this we can predict that the probe will curve to the right and speed up. Furthermore, since the probe's initial velocity is small, we can tell that this segment of the probe's orbit will be an ellipse, not a parabola or hyperbola.

The probe will then enter the region where the two clouds' interiors overlap. In this region, the shell theorem tells us that the probe feels no force from either cloud. It moves in a straight line at constant speed.

It will then emerge into the region that is interior to A but exterior to B. It now feels a force from B but none from A. The force from B is now probably more or less directly backward, which will decelerate the probe somewhat, but without more detailed data there is not much more we can say beyond this point.

10-26 (a) If we divide the earth up into a central sphere that's deeper than P and an outer part that's shallower than P, then the shell theorem tells us that the outer part doesn't contribute anything to the field at P, and that the contribution of the inner part is the same as if it had been concentrated at the center. Because mass is proportional to volume, and volume is proportional to the cube of the linear dimensions, the mass of the inner part is 1/8 the mass of the earth. If this was the only factor, then we would have $g_P/g_S = 1/8$. But P is also closer to the center than S by a factor of 2, and since Newton's law of gravity depends on $1/r^2$, this brings the result back up by a factor of 4. The result is $g_P/g_S = 1/2$.

(b) Generalizing the result of part a, we have $g/g_S = r/r_S$. At $r = 0$, this produces the correct result, $g = 0$, because the field must cancel out by symmetry at the earth's center. At $r = r_S$, it gives $g = g_S$, which also makes sense.

10-27 By the shell theorem, the mantle gives no contribution, and we can take the entire mass of the core (including both its layers) to be concentrated at the center. Let m be the mass of the core and r its outer radius.

$$\begin{aligned}\frac{g_G}{g_s} &= \frac{m}{M}\cdot\left(\frac{R}{r}\right)^2\\ &= (0.307)(1/0.55)^2\\ &= 1.01\end{aligned}$$

Surprisingly, it's almost the same as the field at the surface.

10-28 The acceleration is $a = \omega^2 r = (2\pi/T)^2 r = 4.7\times10^{-5}$ m/s^2. If the astronaut stands on the inside of the outer bulkhead, the force pushing up on her feet is what produces her circular motion. This force is $F = ma = 2.8\times10^{-3}$ N. If she adopts the rotating frame, she interprets this "upward" force as canceling the "downward" fictitious force of apparent gravity. This fictitious force is roughly equivalent to the weight of a raisin.

11-2 No. The mass can't be negative, and neither can v^2.

11-3 If she can run the 100 m dash in about 10 s, then she's going about 10 m/s. If her mass is 60 kg, then her kinetic energy is about 3000 J (one sig fig).

11-4 Converting the speeds to mks, we have 18 m/s and 36 m/s. (a) $(1/2)mv^2 = 2.4\times10^5$ J; (b) Four times greater, 9.6×10^5 J. In this sense, driving twice as fast is four times more dangerous, not twice as dangerous as you might think.

11-5 The total amount of energy required to kill her is

$$(60\text{ kg})\left(\frac{1\text{ J}}{0.24\text{ g}\cdot{}^\circ\text{C}}\right)\left(\frac{10^3\text{ g}}{1\text{ kg}}\right)(6^\circ\text{C}) = 1.5\times10^6\text{ J}$$

Solving the equation $P = \Delta E/\Delta t$ for $\Delta t = \Delta E/P$, we find that she can survive for 7500 s=120 min.

11-6 This is only an order-of-magnitude estimate, so let's make some simplifying approximations. We'll assume the surface area of the oceans is the same as the surface area of the planet ($4\pi R^2$), and that the density of ice is about the same as that of water, 1000 kg/m^3. Then the amount of energy required is

$$\begin{aligned}E &= (1000\text{ kg/m}^3)(10\text{ m})(4\pi R^2)(3\times10^3\text{ J/kg})\\ &\sim 10^{23}\text{ J}\end{aligned}$$

Dividing by 10 years (about 3×10^8 seconds), we find that the power is on the order of 1% of the sun's light output. This is a surprisingly small amount. The equilibrium is extremely delicate.

11-7 Force is a transfer of momentum from one object to another. Equivalently, we can describe force as an interaction between two objects. Whichever way we say it, it takes two to tango — you can't have a force unless two objects are involved. There is no force when the bullet is in the air. There is a force when the bullet is passing through the book. This force will cause the bullet to lose momentum and the book to gain some.

Energy, on the other hand, exists at all times in this story. The bullet has kinetic energy before, during, and after the time when it passes through the book. (While it's passing through the book, some of this energy is converted to heat and sound.)

11-10 The earth is 81 times closer to the center of mass. They both orbit this point once a lunar month, so the earth's speed is 81 times smaller. Kinetic energy is proportional to mv^2, so the earth's kinetic energy is $(81)(1/81)^2 = 1/81$ of the moon's.

11-12 We're testing conservation of energy,

$$E_{total,i} \quad =? \quad E_{total,f}.$$

The only form of energy we can determine from the photo is kinetic energy, so this amounts to testing whether

$$KE_{1,i} \quad =? \quad KE_{1,f} + KE_{2,f},$$

where the initial time is before the collision and the final time is after the collision is over. Using $KE = (1/2)mv^2$, and taking advantage of the fact that the masses are equal, this becomes

$$v_{1,i}^2 \quad =? \quad v_{1,f}^2 + v_{2,f}^2.$$

Now $v = d/t$, where t is the time between flashes, and the common factor of t^{-2} can be divided out on both sides, so we end up with

$$d_{1,i}^2 \quad =? \quad d_{1,f}^2 + d_{2,f}^2,$$

where d is the distance between one position and the next. That is, the square of the distance is our measure of energy.

But rather than measuring directly from one position to the next, it's more accurate to measure the distance over several of the time intervals in succession. In four flashes, the incoming ball moves 6.0 cm. In the same amount of time after the collision, it moves 2.8 cm, and the target ball moves 5.0 cm.

Since we're leaving out lots of factors that have units, I'll write the numbers below without units.

$$\begin{aligned} 6.0^2 \quad &=? \quad 2.8^2 + 5.0^2 \\ 36.0 \quad &=? \quad 7.8 + 25.0 \\ 36.0 \quad &=? \quad 32.8 \end{aligned}$$

There appears to have been a slight loss of energy, but it's really at the limit of our ability to measure things from the figure. We also know that there will be some extra energy after the collision in the form of heat and sound. The only thing that would really disprove conservation of energy in this collision would be if the final KE was *greater* than the initial KE, and if the increase was significant given the limited accuracy of the measurements.

11-13 (a) The total initial kinetic energy is 0.5 J+0 J, and the total final kinetic energy is 0 J+0.5 J. (b) Before the collision, we see the balls moving toward each other, each moving at a speed of 0.50 m/s. The total kinetic energy is 0.12 J+0.12 J. After the collision, the balls are moving apart at 0.50 m/s. Their velocity vectors have been swapped (i.e. each one reverses itself), but this has no effect on the energies, so the total is still the same.

11-14 (a) Plugging the data into $KE = (1/2)mv^2$ gives a kinetic energy of 23 kilojoules. (b) The hundred-times-greater mass increases the KE by a factor of 100, but KE depends on the square of the velocity, so reducing the speed by 1/100 cuts the kinetic energy by 1/10,000. The combined effect is a factor of 100/10,000=1/100. The boulder has 100 times less KE than the foam.

(c) Our intuition doesn't do well with things that are squared. We don't expect the foam to have so much kinetic energy, but its big speed, squared, results in a really really big KE.

11-15 As in the example of the water faucet, conservation of mass says that a narrower stream corresponds to a higher velocity. The water in the discharge pipe is therefore moving faster than the water in the drive pipe, in inverse proportion to its smaller cross-sectional area. A kilogram of water with a high velocity has a higher KE than a kilogram of water with a low velocity, and therefore it will be able to rise higher, converting that KE into a lot of PE. Note, however, that we're not really getting anything for free here. The amount of water that gets up to the top reservoir is a lot less than the amount of water that was allowed to flow into the big pipe. Most of the water is wasted into the ditch way down at the bottom. People use this device in areas where there's a free source of water, but no electricity to run a pump.

11-16 (a) We have to heat it by 20 degrees, melt it, heat it by 100 degrees, boil it, and then heat it by another 37 degrees. The result is

$$\begin{aligned}&(20)(2.05\text{ J})\\&+333\text{ J}\\&+(100)(4.19\text{ J})\\&+2500\text{ J}\\&+(37)(2.01\text{ J})\\&=3.4\times10^3\text{ J}\qquad.\end{aligned}$$

Surprisingly, the majority of the energy is used in a single step of the process, the boiling.

(b) To minimize the amount required, we want the initial temperature of the water to be the maximum, 100°C, and similarly the ice should be at 0°C. At the end of the process, we'll just have one big sample of water at some intermediate temperature, and to minimize the amount of water, we want this temperature to be as low as possible, zero degrees. Conservation of energy gives

$$\begin{aligned}\Delta E&=0\\0&=-m(100^\circ\text{C})(4.19\text{ J/g/}^\circ\text{C})+(1\text{ g})(333\text{ J/g})\\m&=0.795\text{ g}\end{aligned}$$

11-17 As in the examples in ch. 1, we don't try to estimate the mass of the wing directly. Instead, we estimate linear dimensions and get mass from that. Let's approximate a fly's wing as a square 3 mm on a side. I don't know the thickness, but since flies' wings are translucent, it must be much less than the thickness of a piece of paper. I'm going to guess that the thickness is 0.1 mm. This gives a volume of about 1 mm^3, and if we assume the same density as water, we get a mass of 10^{-6} kg.

Let's say that a fly's wing beats 100 times a second, which would be of the right order of magnitude to match the fact that the buzzing is in the audible range of human hearing. Then the speed of a wing would be about $(3\text{ mm})/(10^{-2}\text{ s})\approx0.3$ m/s.

The result for the kinetic energy is about 10^{-7} J.

11-18 As always (see sec. 1.4), we avoid estimating mass or volume directly and instead estimate linear dimensions. Let's say a blade of grass is 0.1 cm thick, 1 cm wide, and 10 cm tall. Then its volume is about 1 cm^3, and if it has the same density as water, then its mass is about 1 g, or 10^{-3} kg.

For the velocity, let's say it grows 1 cm in 10 days. This gives 10^{-8} m/s.

Putting these factors together, we have $KE=(1/2)mv^2\sim10^{-19}$ J.

12-1 Yes, if it's below the reference level you arbitrarily chose as $PE=0$.

12-2 Kinetic energy depends on how fast the ball is moving. It's moving fastest at the bottom, so that's where it has the maximum kinetic energy. Energy is conserved, so it's trading KE and gravitational energy back and forth, and it has the most gravitational energy when it has the least KE, which is at the top when it's momentarily at rest.

12-3 (a) Magnetic energy plus kinetic energy stays the same, so if it's speeding up, the KE is increasing and the magnetic energy must be decreasing. (b) By similar reasoning, magnetic energy must be increasing as they approach each other.

12-4 (a)

$$\begin{aligned}
PE_i &= PE_f + \text{(energy of boiling)} \\
PE_i &= PE_f + mE_b \\
\Delta PE_i &= -mE_b \\
F_{grav}\Delta y &= -mE_b \\
mg\Delta y &= -mE_b \\
\Delta y &= -\frac{E_b}{g}
\end{aligned}$$

(b)

$$\begin{aligned}
\text{m} &\overset{?}{=} \frac{\text{J/kg}}{\text{m/s}^2} \\
&\overset{?}{=} \frac{\text{kg}\cdot\text{m}^2/\text{s}^2/\text{kg}}{\text{m/s}^2} \\
&\overset{?}{=} \frac{\text{m}^2/\text{s}^2}{\text{m/s}^2} \\
&= \text{m}
\end{aligned}$$

12-6 Conservation of energy tells us that the initial energy equals the final energy. What times should we pick as initial and final? Our goal is to find out what height the bike has to start from, so the initial time should be when the bike is starting out, at the top of the ramp. The height is determined by the requirement that the bike not drop off the loop, even at the top, so to connect that to the other information, we should pick the final moment to be when the bike is at the top of the loop, at a height of $2r$.

First let's write conservation of energy in its most general form, and then make it more and more specific:

$$\begin{aligned}
E_{total,i} &= E_{total,f} \\
PE_i + KE_i &= PE_f + KE_f \\
mgh + 0 &= mg(2r) + \frac{1}{2}mv^2 \\
h &= 2r + \frac{v^2}{2g}
\end{aligned}$$

The height depends on the speed v the bike needs at the top of the loop. But how fast *does* the bike have to be moving when it's at the top? If it was just barely about to lose contact with the loop, then the only force on it would be gravity, so its downward acceleration would be $a = F_{grav}/m = (mg)/m = g$. For circular motion, the inward acceleration is v^2/r, so we have $g = v^2/r$, and $v = \sqrt{gr}$. Plugging this into the equation for h, we have

$$\begin{aligned}
h &= 2r + \frac{v^2}{2g} \\
&= 2r + \frac{r}{2} \\
&= \frac{5}{2}r
\end{aligned}$$

12-8 (a) Each atom in the hoop travels the full circumference of the hoop in time T, so its speed is $2\pi r/T$. Plugging this into $KE = mv^2/2$ gives the result claimed.

(b) As the hoop rolls down the slope, the distance its center travels in one period equals the circumference. Therefore the hoop's center-of-mass speed is the same as the speed calculated in part a, $2\pi r/T$. That means that the hoop's KE due to center-of-mass motion is the same as its KE due to rotation. It's as though the hoop had double its normal inertia.

12-10 (a) If the water lost potential energy and the energy was just destroyed, then energy wouldn't be conserved. The only reason Joule thought there would be a temperature effect was that he was already convinced that energy was conserved. By making surprising predictions and then proving them to be correct, he could convince people that his theory was right.

(b) A joule is defined as the amount of heat needed to raise 0.24 g of water by 1°C. If 0.24 g falls 50 meters, the amount of potential energy released is $(2.4\times10^{-4}\text{ kg})(9.8\text{ m/s}^2)(50\text{ m}) = 0.1$ J. If all this energy is converted to heat, then it heats that particular amount of water by 0.1°C.

(c) We had to assume all the energy went into heating. There are three main effects I can think of. First, the water at the top and bottom of the falls might be moving, in which case there would be a third form of energy involved, kinetic energy. If the pool at the bottom is still, whereas the water above the top is moving rapidly before it goes over the edge, then there would be extra heating. On the other hand, waterfalls are noisy, so energy is released into sound waves at the bottom; this effect would tend to reduce the amount of heat production. The third effect is that water might evaporate in mid-air, further reducing the heat produced.

12-12 At the atomic level, the helium's heat energy is the random motion of the helium atoms. Cylinder B's atoms are moving at the same average speed as A's, since they're at the same temperature, but B has twice as many atoms, so its total KE is twice as much.

12-13 Since $KE = (1/2)mv^2$, we have $v \propto 1/\sqrt{m}$, so the ratio of the rates at which the two gases leak out is $\sqrt{14.01/16.00} = 0.9357$, with the oxygen leaking out more slowly.

12-14 When the sweat evaporates, electrical potential energy is increasing, because as the sweat changes from a liquid to a gas, its molecules, which are attracting each other electrically, get farther away. By conservation of energy, some other form of energy must be decreasing: heat.

If you wipe the sweat off with a towel, you lose the cooling effect. The water evaporates after it's out of contact with you, so it doesn't cool you off. As a method of cooling yourself, it's no more effective than spitting!

12-15 Kinetic energy doesn't depend on the direction of the motion, so since both pennies start out at the same speed, they both start out with the same kinetic energy. They also start out at the same height, so they have the same gravitational energy and the same total energy. Energy is conserved, so if they start out with a certain amount of energy, that's the same amount of energy they'll always have; their total energies will always be equal to each other. Although the impacts with the ground will happen at different times, we don't care about that. On impact they have the same gravitational energy and the same total energy, so they must have the same kinetic energy on impact as well. They hit the ground with equal kinetic energies and equal speeds.

12-16 (a) The distance traveled by each atom in one revolution is $2\pi r$, so every atom is traveling at a speed of $v = (2\pi r)/T = \omega r$. Substituting this into $K = (1/2)mv^2$ gives the claimed result.

(b) The quantity ω has units of s^{-1}, inverse seconds, so $m\omega^2 r^2$ has units of $\text{kg}{\cdot}\text{s}^{-2}{\cdot}\text{m}^2$, equivalent to joules.

(c) Let's use lowercase letters for the hoop, uppercase for the block. After descending by a vertical distance h, conservation of energy gives

$$mgh = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}mv^2$$

and

$$Mgh = \frac{1}{2}MV^2 \qquad ,$$

and dividing out the masses and substituting $\omega = v/r$ simplifies these to

$$gh = v^2$$

and

$$gh = \frac{1}{2}V^2 \qquad .$$

These are similar in form to the kinematic equation $v_f^2 = 2ax$ for the final velocity of an object that accelerates from rest, traveling a distance x. Although one equation has h, the vertical distance, and the other has x, the distance along the slope, the relationship between these is the same for the block and the hoop, so it's clear that inserting a factor of 2 on the v^2 side of the equation corresponds to a change in acceleration by a factor of 2.

12-17 (a) First we set up an equation that states conservation of energy:

$$E_{total,i} = E_{total,f}$$
$$KE_{1,i}+PE_{1,i}+KE_{2,i}+PE_{2,i} = KE_{1,f}+PE_{1,f}+KE_{2,f}+PE_{2,f}$$

Subtracting $PE_{1,i}$ and $PE_{2,i}$ from both sides gives:

$$KE_{1,i}+KE_{2,i} = KE_{1,f}+KE_{2,f}+\Delta PE_1+\Delta PE_2$$

Since the masses start from rest, we have:

$$0 = KE_{1,f} + KE_{2,f} + \Delta PE_1 + \Delta PE_2$$

We no longer have any variables that occur in the equation with both i and f subscripts, so we can stop writing them without any risk of confusion.

$$\begin{aligned}
0 &= KE_1 + KE_2 + \Delta PE_1 + \Delta PE_2 \\
0 &= \frac{1}{2}Mv^2 + \frac{1}{2}mv^2 + Mg\Delta y_1 + mg\Delta y_2 \\
0 &= \frac{1}{2}(M+m)v^2 + Mg(-h) + mgh \\
v &= \sqrt{\frac{2(M-m)gh}{M+m}}
\end{aligned}$$

(b) Comparing the result of part a with $v^2 = 2ah$ gives $a = (M-m)g/(M+m)$, which is the result found before using Newton's laws.

12-18 (a) There are three forms of energy present: gravitational PE, electric PE in the rope, and the climber's KE. The climber's KE is zero at both the initial time and the final time, so we can ignore it. We then have

$$PE_{g,i} + PE_{e,i} = PE_{g,f} + PE_{e,f},$$

or

$$\begin{aligned}
PE_{e,i} &= \Delta PE_g + PE_{e,f} \\
0 &= -mgx\sin\theta + \frac{1}{2}kx^2.
\end{aligned}$$

The result is $k = (2mg/x)\sin\theta$.

(b)

$$\frac{\mathrm{N}}{\mathrm{m}} = \frac{\mathrm{kg{\cdot}m/s^2}}{\mathrm{m}}$$

(c) The result increases with m, g, and θ, and decreases with x. A heavier climber, greater gravity, or higher angle will all make it harder to stop the climber in time, so the dependence on these variables makes sense. A greater x makes it easier to stop the climber in time, so this also makes sense.

13-1 (a) The amount of KE to be converted into heat is the same in both cases, so the amount of work that has to be done is the same in both cases. Since work equals force times distance, and the work and force are the same, the stopping distance is the same.

(b) Since the stopping distances are equal, if you watched videos of the two boats stopping, the videos would look exactly the same except for a rescaling of the time. That is, the slower boat's motion would look like the faster one's, but in slow motion. Therefore the time for the slower boat to stop will be greater.

13-2 (a) +, foot moves same direction as foot's force on ground

(b) +, force and motion are both forward

(c) −, force is backward, motion is forward

(d) −, force is up and motion is down

13-3 (a) $KE = \frac{1}{2}mv^2$, so if the average KE is a fixed number, the average velocity is proportional to $m^{-1/2}$. The lighter He atom is moving $\sqrt{8} = 2.8$ times faster.

(b) He atoms are always speeding up and slowing down due to collisions, but because of their small mass, their average speed is very high, and a few unusual ones are moving very very fast — faster than escape velocity, meaning they leave the earth and never fall back.

13-4 (a) As Weiping raises the rock, the sign of the force and the sign of d are the same (both are up), so she does 1 joule of work. On the way down, her force is up and the motion is down, so she does −1 joule of work. The total is zero.

Bubba's force was in the same direction as the motion during both halves of his motion, since, unlike gravity, the force of friction reversed directions on the way back. Bubba did 2 J of work.

(b) Work is defined as a transfer of energy. Weiping added gravitational PE to the rock on the way up, but on the way back down, she took it back out. The rock ends up with the same gravitational PE it started with, so by the definition of work, we verify that her work on the rock is zero.

As Bubba slides the table, he's adding heat energy to it. When he slides the table back in the opposite direction, he's not refrigerating it! Therefore, based on the definition of work, we know that the total work he does is not zero.

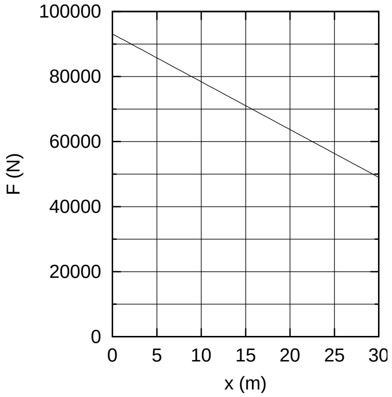
This agrees with the result from part a.

13-5 $PE = -GMm/r$, where r is the distance from the center of the earth. The earth-moon distance is so much greater than the earth's radius that the initial PE is essentially zero, and the result is pretty much the same as falling from infinitely far away. That is, almost all the speed is gained near the end of the drop. The amount of PE lost is

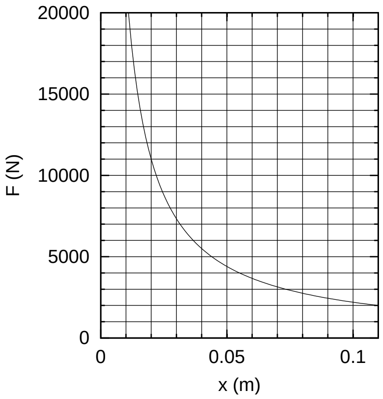
$$\begin{aligned} PE_i - PE_f &\approx 0 - PE_f \\ &= GMm/r_{earth} \qquad . \end{aligned}$$

Whatever is lost as PE is gained as KE, so for the final speed v, we have $\frac{1}{2}mv^2 = GMm/r_{earth}$. The mass, m, of the grasshopper cancels out, since all objects fall with the same acceleration, regardless of their masses. (The acceleration is not a constant 9.8 m/s^2, however, in this situation.) Solving for v, we have $v = \sqrt{2GM/r_{earth}} = 1.1 \times 10^4$ m/s.

13-7 (a) The mass of 30 m of chain is 30 m × 150 kg/m = 4500 kg. The total mass is 5000 kg+ 4500 kg = 9500 kg, so the total weight is that times g, or 9.3×10^4 N.
(b) The mass has decreased by 150 kg, to 9350 kg, giving a weight of 9.2×10^4 N.
(c) There area under the curve is about 43 rectangles, each representing 5 m×10000 N = 5×10^4 J, so the total work done is 2.1×10^6 J.
(d) 2.1×10^6 J/(4186 J/kcal) = 500 kcal.



13-8 (a) See graph.
(b) 47 rectangles × 0.01 m × 1000 N = 470 J.
(c) The engine's power is the work done per unit time, which is 6 × 2400 × 470 J/s = 1.1×10^5 W = 150 horsepower.



(d) Assuming the same value of a, if you doubled both x_1 and x_2, the x axis of the graph would be stretched out to twice the width, but since the force is inversely proportional to x, the force on the piston would be half as much, and the F axis of the graph would be shrunk to half the height. The area would not change. The argument fails for $F \propto x^{-1.4}$, because then the force would decrease by more than 50%.

13-9

$$\begin{aligned} E_i &= E_f \\ KE_i + PE_i &= KE_f + PE_f \\ 0 + PE_i &= KE_f + PE_f \\ KE_f &= PE_i - PE_f \\ &= -\int_{r_i}^{r_f} kr^{-3}\,\mathrm{d}r \\ &= \frac{k}{2}\left(r_f^{-2} - r_i^{-2}\right) \end{aligned}$$

13-10 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2t^2$.
(b) $P = dW/dt = dK/dt = ma^2t$.
(c) Four times more.

13-11 The tricky thing here is that the shell theorem works for forces, but not necessarily for potential energies. The shell theorem tells us that

$$\begin{aligned} F &= \frac{GmM_{inside}}{r^2} \\ &= \frac{Gm}{r^2}\int_0^{r'} \rho\cdot 4\pi r'^2 \,\mathrm{d}r' \qquad , \end{aligned}$$

where the integral can be visualized as dividing the part of the cloud at $r' < r$ into thin spherical shells. This force equals $\mathrm{d}U/\,\mathrm{d}r$, so

$$\frac{1}{4\pi Gm} r^2 \frac{\mathrm{d}U}{\mathrm{d}r} = \int_0^r \rho r'^2 \,\mathrm{d}r$$

and by differentiating on both sides we find

$$\frac{1}{4\pi Gm}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}U}{\mathrm{d}r}\right) = \rho r'^2 \qquad .$$

13-12

$$\begin{aligned} W &= \int_0^\infty F \,\mathrm{d}x \\ &= \int_0^\infty be^{-cx}\,\mathrm{d}x \\ &= b/c \end{aligned}$$

13-13

$$\begin{aligned} E_{total,i} &= E_{total,f} \\ PE_i &= KE_f + PE_f \end{aligned}$$

We can use the equation $PE = -GMm/r$ for the potential energy, and this equation is defined so that $PE = 0$ at $r = \infty$, so actually PE_i is zero. The potential energy is really the sum of the potential energies for the interaction of the comet with the two stars. Writing v for the final velocity, we have

$$\begin{aligned} 0 &= \frac{1}{2}mv^2 - \frac{Gm_1m}{b/2} - \frac{Gm_2m}{b/2} \\ v &= \sqrt{\frac{4G(m_1+m_2)}{b}} \end{aligned}$$

13-14

$$\begin{aligned} W_{wp} &= \int_0^x F_\parallel \,\mathrm{d}x' \\ &= \int_0^x (a+bx')\,\mathrm{d}x' \\ &= ax + \frac{1}{2}bx^2 \end{aligned}$$

By Newton's third law, the plane's force on the air is of the same strength, but in the opposite direction, and therefore $W_{wp} = -W_{pw}$.

13-15 The force is the derivative of the potential energy with respect to distance. (Actually it's minus the derivative, but all that tells you is the direction of the force, and we know it's attractive anyway.)

$$\begin{aligned} \frac{\mathrm{d}PE}{\mathrm{d}r} &= \frac{\mathrm{d}}{\mathrm{d}r}\left(gr^{-1}e^{-r/a}\right) \\ &= g\frac{\mathrm{d}}{\mathrm{d}r}\left(r^{-1}e^{-r/a}\right) \\ &= g\left[r^{-1}\frac{\mathrm{d}}{\mathrm{d}r}\left(e^{-r/a}\right) + e^{-r/a}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^{-1}\right)\right] \\ &= g\left[r^{-1}\left(-\frac{1}{a}\right)\left(e^{-r/a}\right) + e^{-r/a}\left(-r^{-2}\right)\right] \\ &= -gr^{-2}e^{-r/a}\left(\frac{r}{a}+1\right) \end{aligned}$$

13-16 The dot product was defined as

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB} \qquad ,$$

which only depends on the magnitudes of the two vectors and the angle between them. Rotating everything has no effect on the magnitudes, or on the angle of one vector relative to another, so the dot product comes out the same regardless of rotation.

13-17 The definition $\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB}$ implies immediately that $\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$, since the cosine of zero is 1. So for example if two vectors $\mathbf{A}$ and $\mathbf{B}$ only have nonvanishing x components, their dot product equals A_xB_x. Also, the cosine of 90 degrees is zero, so the other possibilities, such as $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}}$, equal zero. Now suppose we have a general dot product such as

$$\mathbf{A}\cdot\mathbf{B} = (A_x\hat{\mathbf{x}}+A_y\hat{\mathbf{y}}+A_z\hat{\mathbf{z}})\cdot(B_x\hat{\mathbf{x}}+B_y\hat{\mathbf{y}}+B_z\hat{\mathbf{z}}) \qquad .$$

The rule $\mathbf{P}\cdot(\mathbf{Q}+\mathbf{R}) = \mathbf{P}\cdot\mathbf{Q}+\mathbf{P}\cdot\mathbf{R}$ allows us to break this into nine terms, of which the three nonvanishing ones add up to $A_xB_x + A_yB_y + A_zB_z$.

13-19 (a) The definition of work is a (mechanical) transfer of energy. Changing the direction of the positive coordinate axis has no effect on the work. Energy is a scalar, so directions are irrelevant.

(b) Reversing the direction of the positive coordinate axis reverses the sign of both F and d, so $W = (-F)(-d) = Fd$ comes out the same.

13-20 In a solid, the molecules are packed in place, and aren't free to twist around very much. In a liquid, they can rotate more freely. The microwaves exert the same force F on a molecule, regardless of whether it's in a solid or a liquid, but it's not just the force that matters, since $W = Fd$. In the solid, d is small, so little energy is transferred.

13-21 To apply conservation of energy, we start by writing down the law in its most generic form,

$$E_{total,i} = E_{total,f} \qquad ,$$

and we then make it more specific by putting in the actual forms of energy involved,

$$KE_i + PE_i = KE_f + PE_f \qquad ,$$

and then substituting in the correct expressions for the kinetic and potential energies,

$$\frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f} \qquad ,$$

Now we have to decide what moments in time will be initial and final. Initial should clearly be the beginning of the motion, when the object is at distance r, so in terms of the variables used in posing the question, we have $r_i = r$ and $v_i = v$, the escape velocity, giving

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f} \qquad .$$

But what should we choose as the final moment in time? We're interested in the case where the object just barely has enough energy to keep from falling back. If the object had less than this amount of energy, it would fly away from the planet, slow down, and finally stop at some distance before falling back down. The greater the energy, the greater the distance at which it stops. When the energy is just barely enough (the velocity is exactly equal to escape velocity), we can think of this distance as being infinite, or at least extremely large. Thus, $r_f = \infty$ and $v_f = 0$. Dividing by a very large number gives a very small result, so dividing by infinity gives zero. That means that both terms on the right are zero. In other words, the total energy is zero.

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0 \qquad ,$$

It may sound strange to say that this object, moving at a gazillion miles an hour, has zero energy, but that's because the object has a huge negative potential energy to start with, which is canceled by its huge initial kinetic energy.

Solving for the escape velocity v, we have

$$v = \sqrt{\frac{2GM}{r}} \qquad .$$

The direction of the velocity doesn't matter, because the only requirement was for the object to have a certain amount of energy, and energy is a scalar. This is counterintuitive: most people would expect that it would be easier to make the object escape if it was shot straight up. One way to think about it is that if the object is launched at an angle just barely above the horizontal, it has the disadvantage that at the beginning, it's hardly getting any farther away from the earth, but it has the advantage that the force of gravity is almost perpendicular to its direction of motion, so gravity is doing almost no (negative) work on it, and is therefore hardly slowing it down.

The mass of the object has canceled out, which is why we can speak of a single escape velocity, rather than different escape velocities for different objects.

No, it is not valid to think of the object as escaping because it is moving too fast for gravity to act on it. The force of gravity only depends on the masses and the distance between them, not on their motion.

13-22 (a) We want to find the area under the graph from $x = 0$ to $x = cL$. This area can be split up into a rectangle plus a triangle sitting on top of it. To make all the math easier, let's start by leaving out the factors of L and T_o, since these just set the scale on each axis; we'll start by calculating everything as if we had $L = 1$ and $T_\text{o} = 1$, and then throw those two factors back in at the end. In these fake units, the point at the graph at the corner, where the muscle can't contract anymore, has coordinates $(c, 1-c)$. The rectange has width c, height of $1 - c$, and area $c(1 - c)$. The triangle has base c, height c, and area $(1/2)c^2$. The total area is $c - c^2/2$. Putting the factors of $L = 1$ and $T_\text{o} = 1$ back in, we find that the work is $W = T_\text{o}L(c - c^2/2)$.
(b) T has units of newtons, L meters, and c is unitless, so the result has units of newton-meters, which is equivalent to joules.
(c) The $c = 0$ indicates a muscle that is "frozen" in its extended position and can't flex at all; physically it shouldn't be able to do any work, and mathematically we do get $W = 0$ in this case. In the $c = 1$ case, the area would be that of a complete triangle with width L and height T_o, giving an area of $(1/2)T_\text{o}L$, and this is indeed what we get if we plug in $c = 1$ into the general equation.
(d) For a cylinder, we have $V = LA$, and with $T_\text{o} = kA$ the result for the work becomes $kV(c - c^2/2)$. The whole thing is strictly proportional to the muscle's volume, V, which is why bodybuilders talk about "bulking up."
(e) Converting the given data to mks, 200 cm^3 = 2×10^{-4} m^3, and $k = 10^6$ N/m^2. Plugging in, we get 96 J.

13-23 (a) The distance traveled in time Δt is $d = v\Delta t$, so the work done is $\Delta E = Fd = Fv\Delta t$, and $P = \Delta E/\Delta t = Fv$.
(b) The vertical forces cancel, so $F_N = mg$, and $F_{s,max} = \mu_s F_N = \mu_s mg$. To burn rubber, the engine has to supply $P > F_{s,max}v = \mu_s mgv$. The critical speed is $v = P/(\mu_s mg)$.
(c) W/N = J/N·s = N·m/N·s = m/s.
(d) Physically, a more powerful engine should be able to burn rubber at even higher speeds; mathematically, increasing P increases v. Physically, better traction makes it harder to burn rubber; mathematically, increasing μ_s decreases v. Increasing m or g also has the effect of increasing traction, so the dependence on these two variables also makes sense, for similar reasons.
(e) (Plug in.)
(f) The power consumed by friction is $Fv \propto v^3$. At top speed, 100% of the engine's power is being consumed by fighting air friction. At the lower speed, this is decreased by $(40/176)^3 \approx .01$.

13-24 The mechanical work done by the hand equals the area under the curve between A and B. I counted about 136 squares. Each square is 20 N tall and 0.05 m wide, so it represents an area of 1 N·m = 1 J, and the total area is 135 J. Accounting for the loss of 30% of this energy, and solving for $v = \sqrt{2K/m}$, we find 85 m/s.

13-25 As suggested in the hint, let $k = 1$, $m = 1$, $g = 1$, and also let the force of kinetic friction equal 1.

When the object is released at a, it travels a distance a while being acted on by a frictional force 1, so the work done by friction is a. This dissipates the entire initial potential energy $(1/2)ka^2 = (1/2)a^2$. This gives $a = 2$.

When the object is released at b, it travels to a position x on the far side of equilibrium at which the force of friction is canceled by the force of the spring. We're given that b is the maximum distance from which this can occur, so the force of friction must be at its maximum. Since we're given that $\mu_s = \mu_k$, this is the same as the force of kinetic friction, which is 1. The magnitude of the spring's force is $|kx| = x$, so $x = -1/$ In summary, the object travels from b to -1.

Applying conservation of energy in this case, we have $PE_i = PE_f + W_{friction}$, or $(1/2)b^2 = (1/2)1^2 + (b+1)\cdot 1$. This gives a quadratic equation, $(1/2)b^2 - b - 3/2 = 0$. The positive solution is $b = 3$.

The result is that $b/a = 3/2$.

13-26 (a) To lift the bag at constant velocity, the person must cancel out the force of gravity mg. The work done is therefore about $mgd = 50$ kJ. (b) The calculation comes out the same, and this is what we expect based on conservation of energy if the pulley is frictionless: the bag still gains the same amount of energy. (In reality there would be some friction in the pulley, so the force would be slightly more than the given value, the work would be slightly greater, and the extra energy would go into heating the pulley and the rope.)

13-27 From the work-kinetic energy theorem, the work you do on the ball equals the change in kinetic energy of the ball. $\Delta\text{KE} = (1/2)m(3v)^2 - (1/2)mv^2 = 4mv^2$, and the work done on the ball is $W = Fx$, where x is the distance over which your foot is in contact with the ball. Setting these quantities equal gives $x = 4mv^2/F$.

14-1 $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$

14-2 The people plus the boat are, roughly speaking, a closed system, since the water does not exert much horizontal force on the boat. Momentum is conserved in a closed system. At the beginning, nothing is moving and there is zero momentum. If the two people's momenta didn't add up to zero, then the boat would have to move to conserve momentum. They should make sure their momenta add up to zero, i.e., they should move in opposite directions, and the heavier person has to move more slowly.

14-3 Horizontal momentum is conserved, since there are no external horizontal forces. Let's pick masses for the planes of 1 kg and 5 kg – although the real masses must be thousands of times bigger, it won't matter as long as the ratio is 5. Let the positive x axis point east and the positive y axis north, and measure angles counterclockwise from the x axis. The magnitudes of their initial momenta are $|p_L| = 300\ \text{kg}\cdot\text{mi/hr}$ and $|p_J| = 750$ kg·mi/hr. The x components are

$$\begin{aligned} p_{L,x} &= 300\ \text{kg}\cdot\text{mi/hr} \qquad \text{and} \\ p_{J,x} &= (750\ \text{kg}\cdot\text{mi/hr})(\cos 225^\circ) \\ &= -530\ \text{kg}\cdot\text{mi/hr} \qquad , \end{aligned}$$

giving

$$p_{total,x} = -230\ \text{kg}\cdot\text{mi/hr} \qquad .$$

Similarly,

$$\begin{aligned} p_{L,y} &= 0 \qquad , \\ p_{J,y} &= -530\ \text{kg}\cdot\text{mi/hr} \qquad \text{, and} \\ p_{total,y} &= -530\ \text{kg}\cdot\text{mi/hr} \qquad . \end{aligned}$$

Since momentum is conserved, their common direction of motion after the crash has to be the same as the direction of their total momentum vector before the crash, which was at an angle of

$$\tan^{-1}(-530/-230) = 67^\circ \text{ or } 247^\circ \qquad .$$

(Both angles are arctangents of -530/-230.) Since the x and y components are both negative, 247° is the correct angle. That's the same as 23° west of south.

14-4 (a) $KE = \frac{1}{2}mv^2$, so $v = \sqrt{2KE/m} = 130$ m/s.
(b) $p = mv = 1.3$ kg·m/s.
(c) same as part b (conservation of momentum)
(d) $KE = \frac{p^2}{2m} = 0.2$ J. The recoiling gun is harmless, because it would take more energy than this to break bones, sever tissues, etc.

14-5 The momentum transferred to the rocket by the exhaust pushing on it equals the area under the curve. Each rectangle represents a momentum of $(0.5\ \text{MN}) \times (1\ \text{s}) = 5 \times 10^5$ N·s, and there are about 54 rectangles under the curve, for a total Δp of $54 \times 5 \times 10^5$ N·s $= 2.7 \times 10^7$ N·s. It starts with zero momentum, so its final velocity equals the momentum transferred divided by its mass, which is 6.8×10^3 m/s.

14-6 We are given that the KE is on the order of that of a well-thrown rock, which might be 10 J. This energy would be dissipated in your body as heat, and you would probably feel it. The momentum would be $p = \sqrt{2\,m\,KE}$, which comes out to be about 10^{-13} kg·m/s if we put in the mass of a proton. This is a tiny amount of momentum, and you definitely wouldn't notice it, i.e. you would not be knocked back perceptibly.

14-7 We start with

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} \qquad .$$

Since each object has the same value of m, m can be factored out of the sums, giving

$$x_{cm} = \frac{m\,(x_1 + x_2 + \ldots)}{m\,(1 + 1 + \ldots)} \qquad .$$

If you add up a series of 1's for each object, you just get the number of objects, N, so this equals

$$x_{cm} = \frac{x_1 + x_2 + \ldots}{N} \qquad ,$$

which is the average of all the x's.

14-9 (a) Let T be the tension in the rope at the corner of the table. For the bottom part of the rope with length x that hangs vertically, we let positive be down (in the direction of motion), so

$$\begin{aligned}
F &= \frac{\mathrm{d}p}{\mathrm{d}t}\\
m_{bottom} g - T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(m_{bottom}\frac{\mathrm{d}x}{\mathrm{d}t}\right)\\
\frac{x}{L} mg - T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{x}{L} mg\frac{\mathrm{d}x}{\mathrm{d}t}\right)\\
xg - \frac{L}{m}T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(x\frac{\mathrm{d}x}{\mathrm{d}t}\right)\\
xg - \frac{L}{m}T &= \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + x\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \qquad .
\end{aligned}$$

For the top part, with length $L - x$, we let positive be toward the edge of the table (again, in the direction of motion).

$$\begin{aligned}
F &= \frac{\mathrm{d}p}{\mathrm{d}t}\\
T &= \frac{\mathrm{d}}{\mathrm{d}t}\left[\left(1 - \frac{x}{L}\right) m\frac{\mathrm{d}x}{\mathrm{d}t}\right]\\
\frac{LT}{m} &= -\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + (L - x)\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \qquad .
\end{aligned}$$

Adding the results of the two calculations, we have $xg = La$, which is the result that was claimed.

(b) For $x = 0$, nothing is hanging over the edge, so the upward normal force of the table completely cancels the force of gravity. For $x = L$, the whole rope is hanging, and there is no upward normal force from the table at all; the result becomes $a = g$. In between these two extreme cases, the upward normal force of the table only partially cancels gravity.

(c)

$$\begin{aligned}
\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} &= \frac{g}{L}x\\
\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(be^{ct}\right) &= \frac{g}{L}be^{ct}\\
bc^2 e^{ct} &= \frac{g}{L}be^{ct}\\
c^2 &= \frac{g}{L}\\
c &= \pm\sqrt{\frac{g}{L}}
\end{aligned}$$

(d) This is true because the derivative of a sum is the sum of the derivatives.

(e) The most general solution is of the form

$$x = qe^{ct} + re^{-ct} \qquad ,$$

where q and r are constants with units of meters, and we'll say c represents the positive root $\sqrt{g/L}$. At $t = 0$, both exponentials equal one, and we have

$$x_\mathrm{o} = q + r$$

and

$$\begin{aligned}0 &= \frac{\mathrm{d}x}{\mathrm{d}t}\\ &= qce^{ct} - rce^{-ct}\\ &= (q-r)c\\ q &= r\end{aligned}$$

The solution is therefore $q = r = x_o/2$, or

$$x = x_o\left(e^{ct} + re^{-ct}\right)/2 \qquad ,$$

which if you like you can write using the hyperbolic cosine.

14-10 The simplest way to do this is by using the center-of-mass frame of reference. If the big object's mass is essentially infinite compared to the little one's ($M >> m$) then the center of mass of the whole system is always located at the big one's center of mass, so the center-of-mass frame is the frame fixed to the big object. (After the collision, the big object recoils a tiny bit from the collision, so you could say that it's no longer at rest in the c.m. frame, but because its mass is assumed to be infinite compared to the little one's mass, its recoil is negligible.)

In this frame, it's not the high-mass object with velocity v coming along and hitting the low-mass object, it's the other way around. The low-mass object, moving at velocity $-v$, hits the high-mass one. This is like hitting a brick wall: the recoil of the wall is negligible. The only thing moving before or after the collision is the small object, and since we're assuming no KE is converted into anything else, the small object must recoil at velocity $+v$, i.e., in the opposite direction but at the same speed. (Reasoning slightly more rigorously, it's not quite true that the big object doesn't recoil at all, but the momentum it absorbs equals $-2mv$, and its KE after the collision is $p^2/2M = 2m^2v^2/M$, which is extremely small because M is so big compared to m.)

In the center-of-mass frame, then, we've proved that the small object enters with velocity $-v$ and exits with $+v$. To convert back to the original frame of reference, we add v to all the velocities. In the original frame, the small object starts with velocity 0, and ends up with $2v$.

14-12 Let's use M for the mass of the ball, U for its initial velocity, and V for its final velocity. Similarly, let m be the mass of the pin, and v its final velocity. Conservation of momentum and energy give

$$MU = MV + mv$$

and

$$\frac{1}{2}MU^2 = \frac{1}{2}MV^2 + \frac{1}{2}mv^2 \qquad ,$$

two equations in the two unknowns V and v. We aren't given V and don't want to find it, so let's eliminate it. Solving the momentum equation for $V = U - (m/M)v$ and substituting into the second equation, we find

$$\begin{aligned}\frac{1}{2}MU^2 &= \frac{1}{2}M[U-(m/M)v]^2 + \frac{1}{2}mv^2\\ U^2 &= [U-(m/M)v]^2 + \frac{m}{M}v^2\\ 2\frac{m}{M}U &= \left(\frac{m}{M} + \frac{m^2}{M^2}\right)v\\ v &= \frac{2U}{1+m/M}\\ &= 3.6\ \mathrm{m/s} \qquad .\end{aligned}$$

14-13 (a) Suppose that at some time, the rocket has mass m remaining. Now an infinitesimal time passes, and the rocket's mass changes to $m+\mathrm{d}m$ (with $\mathrm{d}m < 0$) because it expels $-\,\mathrm{d}m$ worth of exhaust. This causes its velocity to increase by the infinitesimal amount $\mathrm{d}v$. We can now require conservation of momentum, and the algebra is reduced if we do this in the center of mass frame, in which the rocket is initially at rest, i.e., its velocity is increasing from 0 to $0+\mathrm{d}v$.

$$0 = (-\,\mathrm{d}m)(-u) + (m+\mathrm{d}m)\,\mathrm{d}v$$

Discarding the product of the two infinitesimals $\mathrm{d}m\,\mathrm{d}v$, and separating variables, this becomes

$$-\frac{\mathrm{d}m}{m} = \frac{\mathrm{d}v}{u} \qquad ,$$

and integration gives

$$-\ln m + c = \frac{v}{u} \qquad .$$

The interpretation of the minus sign is that as the mass gets smaller, the logarithmic term gets less negative, the left-hand side gets bigger, and the velocity goes up. If we now switch back to the original frame of reference, in which the velocity is not zero but v, the effect is only to add a constant onto v, and that can be absorbed into the constant of integration. Back in this frame, the only time when we have $v = 0$ is initially, when $m = m_i$, and this can be used to determine the constant of integration $c = \ln m_i$. For the final velocity, we have

$$v = u \ln \frac{m_f}{m_i}$$

(b) Solving for m_i/m_f, we have

$$\begin{aligned} m_i/m_f &= e^{v/u} \\ &= \exp(3 \times 10^7 \text{ m/s / 4000 m/s}) \\ &= 2 \times 10^{434} \qquad . \end{aligned}$$

This is a ridiculously huge amount of fuel, vastly more than the mass of the entire galaxy.

14-17 The elegant way to do this is to switch to the center of mass frame. In that frame, the balls are initially moving toward each other at velocities $v/2$ and $-v/2$. After the collision, we can maximize the amount of heat released if we simply make the two masses stick together and stop. That means that *in the center of mass frame*, it's possible to convert all the kinetic energy, $2(1/2)m(v/2)^2 = (1/4)mv^2$, into heat and sound. Back in the original frame of reference, the balls are still moving after the collision, so not all the kinetical energy has been lost, but the amount of heat and sound is still the same amount we found in the c.m. frame: $(1/4)mv^2$, or half the initial kinetic energy of the single moving ball.

As an alternative, a brute force technique would be to require conservation of momentum and energy and crank out the result. Let the final velocities be b and c. Then

$$\begin{aligned} mv &= mb + mc \\ \frac{1}{2}mv^2 &= \frac{1}{2}mb^2 + \frac{1}{2}mc^2 + Q \qquad , \end{aligned}$$

where Q is the amount of heat and sound that comes out. Solving the first equation for $b = v - c$ and substituting into the second, we find $Q = m(vc - c^2)$. Using calculus to find the maximum, we set $0 = \mathrm{d}Q/\,\mathrm{d}c = mv - 2mc$, giving $c = v/2$, and then the rest of the result carries through as in the center-of-mass approach.

14-18 Both energy and momentum are conserved, so neither can have changed after the collision. The kinetic energy has been converted into heat, sound, and the energy required to permanently deform the blobs. The momentum was zero before the collision (it canceled out), and it's zero afterward as well.

14-19 The change in momentum is $\Delta p = \int_0^T F(t)dt$. Also, since the object is at rest at $t = 0$, $\Delta p = p_f$ (which we'll just call $p(t = T)$):

$$\begin{aligned} p(T) &= \int_0^T At^2 dt \\ &= \frac{1}{3}AT^3 \end{aligned}$$

14-20 (a) By drawing a circular arc from where the block was originally hanging to the maximum height it reaches, you should be able to see that the maximum height reached is $H = L(1 - \cos\theta)$.

(b) We will have to do this in two parts: first, find the speed of the bullet/block system after the collision by using conservation of energy, and second, to find the initial speed of the bullet by using conservation of momentum. Note that you cannot just set the final potential energy of the bullet/block system equal to the initial kinetic energy of the bullet because some mechanical energy was lost during the collision.

Call the bullet/block system “BB.” The final PE of BB is equal to the kinetic energy of BB right after the collision. Therefore,

$$\begin{aligned}KE_{\text{i,BB}} &= PE_{\text{final,BB}}\\ \frac{1}{2}(m+M)v^2 &= (m+M)gH\\ \frac{1}{2}v^2 &= gL(1-\cos\theta)\\ v &= \sqrt{2gL(1-\cos\theta)}\end{aligned}$$

Now, using conservation of momentum, we can solve for the initial speed of the bullet, v_0:

$$\begin{aligned}mv_0 &= (m+M)v\\ v_0 &= \frac{m+M}{m}\sqrt{2gL(1-\cos\theta)}\end{aligned}$$

(c) $PE_{\text{final,BB}} = (m+M)gL(1-\cos\theta)$ and $KE_{\text{bullet}} = mgL(1-\cos\theta)$, so the ratio is $(m+M)/m$. For $M \gg m$, this is basically equal to M/m, which is a number much greater than 1. This means that almost all of the mechanical energy of the bullet is lost in such a collision.

14-21 It's best to do part (c) before parts (a) and (b):

(c) Suppose that, initially, the truck is moving with velocity v and the car with velocity $-v$. Then, from conservation of momentum, $(2M)v + (M)(-v) = (2M+M)v_f$, where v_f is the final velocity of both vehicles after the inelastic collision. Solving for v_f, we have $v_f = v/3$.

(a) The change in momentum of the truck is $\Delta p_t = (2M)v_f - (2M)v = -(4/3)Mv$.

(b) The change in momentum of the car is $\Delta p_c = (M)v_f - (M)(-v) = +(4/3)Mv$.

(d) $\text{KE}_\text{i} = (1/2)(2M+M)v^2 = (3/2)Mv^2$; $\text{KE}_\text{f} = (1/2)(2M+M)v_f^2 = (1/9)\text{KE}_\text{i}$. Therefore, $8/9 = 89\%$ of the initial KE was lost as a result of the collision.

15-1 You can get the maximum torque by setting everything up with θ=90°, giving

$$\begin{aligned}\tau &= rF\sin\theta\\ &= rmg\\ &= 270\ \text{N}\cdot\text{m} \qquad .\end{aligned}$$

15-2 The total torque required is the torque needed to lift his arm plus the torque needed to lift the weight. Since his arm is not changing, she needs to tell him to set things up so that the torque needed to lift the weight is also the same as before. Torque is $rF\sin\theta$, and since he is decreasing r by a factor of 17/33, she should tell him to increase the mass by a factor of 33/17 so that the increased weight force will compensate for the decreased r. The new mass should be 2.1 kg × 33/17 = 4.1 kg.

15-3

No. It is momentarily at rest, but not at equilibrium, because there is a force on it, the force of gravity, which is not canceled out by any other force. Another way of putting it is that it has $p = 0$ at that moment, but not $\mathrm{d}p/\,\mathrm{d}t = 0$, which is what equilibrium means.

15-4 No, it just means that any torques that do act are adding up to produce zero *total* torque. For instance, a merry-go-round rotating with constant angular velocity has torques acting on it, but the motor's torque and the torque due to friction are just canceling out to produce zero total torque. Of course it is also *possible* that no torques at all are acting (e.g., the earth has no torques on it at all), but it would be incorrect to infer that the merry-go-round's motion at constant angular velocity implied that there were no torques on it.

15-5 Basically we have a situation with two equations in two unknowns. There is an equation that says the total torque on the foot is zero, and another equation that says the total force on the foot is zero. The two unknowns are the tension in the calf muscle, T, and the force exerted by the shinbone, F_s. We can however simplify the solution by choosing the axis to be the place where the shinbone attaches to the foot. With this choice, the shinbone makes no torque on the foot, and the torque equation thus has only the tension in the calf muscle as an unknown. There are two torques on the foot, a clockwise torque exerted by the muscle and a counterclockwise torque from the floor. The floor's upward force equals the person's weight, so if we use plus signs for clockwise torques the torque equation is

$$Ta - Wb = 0 \qquad ,$$

which is easily solved to give

$$T = W\frac{b}{a} \qquad .$$

Letting positive forces be upward, the force equation is

$$T - F_s + W = 0 \qquad ,$$

so

$$\begin{aligned} F_s &= T + W \\ &= W\frac{b}{a} + W \\ &= \left(\frac{b}{a} + 1\right)W \qquad . \end{aligned}$$

15-6 No. The angular momentum of a particle is $p_\perp r$, so if the particles differ in the distance, r, from the axis or in their direction of motion, they can have different angular momenta. Furthermore, they could even have the same $|\mathbf{r}|$ and $|\mathbf{p}|$, but if they were located in different directions relative to the axis, or their directions of motion were different, they could have different angular momenta. As an example of the effect of the direction, a particle has zero angular momentum if it is moving straight in towards the axis or directly away from it ($\theta = 0$ or $180°$).

15-7 (a) $L = (4\pi/5)MR^2/T = 1.1 \times 10^{42}\ \text{kg} \cdot \text{m}^2/s$.

(b) Angular momentum is conserved because no outside torques are acting, so the angular momentum is the same.

(c) Solving the same equation for T gives $T = 150$ s.

15-8 (a) There are three forces on the ladder, F_1, F_2, and its own weight, which can be thought of as acting at its center for the purpose of calculating torques. If it's going to remain at rest, the total horizontal force, total vertical force, and total torque must all be zero. Taking the axis of rotation as the center of the ladder, the weight force makes no torque. If ϕ was less than θ, both torques would be of the same sign (clockwise), so they could not cancel.

(b) The three conditions for equilibrium are:
zero total horizontal force:

$$F_1 \cos\phi - F_2 = 0 \qquad (1)$$

zero total vertical force:

$$F_1 \sin\phi - mg = 0 \qquad (2)$$

zero total torque:

$$\frac{L}{2}F_2 \sin\theta - \frac{L}{2}F_1 \sin(\phi - \theta) = 0$$

or

$$F_2 \sin\theta - F_1 \sin(\phi - \theta) = 0 \qquad (3)$$

For our numerical check, let's arbitrarily choose $mg = 1.000$ N. L is irrelevant, since it canceled out in eq. (3). Eq. (2) gives $F_1 = 1.118$ N. Eq. (1) gives $F_2 = 0.500$ N. As advertised, eq. (3) checks out.

15-9 In the previous problem, we got three equations, which involved several variables we want to get rid of: F_1, F_2 and m. It should be possible to solve three equations in three unknowns. We can simplify things right off the bat by realizing that eq. (2) is useless; it introduces another variable, m, which we don't know and don't want to know. So we're better off just ignoring it and considering this as two equations, (1) and (3), in two unknowns. The general strategy in solving simultaneous equations like this is to solve one of the equations for one of the unknowns, then substitute into the other equation(s) to eliminate it. Let's solve eq. (1) for $F_2 = F_1 \cos\phi$. Eliminating F_2 from eq. (3), we have

$$F_1 \cos\phi \sin\theta - F_1 \sin(\phi - \theta) = 0 \qquad ,$$

which we can now simplify by dividing by F_1, resulting in

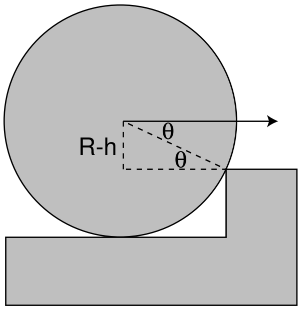
$$\cos\phi \sin\theta - \sin(\phi - \theta) = 0 \qquad .$$

We've eliminated all the unknowns, and we now have a relationship involving only θ and ϕ, the things we're interested in. Using the trig identity as suggested, $\sin(\phi-\theta)=\sin\phi\cos\theta-\sin\theta\cos\phi$, so our equation becomes

$$\cos\phi\sin\theta-\sin\phi\cos\theta+\sin\theta\cos\phi=0 \qquad .$$

Simplifying, we get $\tan\phi=2\tan\theta$, so $\phi=\tan^{-1}(2\tan\theta)$. As claimed, m and L are irrelevant.

15-10 (a) Let h be the height of the step, R the radius of the wheel, m its mass, and F the minimum force required. When the wheel is just about to lift off, the normal force of the ground under it goes to zero, so the only forces involved are gravity, F, and the force exerted by the corner of the step. If we're going to get it over the step, we have to make enough torque on it so that the total torque is clockwise. The minimum force corresponds to the case where the total torque is zero, so that any greater force would do the job. It's natural to consider the corner of the step as the axis, since that's the point around which the wheel will acutally rotate. With this choice, the only nonzero torques are the torque we're applying and the torque of gravity. Defining the angle θ shown in the figure, we have



$$\begin{aligned}
F\sin\theta &= mg\sin(90°-\theta)\\
F\sin\theta &= mg\cos\theta\\
F &= \frac{mg}{\tan\theta}\\
&= mg\frac{\sqrt{R^2-(R-h)^2}}{R-h}\\
&= mg\frac{\sqrt{1-(1-h/R)^2}}{1-h/R}
\end{aligned}$$

(b) For $h=R$, the denominator equals zero, and the force required becomes infinite. This makes sense, because in this situation, you're just trying to pull the wheel straight into the wall of the step.

15-11 Newton's second law is

$$a=\frac{mg-T}{m} \qquad ,$$

where T is the tension in the string. The yo-yo's angular momentum is the same as the angular momentum of a single cylinder of mass m and radius R rotating about its axis:

$$L=\frac{\pi m R^2}{P} \qquad ,$$

where to avoid confusion I've used P rather than T for the period. The period is related to the yo-yo's downward velocity by $v=2\pi r/P$, so we have

$$L=\frac{mR^2v}{2r} \qquad .$$

The torque on the spindle is Tr, so

$$\begin{aligned}
\tau &= \frac{\Delta L}{\Delta t}\\
Tr &= \frac{mR^2a}{2r}\\
T &= \frac{mR^2a}{2r^2}
\end{aligned}$$

We now have two equations in the two unknowns T and a. One equation is Newton's second law, and the other is the equation just derived. Subsituting the latter into Newton's second law to eliminate T, we find

$$\begin{aligned}
a &= g-\frac{R^2a}{2r^2}\\
a &= \frac{g}{1+R^2/2r^2} \qquad .
\end{aligned}$$

Let's check this answer. The units work, because g is an acceleration, and the denominator is unitless. The denominator is greater than one, so $a < g$, which matches up with our real-world experience: a yo-yo doesn't drop like a rock. The acceleration gets smaller if R is bigger in relation to r, and that makes sense too, because increasing R makes it harder to spin (the same torque produces a slower change in the rate of rotation).

15-12 Astrid is right and Michelle is wrong. The force on the ball acts along the line of the string. This line is perpendicular to the ball's motion, so the string does no work on the ball, and the ball must therefore keep the same kinetic energy. After all, if the ball's kinetic energy was going to change, conservation of energy says that some other form of energy would have to change in the opposite direction, but the pole and the string aren't storing or releasing any energy.

The flaw in Michelle's reasoning is that the string's force on the ball is not straight toward the center of the pole, since the string is wrapping around the pole. The string makes a nonzero torque on the ball, and the ball's angular momentum changes. The ball is transferring angular momentum to the planet Earth.

15-13 Conservation of energy gives

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = E \qquad .$$

Intuitively, this tells us that if r is going to get small, v has to get big. But notice that v is squared. That means that v doesn't really increase all that fast, because not much of a change in v is required in order to make v^2 get quite a lot bigger. At the same time, the bomb has to conserve angular momentum, which also requires that as r gets smaller, v has to get bigger to compensate. However, the angular momentum is $L = mvr\sin\theta$, with no squares or anything, and if v isn't getting bigger all that fast, then the only way to pull the angular momentum back up is if $\sin\theta$ increases. But $\sin\theta$ can only get as big as 1, when the bomb is flying horizontally, so that puts a limit on how small r can get:

$$\begin{aligned} L &= mvr\sin\theta \\ &< mvr \end{aligned}$$

From now on, let's just use r to mean the minimum value, rather than the clumsy sumbol r_{min}. Then solving the conservation of energy equation for v gives

$$v = \sqrt{\frac{2E}{m} + \frac{2GM}{r}} \qquad ,$$

so

$$\begin{aligned} L &= mvr \\ &= mr\sqrt{\frac{2E}{m} + \frac{2GM}{r}} \\ 0 &= \left(\frac{2E}{m}\right) + \frac{2GM}{r} - \frac{L^2}{m^2r^2} \\ 0 &= \left(\frac{E}{GMm}\right)r^2 + r - \frac{L^2}{2GMm^2} \qquad , \end{aligned}$$

which we can solve using the quadratic formula to give the result originally claimed.

15-14 Let the forces holding the bridge up on the left and right be L and R, and let the length of the bridge be D. Letting the axis be on the left, force L makes zero torque. There are only two nonvanishing torques on the bridge: a counterclockwise torque DR from the force R, and a clockwise torque from gravity, equal to $(x_{cm})(4W)$. The position of the center of mass is

$$\begin{aligned} x_{cm} &= \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} \\ &= \frac{x_1 + x_2 + x_3 + x_4}{4} \qquad , \end{aligned}$$

where we should think of $x_1, \ldots$ as being at the centers of the pieces, so

$$\begin{aligned} x_{cm} &= \frac{1}{4}\left(\frac{D}{4} + \frac{3D}{4} + \frac{5D}{8} + \frac{7D}{8}\right) \\ &= \frac{5D}{8} \qquad . \end{aligned}$$

From $DR = (x_{cm})(4W)$, we find $R = (5/2)W$, and since the total force is zero, $L+R-4W = 0$, and $L = (3/2)W$.

15-15 The solution follows the steps suggested in the hint. The counterclockwise torque made by the wall at the bottom equals $\sqrt{2}LX$, and the clockwise torque due to gravity is $(L/\sqrt{2})(2W)$. The result is $X = W/2$. Now for the vertical forces, which we label T (at the top joint), B (at the bottom), and M (at the middle). The wall's total vertical force must cancel gravity, so $T + B = 2W$. The vertical forces on the bottom piece must cancel, so $B + M = W$, and similarly for the top piece, $T - M = W$. The solution to these equations is found by the usual process of eliminating one variable after another, and the result is $T = 3W/2$, $M = W/2$, and $B = W/2$. The forces at the three joints have components $(W/2, 3W/2)$ (top), $(W/2, W/2)$ (middle), and $(W/2, W/2)$ (bottom), giving magnitudes of $\sqrt{10}W/2$, $W/\sqrt{2}$, and $W/\sqrt{2}$.

15-16 (a) Let N be the normal force of the cylinder on each bar. The total force on the bars in the vertical direction must be zero, so $2N\cos\theta = 2mg$.

The total torque on each bar must also be zero. Let the axis be at the hinge. The torque due to gravity is $mg(L/2)\sin(90° - \theta) = mg(L/2)\cos\theta$. Constructing the right triangle with vertices at the hinge, the point of contact, and the center of the cylinder, we find that the distance from the hinge to the point of contact is $r\tan\theta$, so the torque of the cylinder on each hinge is $Nr\tan\theta$.

We now have the two equations $2N\cos\theta = 2mg$ and $mg(L/2)\cos\theta = Nr\tan\theta$ in the two unknowns θ and N. Eliminating N, we find the result claimed in the problem.

(b) For large values of r/L, $\tan\theta$ must approach zero, so θ approaches zero. This makes sense, because the cylinder is so huge that the hinge is essentially lying on a flat surface.

For small values of r/L, $\tan\theta$ must approach infinity, so θ approaches 90 degrees. This is intuitively appealing if we imagine the cylinder as a hair-thin wire: the hinge folds flat.

15-17 What makes this a little hard is that not only do we not know the weights w and W of the ship and bottle, we also can't tell the exact positions of their centers of mass. Let the bottle's center of mass lie at a distance H to the right of the fulcrum, and let the ship's center of mass be h to the right of the ship's stern. Suppose that for our first measurement we slide the ship around until its stern is a distance b_1 to the right of the fulcrum. Let the axis be at the fulcrum, so that the (unknown) force from the fulcrum doesn't make a torque. Then setting the total torque equal to zero gives

$$(b_1 + h)w + HW - S_1\ell = 0 \qquad ,$$

where S_1 is the scale's reading in this position, and ℓ is the scale's distance from the fulcrum. This is one equation in four unknowns (h, H, w, and W). It wouldn't do us any good to write down the equation that says that the total force is zero, because that would bring in another unknown: the force at the fulcrum.

It should now be fairly clear that a solution exists. Every time we slide the ship around to a new b, we generate a new equation, without introducing any new unknowns. It is therefore definitely possible to get a unique solution for all four unknowns simply by sliding the ship to locations b_1, b_2, b_3, and b_4.

Actually it turns out that only two locations are necessary. At a second location,

$$(b_2 + h)w + HW - S_2\ell = 0 \qquad .$$

Subtracting the two equations gives $(b_1 - b_2)w - (S_1 - S_2)\ell = 0$, which has only the unknown w in it.

15-18

(a) The equilibrium occurs where the interaction energy has a local minimum. Setting $a = 1$ as suggested, we have

$$\begin{aligned}\frac{\mathrm{d}U}{\mathrm{d}r} &= \frac{\mathrm{d}}{\mathrm{d}r}k\left(r^{-12} - 2r^{-6}\right)\\ &= k\left(-12r^{-13} + 12r^{-7}\right) \qquad .\end{aligned}$$

Setting this equal to zero,

$$\begin{aligned} 12r^{-13} &= 12r^{-7} \\ r^{-13} &= r^{-7} \\ r &= 1 \qquad . \end{aligned}$$

Since $a = 1$, this is the same as saying $r = a$.
(b) Making a graph shows that this is a minimum of U rather than a maximum. (You could also use the second derivative test.)
(c) At $r = \infty$, the equation gives $U = 0$. At $r = a$, we have $U = k(1 - 2) = -k$. The difference in energy is k.

15-19 The equal-area law is just a statement of conservation of angular momentum, which will still be true, because gravity is still directly toward the sun, and still makes zero torque on the planet.

The elliptical orbit law would have to be junked, and there's no easy way to figure out what the shape of the orbits would be instead. The arguments in section 4.1.6 depended completely on the $1/r^2$ nature of the gravitational force.

The law of periods would have to change, but it wouldn't be all that difficult to rework it. If the force law was $F = GMm/r^7$, we would have $a = F/m = GM/r^7 = v^2/r$, and $v = 2\pi r/T$, leading to $T \propto r^4$.

15-21 Let T be the time required for one rotation. Then the velocity of a ball rolling without slipping is $v = 2\pi R/T$, and we have $L/p = (4\pi mR^2/5T)/(2\pi mR/T) = (2/5)R$.

15-22 There is a kinetic frictional force on the ball, which reduces its momentum by Δp. Associated with this force there is also a torque. Just as force is the rate of change of momentum, torque is the rate of change of angular momentum, and since the ratio of torque to force is R, the ratio of ΔL to Δp is also R: $\Delta L = R\Delta p$. The ball has no angular momentum initially, so ΔL is the same as the final angular momentum L. We then have

$$\begin{aligned} \frac{p}{L} &= \frac{5}{2R} = \frac{p_\text{o}}{R\Delta p} - \frac{1}{R} \\ \frac{5}{2} &= \frac{p_\text{o}}{\Delta p} - 1 \\ \Delta p &= \frac{2}{7} p_\text{o} \qquad , \end{aligned}$$

so the ball loses 2/7 of its initial momentum, and is left with 5/7.

15-23 (a) We have two unknowns, F and T, so we're going to need to find two equations to solve for them. Setting the total vertical force on the ruler equal to zero gives one equation (+=up),

$$F - mg + T = 0 \qquad ,$$

and setting the total torque equal to zero gives a second one (axis on the right end, counterclockwise=+),

$$-bF + \frac{L}{2} mg = 0 \qquad .$$

Solving these, we find

$$\begin{aligned} F &= \frac{L}{2b} mg \\ T &= \left(1 - \frac{L}{2b}\right) mg \end{aligned}$$

(b) In the case of $b = L$, we have $F = T = mg/2$, which makes sense because the finger and the string are at the ends of the rod, each supporting half the rod's weight. For $b = L/2$, we get $T = 0$ and $F = mg$, because the rod is balanced on the finger, which means the string is slack.
(c) For $b < L/2$, the tension in the string becomes negative, which doesn't make sense — you can't push with a rope. In this situation, there is no possible equilibrium, and the rod just flips over in the counterclockwise direction.

15-24 For the purpose of calculating gravitational torques, we can treat each branch as if its weight were concentrated at its center of mass. The longer branch's center of mass is twice as far from the trunk, so the r is doubled. The gravitational force is also twice as much, so $\tau = rF$ is four times greater.

15-25 (a) Alice could be in a frame of reference in which Cathy isn't moving. For instance, if Alice and Cathy were in an airplane together, it would be natural for both of them to adopt the frame of reference of the plane. Bob could be using the frame of reference of the dirt.
(b) Your opinions about angular momentum will depend on what point in space you arbitrarily chose as the axis. Let's say Dong is sitting on some grass. Bob might be using the center of the spinning Earth is his axis, while Alice uses the spot Dong is sitting on.

15-29 (a) $\tau = mg(\ell/2+b) - mg(\ell/2-b) = 2mgb$ (b) This is the same as the torque we would get if the full weight $2mg$ acted at the center of mass, which is at a distance b from the fulcrum.

15-30 If the rider speeds up the back wheel, then it's gaining angular momentum. By conservation of angular momentum, this requires that the bike's body "nose up." Slowing down the back wheel makes it "nose down." This allows the rider to control the orientation of the bike so that it makes a two-point landing.

15-31 (a) Let the axis be the hinge, and let the length of the bar br ℓ. The gravitational torque can be calculated by pretending that it acts at the bar's center of mass, with $r = \ell/2$. Cancellation of the torques gives $(1/2)\ell mg = \ell T \sin\theta$, and the result is $T = mg/(2\sin\theta)$.
(b) To minimize T, we want to maximize $\sin\theta$, which means letting θ be 90°. In other words, the cable is vertical.
(c) As θ approaches zero, the tension approaches infinity. No matter how hard you pull, you can't support the weight of the bar with a horizontal cable.

15-32 The rotational analog of $W = F\Delta x$ is $W = \tau\Delta\theta$. Both τ and $\Delta\theta$ are the same for the second revolution as for the first, so the work done in the second revolution is the same.

16-1 (a) According to the ideal gas law, $PV = nkT$, standard values of pressure and temperature give $V \propto n$, since k is also a constant.
(b)

$$\begin{aligned}
V &= nkT/P \\
&= (6.0\times10^{23})(1.38\times10^{-23}\ \text{J/K})(273\ \text{K}) \\
&\qquad /(1.01\times10^{5}\ \text{Pa}) \\
&= 2.2\times10^{-2}\ \text{J/Pa} \\
&= 2.2\times10^{-2}\ \text{N}\cdot\text{m}/(\text{N/m}^2) \\
&= 2.2\times10^{-2}\ \text{m}^3 \\
&= 2.2\times10^{4}\ \text{cm}^3 \\
&= 22\ \text{liters}
\end{aligned}$$

16-3 (a) Looking up the relevant masses, the result is 6.7×10^{-27} kg. (The electrons' masses are nearly negligible.)
(b) n=1 kg/(6.7×10^{-27} kg)= 1.5×10^{26}
(c) Approximating helium as an ideal gas, we can ignore the interactions between atoms, so there is no electrical energy to take into account, only kinetic. The thermal energy of the gas is $E = n\bar{K} = (3/2)nkT$. A change in temperature of one degree requires a change in energy of $(3/2)nk(1\ \text{K}) = 3.1\times10^3\ (\text{J/K})\text{K} = 3.1\times10^3$ J.

16-4 (a)

$$\begin{aligned}
\text{eff} &= \frac{Q_L}{W} \\
&= \frac{Q_L}{Q_H - Q_L} && \text{[cons. of energy]} \\
&= \left(\frac{Q_H}{Q_L} - 1\right)^{-1} \\
&= \left(\frac{T_H}{T_L} - 1\right)^{-1} && \text{[Carnot engine]} \\
&= \left(\frac{308\ \text{K}}{300\ \text{K}} - 1\right)^{-1} \\
&= 38 \\
\text{EER} &\approx (10/3)(\text{eff}) \\
&\approx 150
\end{aligned}$$

(b)

$$\begin{aligned}\Delta S &= \frac{Q_H}{T_H} - \frac{Q_L}{T_L} \\ &= \frac{W+Q_L}{T_H} - \frac{Q_L}{T_L} \qquad \text{[cons. of energy]} \\ &= W\left(\frac{1+\text{eff}}{T_H} - \frac{\text{eff}}{T_L}\right) \\ &= 3\times 10^4 \text{ J/K}\end{aligned}$$

The result is positive, so it's consistent with the second law.

16-5 (a)

$$\begin{aligned}P_{surface} - P_{center} &= \int_{r=0}^{b} \mathrm{d}P \\ &= -\int_{r=0}^{b} \rho g\, \mathrm{d}r\end{aligned}$$

The pressure at the surface is essentially zero, so

$$\begin{aligned}P_{center} &= \int_{r=0}^{b} \rho g\, \mathrm{d}r \\ &= \int_{r=0}^{b} \rho \frac{Gmr}{b^3}\, \mathrm{d}r \\ &= \left.\frac{G\rho m r^2}{2b^3}\right]_{r=0}^{b} \\ &= \frac{G\rho m}{2b} \\ &= \frac{3Gm^2}{8\pi b^4}\end{aligned}$$

(b) The units of Gm^2/b^2 are newtons, so the units of Gm^2/b^4 are N/m^2.

(c) 1.7×10^{11} Pa.

(d) Atmospheric pressure is 10^5 Pa, which is, as expected, about six orders of magnitude less than the answer to part c.

16-6 (a) The condition for a molecule to escape is that its kinetic energy has to equal the amount of gravitational energy it would gain while escaping. The kinetic energy depends on v^2, and the gravitational energy on M/r, so the ratio of the escape velocities is

$$\begin{aligned}\frac{v_m}{v_e} &= \sqrt{\frac{M_m r_e}{M_e r_m}} \\ &= 0.22 \qquad \text{[ratio of escape velocities]}\end{aligned}$$

(b) The absolute temperature is proportional to the average kinetic energy per molecule, so

$$\begin{aligned}\frac{v_m}{v_e} &= \sqrt{\frac{403\text{ K}}{273\text{ K}}} \\ &= 1.21 \qquad \text{[ratio of average velocities]}\end{aligned}$$

(c) For a given temperature, the average kinetic energy is a fixed value, so the mass is proportional to the inverse square of the velocity. If the temperatures on the earth and moon were the same, then the mass of the fluorocarbon molecule would have to be greater by a factor of $(0.22)^{-2}$. The higher temperature on the moon means we need the fluorocarbon molecules to be heavier by an additional factor of $(1.21)^2$. The result is that we need a mass ratio of $(0.22)^{-2}(1.21)^2 = 30$.

The atomic masses involved are N=14.0, F=19.0, C=12.0.

$$30(2\times 14) = (12.0)n + (19.0)(2n+2)$$

$$n \approx 16$$

So if some super-advanced alien civilization decided to cover the moon with an atmosphere of $C_{16}F_{34}$, you'd be able to walk around on the surface with only an oxygen mask!

16-7

$$\begin{aligned}P &= \frac{n}{V}kT \\ &= \left(10^{-5}\text{ cm}^{-3}\right)\left(1.38\times 10^{-23}\text{ J/K}\right)\left(10^3\text{ K}\right) \\ &= \left(10\text{ m}^{-3}\right)\left(1.38\times 10^{-23}\text{ J/K}\right)\left(10^3\text{ K}\right) \\ &\sim 10^{-19}\text{ J}\cdot\text{m}^{-3} \\ &= 10^{-19}\text{ N}\cdot\text{m}\cdot\text{m}^{-3} \\ &= 10^{-19}\text{ Pa}\end{aligned}$$

16-8 The ideal gas law says $PV = nkT$. The volume is the same as before, and the temperature is the same, since the gas is back in thermal equilibrium with the room. Since n is doubled, P must be doubled as well.

16-15 The energy of 1.3 J equals the amount Q gained by the spoon and minus the amount $-Q$ lost by the coffee. The change in entropy is

$$\Delta S = \frac{Q}{T_{\text{spoon}}} - \frac{Q}{T_{\text{coffee}}} = 7.2 \times 10^{-4} \text{ J/K}.$$

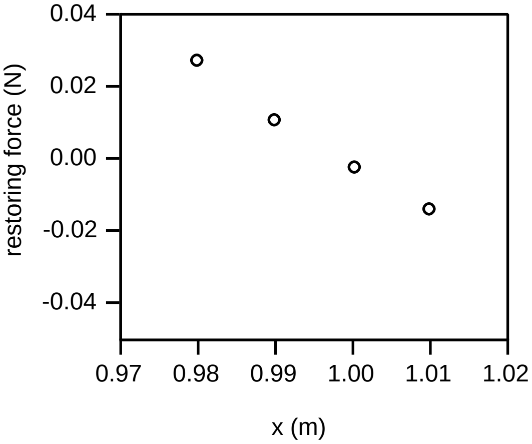
17-1 $f = 1/T = 1/\left(2\pi\sqrt{m/k}\right) = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

17-2 0.07 to 0.10 s $\left(\frac{1}{15 \text{ Hz}} \text{ to} \frac{1}{10 \text{ Hz}}\right)$

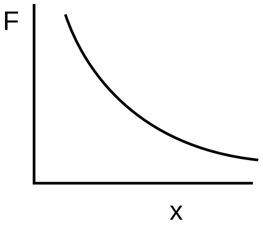
17-3 (a) Pendulum 2 has then same total force on it when it is at the same angle, but when the two pendula are at the same angle, pendulum 2 is at twice the distance from equilibrium. Pendulum 2's $F-x$ graph will therefore look like pendulum 1's, but stretched out twice as wide. Since k is the slope of this graph, pendulum 2's k is half as much.

(b) $T = 2\pi\sqrt{m/k}$, so if pendulum 2's k is half as much, its period is greater by a factor of $\sqrt{2}$.

17-4 I just picked a few points to graph. The graph is so nearly linear over this range that you can't even see that it's not a straight line. Picking two points and taking the rise over the run, the slope is -1.4 N/m, so within this restricted range, the restoring force has a "spring constant," k, equal to 1.4 N/m. The mass is $F_W/g = 0.10$ kg. The period of oscillation will be $2\pi\sqrt{m/k} = 1.7$ s.

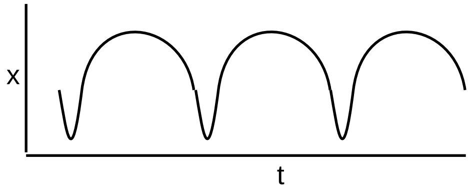


17-5 The $F-x$ graph will look like this:



(Either graph a few points, or use your general math experience.) The graph in the previous problem only covered a small portion of this range, so it looked linear.

We only get simple harmonic motion (sinusoidal motion) when the $F-x$ graph is linear, so for large vibrations, where the $F-x$ graph can no longer be considered approximately linear, there will not be simple harmonic motion. Using the hint, we find that the motion must look something like this:



17-6 (a) The volume of the displaced water is $h_o b^2$, so its mass is $h_o b^2 \rho$, and its weight is $h_o b^2 \rho g$. Setting this equal to mg gives $h_o = m/b^2\rho$.

(b) A calculation similar to the one in part a gives $hb^2\rho g$ for the weight of the displaced water. Letting positive forces be upward, we have $F = hb^2\rho g - mg$.

(c) The equation $F = hb^2\rho g - mg$ is a line of slope $b^2\rho g$ when F is graphed against h, so the "spring constant" is $k = b^2\rho g$. The period is

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ &= 2\pi\sqrt{\frac{m}{b^2\rho g}} \\ &= 2\pi\sqrt{\frac{h_o}{g}} \end{aligned}$$

17-7 This is similar to the spring-and-lever example in section 16.2. Let x be the vertical position of one of the kids relative to equilibrium. The distortion of each spring is $(b/a)x$, so the total energy in the two springs is $PE = (2)(1/2)k[(b/a)x]^2$. The second derivative of this quantity gives the effective spring constant $k' = 2k(b/a)^2$. The moving mass is $m' = 2m$, so $\omega = \sqrt{k'/m'} = (b/a)\sqrt{k/m}$. (We have canceling factors of 2 due to the two kids and the two springs.) The resulting period is $T = 2\pi/\omega = 2\pi(a/b)\sqrt{m/k}$.

17-8

The 2π is unitless. The rest of the right-hand side has units of $[\text{kg}/(\text{N/m})]^{1/2} = [\text{kg}\cdot\text{m}/(\text{kg}\cdot\text{m/s}^2)]^{-1/2} = \text{s}$, which matches the units on the left.

17-9 $T \propto k^{-1/2} \propto g^{-1/2}$, so $g \propto T^{-2}$. Therefore a fractional change in T causes twice the fractional change in g. For example, a 1% increase in T will cause a 2% decrease in the inferred value of g. In symbols, $\Delta g/g = 2\Delta T/T$. The ΔT that would cause a $\Delta g = .024\ \text{m/s}^2$ is about 1 ms.

18-1 Power is proportional to energy, and energy is proportional to A^2, so $A \propto \sqrt{P}$. The ratio of the amplitudes is $\sqrt{50/20} = 1.6$.

18-2 $FWHM = f_{res}/Q = 100$ Hz, so since the curve is approximately symmetric, the range will be ±50 Hz, i.e., 250-350 Hz.

18-3 To find the minimum of the quantity inside the square root, we set its derivative equal to zero,

$$\begin{aligned} 0 &= \frac{\mathrm{d}}{\mathrm{d}f}\left(4\pi^2 m^2\left(f^2 - f_\text{o}^2\right)^2 + b^2 f^2\right) \\ 0 &= 8\pi^2 m^2\left(f^2 - f_\text{o}^2\right)(2f) + 2b^2 f \end{aligned}$$

One solution is $f = 0$, but that's normally a minimum. We're interested in solutions with $f \neq 0$, so we can divide by $2f$ to simplify:

$$\begin{aligned} 0 &= 8\pi^2 m^2\left(f^2 - f_\text{o}^2\right) + b^2 \\ f &= \sqrt{f_\text{o}^2 - \frac{b^2}{8\pi^2 m^2}} \end{aligned}$$

18-4 (a) If W/E is the fraction of energy lost to heat per cycle, then the fraction remaining after one is $(1 - W/E)$. After two cycles, the fraction remaining is $(1 - W/E)^2$, and so on. After n cycles, the fraction remaining is $(1 - W/E)^n$.
(b) The definition of Q amounts to $(1-W/E)^Q = 1/535$, or

$$(1 - W/E)^Q = e^{-2\pi} \qquad .$$

(c) Taking natural logs of both sides,

$$Q\ln(1 - W/E) = -2\pi \qquad .$$

Since W/E is small, we can use the approximation $\ln(1 + x) \approx x$, so $QW/E = 2\pi$, or $1/Q \approx (1/2\pi)W/E$.

18-5 (a) The force is not constant, so we have to do the integral

$$\begin{aligned} \frac{1}{2}W_{damp} &= \int_{-A}^{A} F\,\mathrm{d}x \\ &= \int_{-A}^{A} -bv\,\mathrm{d}x \end{aligned}$$

To find v as a function of x, we can use conservation of energy:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

Solving this for v gives

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} \qquad ,$$

so we now have

$$\begin{aligned} \frac{1}{2}W_{damp} &= -\int_{-A}^{A} b\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}\ \mathrm{d}x \\ &= -b\sqrt{\frac{k}{m}}\int_{-A}^{A}\sqrt{\left(A^2 - x^2\right)}\ \mathrm{d}x \end{aligned}$$

The definite integral can be done by a change of variable and a trig substitution, or by looking it up in a table of integrals, or, most simply of all, by recognizing the integrand as the equation of a semicircle of radius A, so that the area under the curve equals half the area of a circle, $\pi A^2/2$.

$$\frac{1}{2}W_{damp} = -\frac{\pi}{2}A^2 b\sqrt{\frac{k}{m}}$$

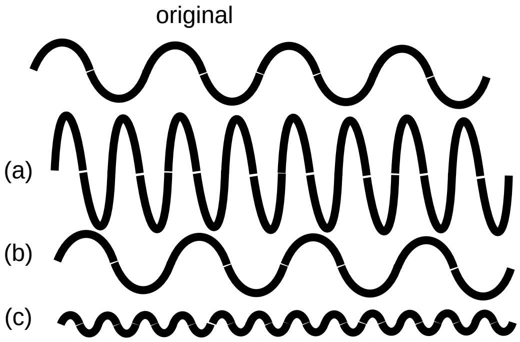
We wish to get this in terms of f, which equals $(1/2\pi)\sqrt{k/m}$, so

$$W_{damp} = -2\pi^2 A^2 bf$$

(b,c,d) The energy is $kA^2/2$, so the rest follows immediately.

18-6 (a) The definition of Q says that when $n = Q$, the energy falls off by a factor of $e^{2\pi}$. Since energy is proportional to the square of the amplitude, this corresponds to a factor of $Z = e^{\pi}$ in amplitude. Now we need to generalize this to the case where the number of oscillations we actually observe isn't actually Q. If the amplitude drop in n oscillations is Z, then the drop in one oscillation is $f^{1/n}$. The drop in Q oscillations is then $(Z^{1/n})^Q$. Setting this equal to e^{π} and solving for Q, we find $Q = n\pi/\ln Z$.
(b) The figure is pretty small, so this will be a fairly rough estimate. After $n = 2$ cycles of oscillation, it looks like the amplitude falls off by a factor of $Z = 2$. Applying the result from part a, we have $Q \approx 9$.

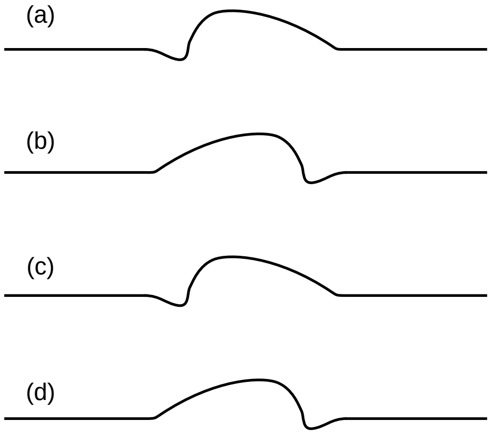
19-1 Since it's a graph of the wave as a function of position, what it shows is amplitude and wavelength. (a) The amplitude is doubled, and $\lambda = v/f$, so the wavelength is cut in half. (b) Since $v = f\lambda$, doubling both velocity and frequency leaves the wavelength unchanged. This is exactly the same as the orginal wave. (c) The information about the velocity is irrelevant. We're given the wavelength and amplitude directly, so that's all we need to show. (The change in velocity implies an additional change in frequency for the given wavelength, but the wavelength is 1/3 the original, not 1/6.)



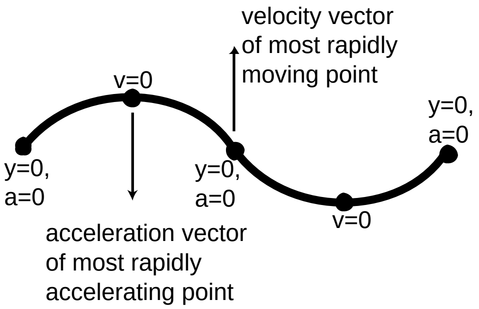
19-2 (a) The scale on the time axis is not comparable to the scale of the distance axis, so there is no reason why your answer has to be the same width on the page. As the pulse passes through a certain point traveling to the right, the water will first go down a little, then up, so the graph should look similar, but reversed.

(b) As the pulse travels to the left, the water will first do the big, slow rise, then the small, short dip. The graph is not reversed compared to the original.

(c) and (d) are the same idea – the roles of question and answer are just reversed.



19-3 The points with $v = 0$ are the ones that have gone as far out to one side as they'll go, and are now ready to come back. The points that are whipping across the center line are the ones that are going the fastest, and they also have zero acceleration because their v vs t graph is at a peak, where the tangent line has zero slope.



19-4 (a) In the expression

$$\lambda' = \left(1 - \frac{v_s}{v}\right)\lambda \qquad ,$$

we eliminate wavelength λ and replace it with v/f:

$$\frac{v}{f'} = \left(1 - \frac{v_s}{v}\right)\frac{v}{f} \qquad ,$$

We simplify by dividing out the factor of v on both sides, and inverting both sides:

$$f' = \frac{f}{1 - \frac{v_s}{v}}$$

(b) The ratio v_s/v is unitless, so it makes sense to subtract it from a unitless 1. Dividing a frequency by a unitless number gives a frequency.

(c) If $v_s = 0$, then $f' = f$, which makes sense. For v_s greater than zero (but less than v), the denominator is less than one, so $f' > f$, indicating an upward frequency shift, which makes sense if positive v_s indicates motion toward the observer.

19-5

There are a couple of things that make it difficult to devise an accurate experiment: (1) it's hard to produce waves in a controlled way, and (2) it's hard to measure the height of the superposing waves in real time.

Possible solution: Find a machine that's capable of making spreading ripples of a consistent height, e.g., a wind-up toy car whose wheels go around. This takes care of problem 1. You could put two such cars in the water, and see if the peaks formed by constructive interference become twice as tall at a point equidistant from the two cars, compared to their height when only one car was turned on.

The toy cars also help to solve problem 2, because the waves are steady. You can put a ruler in the water, and watch the steady up-and-down vibration of the surface, measuring the height of the wave. This could be made easier by taking a video on a cell phone and then clicking through it frame by frame.

19-6 $T = 1/f = 3.82$ ms, and $\lambda = v/f = (340\ \text{m/s})/f = 1.3$ m.

19-7 1% of the speed of sound, or about 3 m/s

19-8 When a gas gets hot, it expands. For instance, that's why a hot-air balloon works: the hot air is less dense than the surrounding cold air. A hotter gas is less dense, and its lower inertia makes it easier for it to vibrate rapidly. Liquids and solids don't contract or expand nearly as much with changes in temperature, so the speed of sound in liquids and solids doesn't change much with temperature. (The role played by tension in waves on a string is played by pressure in sound waves. Pressure is basically irrelevant here. The pressure inside a solid or liquid doesn't vary with temperature, and the pressure in a gas only varies with temperature if the gas is confined to a fixed volume. This is not the case for sound waves in open air; the air pressure is essentially the same on a cold day as on a hot day.)

20-1 Due to the difference in speed, there will be partial reflection of the sunlight.

20-2 (a) Since the result only depends on the ratios of speeds, we can drop the powers of 10:

$$R = \frac{2.2 - 3.0}{2.2 + 3.0} = -0.15$$

(b) Energy is proportional to the square of amplitude, so the fraction of the energy reflected is $(0.15)^2 = 0.022$.

20-3 $L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{340\ \text{m/s}}{2f} = 65$ cm

20-4 (a) These are all different standing wave patterns that fit within the tube. Notice how all the frequencies are multiples of 150 Hz. (Since we're getting all the multiples, not just the odd multiples, the saxophone apparently acts like a symmetric air column, and we can visualize the series of wave patterns as the one in which we fit some number of half-wavelengths in the length of the tube.)

(b) Higher frequencies correspond to shorter wavelengths. You can fit more than one half-wavelength in the tube, but you can't put less than one half-wavelength in, because the wave wouldn't be doing the right things at the ends.

20-5 (a)

C:	261.6	523.2	784.8	1046.4	1308.0
G:	392.0	784.0	1176.0	1568.0	1960.0

The 784.8 and 784.0 Hz frequencies differ by much less than 1%, and the 1046.4 and 1176.0 Hz frequencies differ by 12%. Neither combination is really in the 1-10% range that causes dissonance.

(b)

C:	261.6	523.2	784.8	1046.4	1308.0
Bb:	466.2	932.4	1398.6	1864.8	2331.0

The 1308.0 and 1398.6 Hz frequencies differ by about 7%, which is clearly going to make them dissonant.

20-6 (a) The increase in the size of the instrument will cause an increase in the wavelength. Since we're analyzing the two effects separately in parts a and b, we'll assume that the wave velocity stays the same here, so that $v = f\lambda$ gives $f \propto 1/\lambda$. A change in temperature of ten degrees would only give an effect on the order of parts in ten thousand, which is much too small to account for the observed effect. Not only that, but this would be a reduction in frequency, whereas what we actually observe is an increase.

(b) Now we assume that λ is constant, and consider only the effect of changing v, in which case $f \propto v \propto \sqrt{T}$. If we take the same 10-degree increase in temperature assumed in part a, but now apply it to the air rather than the brass, we expect the frequency to increase by a factor of $\sqrt{(T+10)/T}$. Assuming an absolute temperature of around 300, the result comes out to be about one and a half percent, which is in exactly the right range to cause an effect of the observed size.

20-7 In the equation $v = f\lambda$, the wavelength is fixed by the length of the instrument, so $f \propto v \propto 1/\sqrt{\rho}$. Increasing the density by a factor of 1.023 should decrease the frequency by a factor of $1/\sqrt{1.023} = .989$, a change of -1.1%.

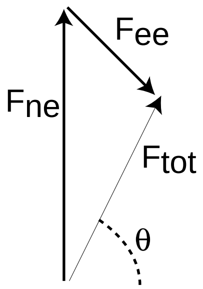
20-8 (a) We have $R = (\alpha - 1)/(\alpha + 1)$. Inverting α gives $R = (1/\alpha - 1)/(1/\alpha + 1) = (\alpha - 1)/(1 + \alpha)$, which means we've simply negated R. The interpretation is that the energy of the reflected pulse is unchanged, but the reflection's inverting or uninverting character is flipped.

(b) Based on part a, we expect that there will be two values of α that will do the job, and that they will be inverses of each other. Solving for α in terms of R gives $\alpha = (1 + R)/(1 - R)$. Taking $R = 1/2$ gives $\alpha = 3$, and without having to do the arithmetic for the case of the inverting reflection, we can tell that it will happen for $\alpha = 1/3$.

21-1 The positive and negative charges attract each other.

21-2 Let's use positive numbers for forces to the right, and negative numbers for forces to the left. There are two forces on the right-hand electron: a repulsive force from the other electron, and an attractive force from the nucleus. These will be represented as positive and negative numbers, respectively. The charges of the electrons are $-e$, and the charge of the nucleus is $+2e$, so the electron-electron force is $+ke^2/(0.2\text{ nm})^2$, and the nucleus-electron force is $-k(e)(2e)/(0.1\text{ nm})^2$, for a total force of -4×10^{-8} N.

21-3 You should always start this type of vector addition problem by drawing a rough diagram first so that you can check that your results make sense at every step.



The nucleus's force on the bottom electron, F_{ne}, is attractive, so it points up. The other electron's force on the bottom electron, F_{ee}, is repulsive, so it points down and to the right at a 45-degree angle. We'll state the final result using the angle θ defined in the figure. The magnitude of F_{ne} is given by Coulomb's law, $|F_{ne}| = k(e)(2e)/(0.1\text{ nm})^2 = 4.6 \times 10^{-8}$ N. The magnitude of F_{ee} is smaller by a factor of four: a factor of two because of the smaller charge, and another factor of two because its distance from the bottom electron is greater by a factor of $\sqrt{2}$. Thus, $|F_{ee}| = 1.1 \times 10^{-8}$ N. To do vector addition, we need to add components. Let positive x components be to the right, positive y components up. $\mathbf{F}_{ne}$ has components $F_{ne,x} = 0$ and $F_{ne,y} = 4.6 \times 10^{-8}$ N. $\mathbf{F}_{ee}$ has components

$$F_{ee,x} = +(1.1 \times 10^{-8}\text{ N})(\cos 45°) \qquad \text{and}$$
$$F_{ee,y} = -(1.1 \times 10^{-8}\text{ N})(\sin 45°) \qquad .$$

At this point, you can check the + and − signs of the components against the figure. The total force has components

$$F_{tot,x} = F_{ne,x} + F_{ee,x} = 7.8 \times 10^{-9}\text{ N} \qquad \text{and}$$
$$F_{tot,y} = F_{ne,y} + F_{ee,y} = 3.8 \times 10^{-8}\text{ N} \qquad .$$

Again, you can check the + and - signs against the figure. The magnitude of the total force is $\sqrt{F_{tot,x}^2 + F_{tot,y}^2} = 4 \times 10^{-8}$ N (one sig fig). The angle θ equals $\tan^{-1}(F_{tot,y}/F_{tot,x}) = 80°$ (one sig fig).

21-4 (a) Your body is mostly water, and water is mosly oxygen by weight, so for the purposes of this estimate, let's pretend that your body just consists of oxygen atoms. The mass of an oxygen atom is about 16 times the mass of a proton, which is 1.7×10^{-27} kg. If your hand has a mass of about 0.2 kg, then the number of oxygen atoms in a hand is about 0.2 kg/$(16 \times 1.7 \times 10^{-27}$ kg). The number of electrons is eight times that, $(8) \times (0.2\text{ kg})/(16 \times 1.7 \times 10^{-27}\text{ kg}) = 6 \times 10^{25}$, or call it 10^{26} electrons since its only an estimate.
(b) Coulomb's law gives $F = k(10^{26}e)(10^{26}e)/(0.1\text{ m})^2 = 2 \times 10^{26}$ N, or call it 10^{26} N.
(c) Each hand has zero net charge, and the negative charge is distributed within each hand in almost the same way as the positive charge. Another way of looking at it is that there are repulsive forces acting from one set of electrons on the other set of electrons and from one set of protons on the other set of protons, but there are attractive forces from each set of electrons acting on the protons of the opposite hand.
(d) A force of 10^{26} N is enough to pulverize a planet, so the cancellation is obviously very nearly perfect; even a very tiny percentage difference in charge would lead to a huge force. If the fractional difference was ϵ, then the total charge on each hand would be ϵq, where $q = 10^{26}e$ as estimated above. The total force would be $k\epsilon^2 q^2/r^2 = F$. You could easily notice a force of as little as 0.1 N, so $\epsilon \lesssim (r/q)\sqrt{F/k} \sim 10^{-14}$.

21-5 There are 82 protons in a lead nucleus. Since the lead nucleus is spherical, the electric force from the 81 other protons is the same as that of an equal amount of charge (81 times the charge of a proton) concentrated at the nucleus' center. The force is

$$k(q_{proton}) \times (81 q_{proton})/(6.5 \times 10^{-15}\text{ m})^2 = 440\text{ N} \qquad ,$$

which is less than the attractive force of 8 kN.

21-6 Electricity is an inverse square force, so if r is smaller by a factor of 206.77, the force will be greater by a factor of $206.77^2 = 42754$.

21-7 Let q stand for the absolute values of the charges, which are equal to each other. We can find the force either by using Newton's law of gravity or by using the kinematical equation $a = v^2/r$ for the moon's acceleration. Choosing the first approach, we have

$$k\frac{q^2}{r^2} = G\frac{m_1 m_2}{r^2} \qquad ,$$

which we solve for q:

$$q = \sqrt{\frac{G m_1 m_2}{k}}$$
$$= 5.70 \times 10^{13}\text{ C}$$

This may sound like a big number, but it's actually not that much charge at all, on the same order of magnitude as the amount of charge in a human-scale object. It's just that in real life, the charges balance out perfectly, giving zero total charge on each body. Note that the reason the charge comes out fairly small, despite the large masses, is that G is much smaller than k, i.e. gravity is in some sense a much weaker force than electricity.

21-8 The lithium is located nice and symmetrically in the lattice, so if there was no defect, the total force on it would be zero. Removing a Cl^- to make the hole removes a certain amount of force, so the total force without the Cl^- is equal to minus the force the Cl^- would have made, which we can find from Coulomb's law. The distance between the lithium and the hole is $\sqrt{(2.5\times 0.3\ \text{nm})^2+(0.5\times 0.3\ \text{nm})^2} = 0.77$ nm, so the magnitude of the force is $ke^2/(0.77\ \text{nm})^2 = 3.9\times 10^{-10}$ N, or call it 4×10^{-10} N. The Li^+ would have been attracted by the Cl^-, so removing that attractive force leaves a total force that points directly away from the hole.

21-9 (a) Let each fixed charge be Q, and let the charge of the ball be q. When the ball is at position x, its distance from one of the fixed charges is $\sqrt{b^2+x^2}$, so the magnitude of each force on it is $|\mathbf{F}| = kQq/(b^2+x^2)$. There are two such forces, acting at different angles, so we have to add them using vector addition. Their y components cancel, and their x components are the same, so for the total force on the ball, we have

$$\begin{aligned} F &= 2F_x \\ &= \frac{2kQq}{b^2+x^2}\cdot\cos\theta \\ &= \frac{2kQq}{b^2+x^2}\cdot\frac{x}{\sqrt{b^2+x^2}} \\ &= 2kQq\left(b^2+x^2\right)^{-3/2}x \end{aligned}$$

(b) By $a = F/m$, the maximum acceleration occurs at the same point as the maximum force. To find the maximum force, we have to set the derivative of the force equal to zero and solve for x. The constants out in front don't affect the location of the maximum, so we can ignore them.

$$\begin{aligned} 0 &= \frac{\mathrm{d}}{\mathrm{d}x}\left[\left(b^2+x^2\right)^{-3/2}x\right] \\ &= -\frac{3}{2}\left(b^2+x^2\right)^{-5/2}(2x)x+\left(b^2+x^2\right)^{-3/2} \end{aligned}$$

We can simplify by multiplying both sides by $\left(b^2+x^2\right)^{5/2}$.

$$\begin{aligned} 0 &= -3x^2+b^2+x^2 \\ x &= \frac{b}{\sqrt{2}} \end{aligned}$$

21-11 $I = \Delta q/\Delta t = (10^{10})(e)/(5\times 10^{-4}\ \text{s}) = 3\ \mu\text{A}$

21-12 $R = \Delta V/I = (110\ \text{V})/(0.9\ \text{A}) = 120\ \Omega$

21-13 (a) $P = I\Delta V = I(IR) = I^2R$

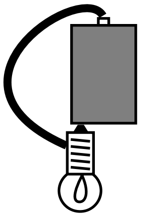
(b) Energy dissipated in those wires is wasted in heating the neighborhood, so we want R to be small.

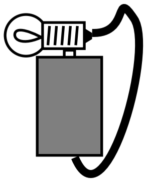
21-14 (a) $P = I\Delta V = (\Delta V/R)\Delta V = \Delta V^2/R$

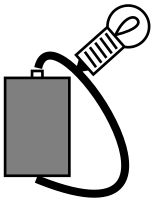
(b) The power is proportional to ΔV^2, so cutting ΔV in half reduces the power by a factor of 4, to 50 W.

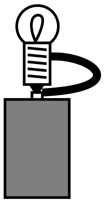
21-15

21-16


(a) good


(b) good


(c) no good


(d) no good

The bulb has two terminals. One is the bump on the end, and the other is the screw-threaded metal jacket. The battery also has two terminals: the bump on one end and the flat surface on the opposite end. In the two good methods, a and b, current can flow out of one terminal of the battery, into one terminal of the bulb, through the filament, out the other terminal of the bulb, and back in to the opposite terminal of the battery. In method c, there is a dead end in the bulb: the current could get in through the bump-shaped terminal, but there would be no way for it to get out of the screw-threaded terminal once it had gone through the filament. The other problem with *c* is that there is an easy way for current to go through the low-resistance wire, so the wire will get very hot and the battery will burn out. In method d, there is a complete loop of wire consisting of the bump-shaped terminal of the bulb, the filament, the screw-threaded terminal, and the wire that comes back to the bump-shaped terminal. This loop is all made out of conductors, and the voltage is equal at all points within a piece of conducting material. Therefore the voltage difference between the two ends of the filament is zero, and by Ohm's law, $V = IR$, no current will flow through the filament. Also, method d gives no way for current to get to the other terminal of the battery.)

21-18

21-19 (a) In one revolution, the amount of charge passing by a certain angular position is 1 nC. The time required for one revolution is $1/(33.3\ \text{min}^{-1})$, so $I = \Delta q/\Delta t = 3.3 \times 10^{-8}$ C/min $= 600$ pA.
(b) The idea is the same. The time for one orbit is $\Delta t = 2\pi r/v$, so $I = e/\Delta t = ve/2\pi r = 1$ mA.

21-20 The basic idea is to find the voltage V at the center of the triangle (defining the voltage infinitely far away to be zero, as usual). We then use conservation of energy to find the minimum initial kinetic energy, which corresponds to some minimum initial speed. A little trig shows that the distance r from the center to one of the vertices is given by $r = (b/2)/\cos 30° = b/\sqrt{3}$, so

$$\begin{aligned} V &= \frac{3kQ}{r} \\ &= \frac{3\sqrt{3}kQ}{b} \\ qV &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2qV}{m}} \\ &= \sqrt{\frac{6\sqrt{3}kQq}{mb}} \end{aligned}$$

21-21 If there was no hole in the lattice, then by symmetry, the potential at any of the available gaps would be the same. Removing the Cl^- to make a hole will then change the potentials by an amount equal to minus its original contribution. Its original contribution was $kq/r = -ke/r$, so the potential at each point in space is changed by $+ke/r$. Now the lithium is going to jump from the gap it occupies in the figure, where the potential is V_1, to some neighboring gap with potential V_2, losing potential energy of $e(V_1 - V_2)$. Without the hole, we would have had $V_1 - V_2 = 0$, but removing the Cl^- to make the hole changes both V_1 and V_2. V_1 changes by an amount ke/r_1, where r_1 is the distance from the original gap to the hole, and similarly for V_2. The potential energy lost by the lithium is therefore $ke^2(1/r_1 - 1/r_2)$, so that is also the amount of KE it gains. To get its speed, solve $KE = (1/2)mv^2$ for $v = \sqrt{2KE/m} = e\sqrt{2k(1/r_1 - 1/r_2)/m}$. It is going to jump to the gap to the left, since that is the direction closest to the direction of the force on it as found in the previous homework problem. The Pythagorean theorem gives $r_1 = 0.77$ nm and $r_2 = 1.06$ nm, the mass of a lithium atom is about 7 times the mass of a proton, and plugging in to the equation for the velocity gives 3.7×10^3 m/s, or call it 4000 m/s.

21-22 To get negative values of P, you'd have to have opposite signs for I and ΔV. But you can't control the signs of I and ΔV independently — switching the wires reverses both. The current flows in response to the electric force, whose direction is related to the direction of the voltage drop. Another way of looking at it is that the electrons collide with the atoms, causing them to vibrate, and the vibration is what heat is. There is no way that randomly vibrating atoms will all hit the electrons in the same direction, producing a current.

21-23 (a) If the power is the same, then $P = I\Delta V$ tells us that tripling the voltage has to go along with cutting down the current to one third of what it was.
(b) By $P = I\Delta V$, the only way to get more power with the same voltage would be to increase the current. This would require fatter wires, which would be more expensive.

21-24 (a) The loop rule says that when you add up the voltage drops across the batteries plus the (opposite in sign) voltage drop across the bulb, you get zero. The batteries work the same as a single battery with a voltage equal to their sum.
(b) It apparently has thousands of cells in series. The "resistor" would be its prey.

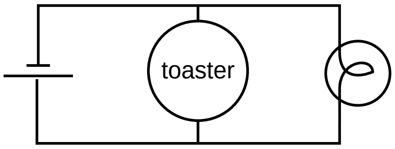
21-25 This setup could be made to work just fine at the extreme settings. Let R_k be the (variable) resistance of the knob, and R_h the resistance of the heater.

The "off" position would have to have $R_k = \infty$, i.e., it would be like an open circuit. The total resistance $R = R_k + R_h$ would be infinite as well, zero current would flow, and no power would be dissipated anywhere.

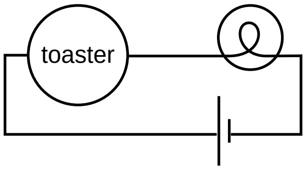
The full-power position would have $R_k = 0$, i.e., it would be like a piece of wire. Current would flow, but there would be no voltage drop across R_k, so the only heating would be in R_h.

The problem occurs at intermediate settings, where R_k is neither zero nor infinity. Here R_k would get hot, wasting power by heating up the knob. At $R_k = R_h$, for example, the knob would be dissipating the same amount of heat as the heating element itself! This would presumably melt the knob, or burn your hand.

21-26



(a)



(c)

(a) See figure.
(b) The E-shaped piece of wire across the top is one big piece of conducting material, so it must have the same voltage throughout it. Similarly, the whole E-shaped piece on the bottom is at the same voltage. Therefore, the voltage difference across each of the three components is the same. Since we know the supply is 110 V, the voltage drops across the toaster and lamp must also be 110 V. Ohm's law gives $I_{toaster} = (110\text{ V})/R_{toaster} \approx 100$ A, and $I_{lamp} = (110\text{ V})/R_{lamp} \approx 50$ A.
(c) In this setup, which is not what's really used in your house, it's not the voltage drops that are equal but the currents — whatever current goes through one component also has to go through the others. The two resistances in series add together to make a total resistance of 3 Ohms, so $I_{toaster} = I_{lamp} = V/3\ \Omega \approx 40$ A. Ohm's law can now be used to find the voltage drop across each appliance, $V_{lamp} = I_{lamp}R_{lamp} = 73$ V, and $V_{toaster} = I_{toaster}R_{toaster} = 37$ V. (I didn't round the voltages off to one sig fig because I wanted to make it clear that they add up to 110 V.)

21-27 Resistance is inversely proportional to the cross-sectional area. The area depends on the square of the diameter, so the ratio of resistances is $(1.2)^2 = 1.4$.

21-28 An open switch in (b) is the same circuit as a closed switch in (a): the light bulb is lit, and there's no problem. When you close the switch in (b), it does cause the bulb to go out (basically because the two sides of the bulb are connected with wire and therefore have no voltage difference, but see the more detailed note below).

But huge amounts of current will now flow through the middle branch of the circuit. This is a short, which will certainly kill the battery, and may also cause enough heating to burn up the flashlight.

One subtle issue about this is that you might think closing the switch in (b) would have no effect on the bulb, since you're just adding a new parallel branch to a circuit. This would lead you to believe that although the short would kill the battery and burn things up, the bulb would stay on, contrary to the claim in the problem that it would go out. In fact, the bulb will go out in (b), because it's not a pure parallel circuit. You have to readjust your thinking a little, because ordinarily we assume that wires have negligible resistance, but in a short circuit their resistances aren't negligible anymore, because the equivalent resistance of the whole circuit is so low. That means you can't neglect the resistance of the wires going from the battery to the junctions. You basically have a series circuit now, consisting of the wire from the battery to the junction, the wire through the switch, and the wire from the switch back to the battery. This series circuit shares the voltage drop of the battery three ways, which means that the voltage across the middle piece is greatly reduced.

21-29 (a) Call the voltages V_5 and V_{10}. All three currents are the same, since it's a single closed loop, and there is no place to store charge. We therefore only need one symbol for current — just call it I. Kirchoff's loop rule says $V_5 + V_{10} = 9$ volts. Ohm's law tells us $V_5 = (I)(5\ \Omega)$ and $V_{10} = (I)(10\ \Omega)$. This means that V_{10} must be twice as big as V_5, and since they have to add up to 9 volts, they must be $V_5 = 3$ V and $V_{10} = 6$ V.

(b) As explained above, all three currents are the same. One way to find the current is by solving the equation $V_5 = (I)(5\ \Omega)$ for I, giving $I = 0.6$ A. Another way would be to divide 9 V by the series resistance of 15 ohms.

(c) The 10-ohm bulb will go out, because its two sides are now connected with wire, and therefore there is no voltage difference between the two sides. The 5-ohm bulb now has the full voltage drop of the battery across it, i.e. 9 volts, and its current will be $(9\text{ V})/(5\ \Omega) = 1.8$ A. This means it will be brighter than it was before.

(d) You'll see exactly the same thing as in the original circuit. None of the reasoning used in solving part (a) made any assumption about what order the bulbs were in. The current does not "get tired" or "get used up" by having to go through one bulb first before it gets to the other. Conservation of charge says that charge can't "get used up."

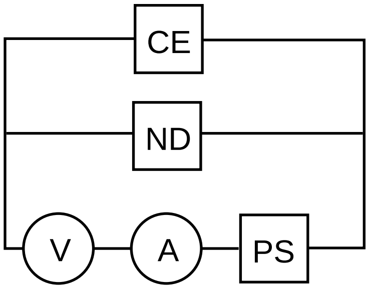
21-30 (a) This has to be true based on the junction rule, so it doesn't tell you anything about the resistance values.

(b) We already knew without even touching the circuit that the currents were going to be equal, and if we find that the voltages are also equal then $V = IR$ says that the resistances must be equal.

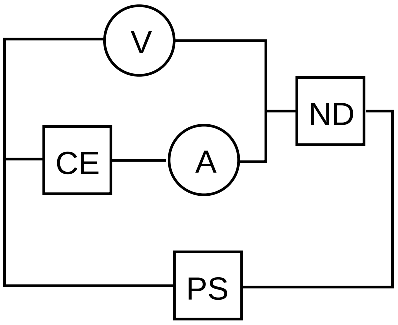
(c) The loop rule tells us that the voltage drops are equal in the parallel circuit, and if we also find out the currents are equal, then $V = IR$ says the resistances must be equal.

(d) This is not useful information for finding out about the resistors, since it has to be true based on the loop rule.

21-31



(a) They have correctly set up the CE and ND in parallel. The ammeter is measuring the total current out of the power supply, however, rather than just the current that is going to go through the ND. The voltmeter is hooked up in series, but it should be in parallel. A voltmeter has a very high resistance, so the circuit will not function when the voltmeter is hooked up this way.



(b) The meters are hooked up correctly, but the CE and ND are in series rather than in parallel. (If they had hooked up the CE and ND in parallel, then they would have had to get the ammeter hooked up correctly to measure the current through the ND rather than the CE, but in this series circuit, both the CE and the ND have the same current through them.)

21-32 All three in series gives one possibility. All three in parallel: one possibility. Two in series, in parallel with the third: three possibilities. Two in parallel, in series with the third: three possibilities. The total number of resistance values you can make is eight.

21-33 The total resistance seen by the voltage source is $x+y$, so the current that flows is $I=\Delta V/(x+y)$. Using the result of problem 1 from ch. 3, the power dissipated in the lamp is $P=I^2y=\Delta V^2y(x+y)^{-2}$. Thus we want to find the value of y that maximizes the quantity $y(x+y)^{-2}$. To find a maximum, we set the derivative equal to zero:

$$\begin{aligned}0&=\frac{\mathrm{d}}{\mathrm{d}y}y(x+y)^{-2}\\&=(x+y)^{-2}-2y(x+y)^{-3}\end{aligned}$$

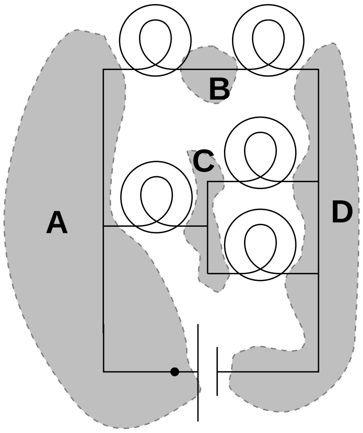
Multiplying both sides by $(x+y)^3$ simplifies this to

$$0=(x+y)-2y \qquad ,$$

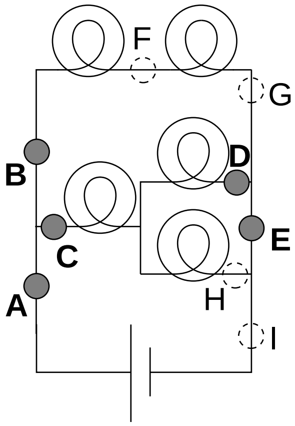
which gives $x=y$.

Physical explanation: Very large values of y are like an open circuit, e.g. like opening a switch. Very little current flows, so no power is dissipated in the lamp. Very small values of y result in more current, but the current is still limited by the resistance of the cord, and this current, passing through a small resistance y, dissipates very little power. This is a specific example of something known as impedance matching: the maximum power is transmitted when the resistance of the load is matched to the resistance of the transmission line.

21-36 There is constant voltage throughout a conductor. The wires are good conductors, but the bulbs aren't, so all we have to do is count how many parts there are in the circuit that consist of nothing but wire. As shown in the figure, there are four such parts. For instance, if we connect one probe of the voltmeter at the dot, and one in region D, we'll get a certain voltage reading, and we'll get the *same* voltage reading even if we move the second probe to some other place in region D. We can get a total of four unique voltage readings, but region A gives zero, so there are only three unique, nonzero readings we can get.



21-37 There are five unique currents you could measure, by inserting an ammeter at A, B, C, D, and E.



The key here is that charge is conserved. That means that, for example, you'll get exactly the same current reading at B, F, and G, because any charge that flows up through B will also have to go through F and G; it can't "drive off the road," and it can't get used up in the bulbs, because that would violate conservation of charge. Likewise, I and A give the same reading.

You also can't get two different results from D and H. This isn't because of conservation of charge, it's because the two bulbs are identical, and have the same voltage drop across them.

21-38 This is a problem that has a trick to it. If you don't think of the trick, it's very difficult, because the circuit can't be broken down into parallel and series parts. The trick is to realize that although the circuit has been drawn in an asymmetric way, it's actually very symmetric. The two rightmost bulbs play identical roles in the circuit, as do the two leftmost one. You can see this if you redraw the circuit so that the top left bulb and the top right bulb are on one horizontal line, and similarly for the bottom left and bottom right. The bulb in the middle has zero current through it by symmetry.

21-39 Like the preceding problem, this one can't be solved by breaking the circuit down into parallel and series parts. Unlike that one, there is no easy way to solve it by symmetry. We simply have to use the loop rule and the junction rule to write down enough equations to solve for all the unknowns. Let's label the bulbs A, B, C, ... going from top to bottom and left to right, like reading a page of text. For simplicity, we think of the bulbs at the bottom as a single bulb, E, with a resistance of two ohms. (This is the only part of the circuit that can be simplified in this way.) Let A stand for the current through bulb A, and so on. The junction rule then gives

$$\begin{aligned} D &= B + E \qquad \text{and} \\ A + B &= C \qquad . \end{aligned}$$

Since every bulb except for E has a resistance of one ohm, the voltage drop across each bulb equals the current through it. That is, we can use the symbol A interchangeable for both the current through A (in amperes) and the voltage drop across A (in volts). The only exception is the double bulb E, which has a voltage $2E$ across it. We use the loop rule three times:

$$\begin{aligned} A + C &= 1 \\ D + 2E &= 1 \\ A &= D + B \qquad . \end{aligned}$$

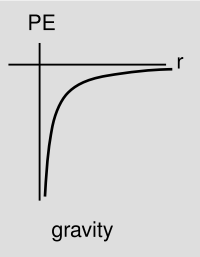
The result is five equations in five unknowns, which we solve by eliminating one variable after another. The final results are $A = 6/13$, $B = 1/13$, $C = 7/13$, $D = 5/13$, and $E = 4/13$. The total current is $A + D = 11/13$ amps, so the power drawn from the one-volt battery is (11/13 A)(1 V) = 11/13 W.

21-40 Using the junction rule at the junction A feeds into, we find that the current through A is 300 mA. Using the junction rule at the junction on the left, we then have 600 mA for the total current drawn from the battery, which is also the current through D. Applying the junction rule at the junction in the middle, we find a current of 100 mA for both B and C. All our unknowns are now determined, so we can use the junction on the right side to check that our solution is correct: 500 mA plus 100 mA gives 600 mA, which is indeed what we had for D.

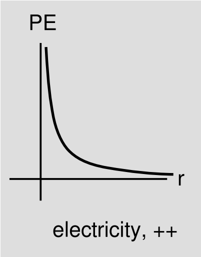
21-41

$$\begin{aligned} I &= \mathrm{d}q/\,\mathrm{d}t \\ &= 2ct \\ P &= I\Delta V \\ &= I\cdot(IR) \\ &= I^2R \\ &= 4c^2t^2 \end{aligned}$$

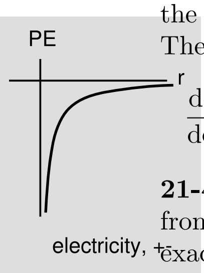
21-42 (a) Dividing by a small r gives a large negative result, while dividing by a big r gives a small negative result. The graph for gravity looks like the one on the left:



gravity



electricity, ++



electricity, +-

As the rock falls to smaller values of r, gaining KE, the graph shows its PE decreasing, which is what we expect from conservation of energy.

(b) Coulomb's law is directly analogous to Newton's law of gravity, except for the fact that there are two types of charge, and the forces can be either repulsive or attractive. Se we can translate directly from the gravitational case to the electrical case just by changing each symbol to the analogous electrical quantity, as long as we make sure to put in the plus and minus signs by hand. Regarding the sign, two positive masses (which is the only kind that exist!) attract, but two positive charges repel. This means that, at least in the case of two positive charges, the sign has to be reversed for the electrical equation: $PE = kq_1q_2/r$. To check that this makes sense, we can imagine two positive charges being released, and accelerating away from each other because of their mutual repulsion. As r gets bigger, dividing by r gives a smaller result for the potential energy, and that makes sense, because PE has to be converted to KE. Two negative charges would also accelerate away from each other and convert PE to KE as r increased, so the same equation still works ($q_1q_2 > 0$). In fact, the equation even works when one charge is positive and one is negative; here, the product q_1q_2 is negative, the charges would accelerate toward each other, and the PE would decrease (become more negative) as r got smaller.

21-43 (a) The voltage at the surface of the large sphere is kq_a/a, and on the small one kq_b/b. Since they're connected by a wire, they must be at equal voltage. Equating the two expressions gives $q_a/q_b = a/b$.
(b) The area of a sphere is proportional to r^2, so the charge per unit area is proportional to qr^{-2}. Therefore

$$\frac{\text{density of charge on sphere } b}{\text{density of charge on sphere } a} = \frac{q_b b^{-2}}{q_a a^{-2}} = \frac{a}{b}$$

21-44 The resistor on the left looks different from the others, but actually it's connected to exactly the same things they are. The whole circuit is just four resistors in parallel, but drawn in a funny way. By the loop rule, the voltage across each of the resistors is the same as the battery's voltage V, and therefore each resistor has a current $I = V/R$. The total current is $4V/R$.

21-45 (a) Let the voltage of the battery be V. In the original circuit, we have $P = IV = V^2/R$, while in the new one $P' = V^2/(R+r)$. By forming the ratio of these two quantities, we can eliminate the voltage, and $P/P' = 1 + r/R$. The result is $r = (P/P' - 1)R$.
(b) The ratio P/P' is unitless, which makes sense because we're subtracting a unitless 1 from it. The result is that ohms equal ohms, which makes sense.
(c) For $P' = P$, we have $r = 0$, which makes sense: no additional resistance was introduced, no no change occurred. For $P' = 0$, we have $r = \infty$, which also makes sense. For example if the dirt is a perfect insulator, then we have an open circuit.
(d) The current in the modified circuit is $I =$

$V/(R+r)$, which gives $P' = I^2R = V^2R/(R+r)^2$. After a little algebra, the result is

$$r = \left(\sqrt{\frac{P}{P'}} - 1\right) R,$$

which, remarkably, differs from the result of part a only by the presence of the square root.

22-1 $E = V/d = 70 \text{ mV}/6.0 \text{ nm} = 1.2 \times 10^7$ V/m

22-2 (a) $V = Ed = 1.8 \times 10^3$ V

(b) Since $V = Ed$, a larger d would require a larger V.

22-3 (a) $F = ma$ and $F = qE$, and since the field is uniform, the force and acceleration will be constant. For motion with constant acceleration, we have $v = at$, and eliminating the unknown F gives $v = qEt/m$.

(b) You can conclude that they have the same ratio of q/m, but you can't tell whether they have the same q and the same m. One could have twice the charge and twice the mass compared to the other, for instance.

22-5 Let the dipole consist of a charge q at a distance $\ell/2$ to the right of the origin, and a charge $-q$ at $\ell/2$ on the other side. The field at a point lying at a distance r to the right of the origin is then

$$\begin{aligned} E &= \frac{kq}{(r-\ell/2)^2} - \frac{kq}{(r+\ell/2)^2} \\ &= \frac{kq}{r^2}\left[\frac{1}{(1-\ell/2r)^2} - \frac{1}{(1+\ell/2r)^2}\right] \\ &\approx \frac{kq}{r^2}\left[\left(1+\frac{\ell}{r}\right) - \left(1-\frac{\ell}{r}\right)\right] \\ &= \frac{2kq\ell}{r^3} \\ &= \frac{2kD}{r^3} \end{aligned}$$

22-6 (a) Imagine breaking the ring down into many tiny pieces, like a necklace. These pieces all contribute to the field at the point we're talking about, and their contributions add like vectors. The components of these vectors parallel to the plane of the ring all cancel out, but the components along the axis don't. Each contribution to the field is at an angle to the axis, which we call θ, and which can be visualized as the angle halfway across the mouth of the cone formed by the ring and the point. The contribution of each vector to the field equals the vector's magnitude multipled by $\cos\theta$. Letting r be the distance from the ring to the point, we have, e.g., $E_{37} = (kq_{37}/r^2)\cos\theta$ for the contribution of the 37th piece of charge to the total field along the axis. Adding up $E_1 + E_2 + \ldots$, everything on the right side is the same for every term, except for the individual charges, $q_1, \ldots$ which just add up to q, so we have $E = (kq/r^2)\cos\theta = kqa/r^3$, which is the same as the result claimed in the problem.

(b) For $a = 0$, the field is zero, which makes sense, because we expect perfect cancellation at the center of the ring. For very large values of a, b becomes negligible compared to a, so the field is $kqa(a^2)^{-3/2} = kq/a^2$, which is just what we'd expect for any charge q at a distance a. In other words, the field doesn't depend on the details of how the charge is distributed inside a tiny region very far away.

22-7 In the earlier example, we considered the field of a finite-length segment of the line of charge. Analogously, we now consider the field of a finite-width piece of the plane. This ribbon-shaped piece is infinite along one axis, but finite in the perpendicular direction. The ribbon piece can be thought of as being composed of lines, each of whose field falls off like $1/r$. Therefore the field of the ribbon must fall off like $1/r$. Now we perform a scaling of r by a factor of 2, as in the earlier argument, while keeping the angles the same. The new ribbon is twice as wide, and this doubles its charge. The rest of the argumet proceeds in the same was as the earlier one.

22-8 The voltage is the integral of the field with respect to position. We can use an indefinite integral and not worry about the constant of integration, since physically it just relates to the arbitrary choice of a reference point.

$$\begin{aligned} V &= -\int E \; \mathrm{d}r \\ &= -\int \frac{d}{r^3} \; \mathrm{d}r \\ &= \frac{d}{2r^2} \end{aligned}$$

22-9 This is the opposite of the previous problem. To find the field given the voltage, we take the derivative:

$$\begin{aligned} E &= -\frac{\mathrm{d}}{\mathrm{d}r}\frac{k}{r^3} \\ &= \frac{3k}{r^4} \end{aligned}$$

22-10 The voltage is

$$\begin{aligned} V &= -\int E \, \mathrm{d}x \\ &= -ax - \frac{1}{4}bx^4 \qquad , \end{aligned}$$

where the constant of integration is irrelevant because the electrical energy is only meaningful up to an arbitrary additive constant. Conservation of energy gives

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 &= \frac{1}{2}mv^2 + eV(x_2) \\ v &= \pm\sqrt{-\frac{2eV(x_2)}{m}} \\ &= \pm\sqrt{\frac{2e(ax_2 + \frac{1}{4}bx_2^4)}{m}} \qquad . \end{aligned}$$

Note that for certain values of a, b, and x_2, we could end up with a negative quantity inside the square root. If so, then that would be Mother Mathematics' way of telling us that the proton would turn around before reaching position x_2.

If the quantity inside the square root is positive, then we have two square roots, a positive one and a negative one. This tells us that the proton could oscillate back and forth through position x_2 more than once.

22-11 Let the cylinder's mouth be centered on the origin, and let the cylinder extend off along the positive x axis. Call its radius b. We imagine slicing the cylinder like a salami into a series of rings, with each ring extending from x to $x+\mathrm{d}x$. The area of such a ring equals its circumference times its length, $(2\pi b)(\mathrm{d}x)$, so its charge is $\mathrm{d}q = 2\pi\sigma b \, \mathrm{d}x$. Using the technique demonstrated in the solution to problem 9,

$$E = \int \mathrm{d}E \qquad ,$$

and inserting the result of problem 8, we have

$$\begin{aligned} &= \int_0^\infty \frac{kx \, \mathrm{d}q}{\left(b^2 + x^2\right)^{3/2}} \\ &= 2\pi k\sigma b \int_0^\infty \frac{x \, \mathrm{d}x}{\left(b^2 + x^2\right)^{3/2}} \qquad . \end{aligned}$$

This integral can probably be done using trig substitutions or integration by parts, but in real life you'd want to look it up in a table or evaluate it on a computer. To get it into the form you'd find in a table, we do the change of variable $u = x/b$ to clear the inside of the integral of everything but x stuff.

$$E = 2\pi k\sigma \int_0^\infty \frac{u \, \mathrm{d}u}{\left(1^2 + u^2\right)^{3/2}}$$

Before we even look up the integral, we see something interesting: the constant b has gone away! To see why this is, imagine doubling b. Consider a piece of the cylinder near the mouth. This piece has now been made twice as large in radius and also twice as long (as has every other, similar cylindrical piece farther down the line). Its surface area has been quadrupled, so it has four times as much charge as before. However, it is now twice as far from the origin, and electric fields scale with the inverse square of the distance, so the greater charge is compensated for by the greater distance. The same argument holds for every other part of the cylinder, so we find that the strength of the field doesn't depend on b. This answers part (b) of the problem.

Now we go back and finish the integral to answer part (a). Going to integrals.com, I found that the indefinite integral was $-1(1+u^2)^{-1/2}$, and plugging this in for the definite integral we find that it equals 1. Tacking on the constants out in front, we have the desired result.

22-12 (a) The energy per unit volume is

$$\frac{E}{V}=\frac{1}{8\pi k}|\mathbf{E}|^2 \qquad .$$

(E is energy, $\mathbf{E}$ is electric field.) At the moment when the lightning strikes, the field equals E_c. The volume of the air in which the field exists is L^2h.

$$E=\frac{1}{8\pi k}E_c^2L^2h$$

(b) A greater h gives a greater energy, so it's more dangerous if it's higher.

(c) Let's use 1000 m for both L and h. Plugging in, the result is about 10^{10} J.

22-13 (a) Its volume is (length) × (circumference) × (thickness) = $(L)(2\pi r)(t)$. The electric field is $E=V/t$. The energy stored is

$$\begin{aligned}\frac{1}{8\pi k}|\mathbf{E}|^2\times(\text{volume}) &= \frac{1}{8\pi k}\left(\frac{V}{t}\right)^2(2\pi rLt)\\ &= \frac{V^2rL}{4kt}\end{aligned}$$

(b) Making r small means that the neuron needs less energy stored in it to have voltage V across its membrane, which may explain why they evolved to be so skinny. There is a limit on how small r can be, however, because the cell can't be so skinny that it lacks room inside for its machinery. Having a thicker membrane would also reduce the energy needed (dividing by a larger number gives a smaller result), but a cell membrane can't be arbitrarily thick, because it needs to take in nutrients and expel wastes through it.

22-14 (a) Consider a ring extending from r to $r+\mathrm{d}r$ around the wire. Its volume is proportional to its circumference and to $\mathrm{d}r$, so $\mathrm{d}V=(\ldots)r\,\mathrm{d}r$, where the ... stands for stuff that we don't care about because it won't affect whether the result is infinite. The magnetic field varies as $1/r$, and to integrate the energy, we have to integrate something that has $B^2\,\mathrm{d}V$, which varies as $(1/r^2)r$, or $1/r$. When you integrate $1/r$, you get a logaritm, which blows up both at $r=0$ and at $r=\infty$.

(b) The divergence at zero assumes an infinitely thin wire, which doesn't exist. The divergence at infinity also makes an unrealistic assumption, which is that the wire is infinitely long. The equation for the field is only a good approximation at points whose distance from the wire is small compared to the length of the wire.

(c) A spherical shell extending from r to $r+\mathrm{d}r$ has a volume proportional to its surface area and to $\mathrm{d}r$, so $\mathrm{d}V=(\ldots)r^2\,\mathrm{d}r$. The field depends on $1/r^2$, so the energy density goes like $1/r^4$, and the thing we're integrating has a $1/r^2$. Integrating this gives $1/r$, which diverges at zero, but not at infinity.

22-15 (a) The energy stored in each one is $(1/8\pi k)E^2V$, and all the factors equal one except for the $(1/8\pi k)$ in front, which gives a numerical value of 4.42×10^{-12}, in units of joules.

(b) The vectors are perpendicular, so putting them tip-to-tail for vector addition results in a 45-45-90 right triangle. The magnitude of the total field is $\sqrt{2}$ times the original value, or 1.41 V/m.

(c) In the original, separate configuration, the total energy is $2\times4.42\times10^{-12}$ J $=8.84\times10^{-12}$ J. In the put-together setup, the energy density of the field is doubled $(\sqrt{2}^2)$, but the volume is halved, so the energy is exactly the same as before.

22-16 The energy density is proportional to the square of the field strength, so decreasing the field by a factor of 10 is equivalent to cutting the energy by a factor of 100.

23-1 The equation for gamma is $\gamma=1/\sqrt{1-v^2}$. If v is negative, v^2 still comes out positive. Negative and positive values of v give the same results for gamma.

This makes sense, because positive and negative values of v indicate motion in opposite directions. Relativistic effects like time dilation and length contraction shouldn't come out different if you travel west rather than east, or north rather than south. Gamma indicates how strong these relativistic effects are, so gamma shouldn't depend on the direction of motion. This relates directly to the second postulate stated in section 7.2, that all directions in space have the same properties.

Note that it *is* OK for a directional (vector) quantity like velocity to be connected to some other direction quantity, such as acceleration, by a relation such as $a = dv/dt$. Then an observer who chooses the opposite orientation for their coordinate system will simply flip all the signs of the velocities and accelerations, and there is no way to tell in experiments which choice is "special." That's different from the situation with γ, which is not directional and can be measured without even choosing a coordinate system.

23-2 (a) $v = (\text{slope}) = (\text{rise})/(\text{run}) = x/\gamma$, so $x = v\gamma$. (b) Because we're using units in which $c = 1$, the diagram has symmetry with respect to reflection across the 45-degree diagonal. Therefore the (t, x) coordinates of P and Q are simply related by a swap of t and x, and Q has coordinates $(v\gamma, \gamma)$. (c) Points P and Q are separated by $|\Delta t| = (1 - v)\gamma$, and $|\Delta x| = (1 - v)\gamma$ as well. The Pythagorean theorem results in $\text{PQ} = \sqrt{2}(1 - v)\gamma$. (d) Averaging the coordinates of points P and Q, we find that C has coordinates $x = t = (1 + v)\gamma/2$. The Pythagorean theorem then gives $\text{OC} = (1 + v)\gamma/\sqrt{2}$. (e) The area of triangle PQO is $(1/2)(\text{base})(\text{height}) = (1/2)\text{PQ} \cdot \text{OC} = (1/2)(1 - v)(1 + v)\gamma^2$, which simplifies to $(1/2)(1 - v^2)\gamma^2$. Doubling this to find the area of the quadrilateral, we obtain $(1-v^2)\gamma^2$, and setting this equal to 1 and solving for γ gives the desired result.

23-3 (a) She says her own velocity is zero.

Time dilation happens because of relative motion. We can conclude that A and B are not in motion relative to one another, but that there is relative motion between them and C.

(b) Alice says B is at rest, but C is moving. (c) Betty says A is at rest, but C is moving. (d) Cathy says both A and B are moving, and she sees them moving in the same direction at the same speed.

23-4 (a)

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-(v/c)^2}} \\ &= \frac{1}{\sqrt{1-\left(\frac{17000\ \text{m/s}}{3\times10^8\ \text{m/s}}\right)^2}} \\ &= 1.0000000016\end{aligned}$$

(b) Gamma gives us a comparison of the rate at which time flows on earth and on the probe. A gamma of one would indicate equal rates. Since the gamma is a tiny bit greater than one, the ratio is a tiny bit different from a one-to-one ratio. Gamma differs from one by 1.6×10^{-9}, i.e., 1.6 parts per billion. The disagreement between clocks on the probe and on earth that accumulates over each year is therefore

$$(31 \times 10^6\ \text{s})(1.6 \times 10^{-9}) = 0.050\ \text{s} \qquad .$$

This is a fairly big discrepancy. You might even be able to detect it with a consumer-grade clock. (The way NASA really notices the effect is that when the probe signals back to earth by radio, the radio waves vibrate a little more slowly than they should, i.e., the signal's frequency is shifted a tiny bit.)

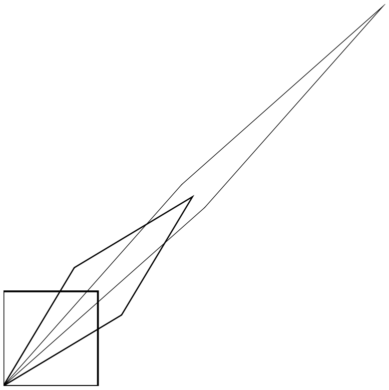
23-5 Let $r = 6.4 \times 10^6$ m be the radius of the earth, $R = 1.5 \times 10^{11}$ m the radius of its orbit, and $T = 1$ year $= 3.2 \times 10^7$ s the period of its orbit. Then the velocity is $v = (2\pi R)/T$, and we have $\gamma = 1/\sqrt{1 - v^2/c^2} = 1 + 4.8 \times 10^{-9}$. The resulting contraction of the diameter is about $(4.8 \times 10^{-9})(2r) = 6$ cm.

23-6 We can do this by using postulates 1-4 from the original list, plus the observation of length contraction. Postulates 1-4 are enough to prove that the only possible methods of converting between frames of reference are those labeled I and III in the text, i.e., Galilean relativity and the Lorentz transfomation. Length contraction doesn't occur in Galilean relativity, so the Lorentz transformation is the only one left.

23-8 Unlike the example of the school bus paradox, the answer here is not frame-dependent. We are not asking whether all parts of the ball fit in the hole simultaneously (which would be a frame-dependent question because simultaneity is frame-dependent). We are simply asking whether it is possible that no part of the ball ever intersects any part of the wall. Intersections of world-lines are frame-independent events.

Length contraction will not help the ball to fit through the hole, because length contraction occurs only along the direction of motion, not along the transverse axes.

24-1 To get good precision, you need to do this on a full-size printout, as suggested by the problem. To keep from using up a whole page on my solutions handouts, I've shown the idea below at about 25% of the actual size.



First I drew the long diagonal twice as long again, meaning about 276 mm. To keep the area the same, I had to make its "cross-bar" half as long as on the preceding parallelogram, or about 17.5 mm. The speed represented by this new parallelogram is equal to the slope of its bottom edge. To get this slope, which is the rise divided by the run, we have to draw the rise and run accurately, i.e., lined up straight with the edges of the paper. I did this by using a second piece of paper as a right angle. The result is that the slope is (rise)/(run) = 91 mm/105 mm = .86, representing 86% of the speed of light.

24-2 (a) As the relative velocity gets closer and closer to c, the length of the long diagonal of the parallelogram approaches infinity. But the area is supposed to stay the same, so this means the short diagonal's length must approach zero. The parallelogram becomes a line. But one of our basic rules here is that observers in different frames of reference must always agree on whether or not events are the same, and this rule is being violated here, because points that were distinct on the original square are on top of each other when the square is compressed to a line. The conclusion is that we can never have a frame of reference that moves at c relative to another frame of reference.

(a) The paradox implicitly assumes the existence of a frame of reference that moves along with beam number 1, so that we can then talk about the velocity of beam 2 in beam 1's frame of reference. But there aren't frames of reference that move at c.

24-3 If the little solenoid's current goes the same way as the big one's, then their fields reinforce and the energy density is higher than if they were separate. It's true that the fields are filling some of the same space, which would tend to reduce the total energy, but since the energy density depends on the square of the field, the increase in energy density is a bigger effect than the decrease in the amount of space. For instance, if the two fields are equal, and the volumes are nearly equal, then the combined field has an energy density four times greater, which more than compensates for the fact that the fields are occupying half the volume compared to the separate setup.

If the little solenoid's current goes the opposite way, their fields cancel at least partially, and the energy is less than when they were separate. The stable configuration is the one with the opposite currents.

24-4 (a) Using the equation for the magnetic field at the center of a current loop, we get 0.063 mT.

(b) Along the positive z axis.

(c) It would make a field with the same magnitude along the negative x axis.

(d) The total field is the vector sum of the two. Since they're perpendicular, the magnitude of the sum is given by the Pythagorean theorem, coming out to be 0.09 mT. It points at a 45° angle between the two, in the $x - z$ plane.

24-5

24-6 The electric force is either in the same direction as the electric field or in the opposite direction, depending on whether the particles are positively or negatively charged. Since the magnetic force is perpendicular to both **v** and **B** and the question states that the electric field is perpendicular to both **v** and **B**, the magnetic force must be either in the same direction as the electric force or in the opposite direction. Since the goal is to make the forces cancel, we'll assume the directions of the fields are chosen so the forces are in opposite directions. That means the forces cancel if they're chosen to have equal magnitudes. The electric force equals $q\mathbf{E}$. The magnitude of the magnetic force equals $qvB\sin\theta$, and since the **v** and **B** vectors are perpendicular, $\sin\theta = 1$. Therefore we need $qE = qvB$. Canceling q from both sides and solving for v, we have $v = E/B$.

24-7 Four times the current makes four times the magnetic field, which makes $4^2 = 16$ times the energy.

24-8 The compass will respond to the combined fields of the magnet and the earth. There is no fundamental difference between the two- and three-dimensional cases, so for simplicity let's imagine the mystery magnet and the compass both being glued to a flat piece of wood, which we can orient at any angle in a horizontal plane. Before gluing the compass to the wood, we determine the direction of the earth's field. Then, with the compass glued down, we fiddle with the board's orientation until we get the compass to point in the direction of the earth's field again. In this orientation, the mystery magnet's field is along the same line as the earth's field. But is it north (reinforcing the earth's field) or south (opposing it)? If the magnet's field is stronger than the earth's, then we can easily tell by flipping the board 180 degrees; the compass will point in the direction of the magnet's field in both cases. If the magnet's field is weaker than the earth's, it's a little harder. The simplest technique is to turn the magnet away from the aligned direction by some small angle, say 5 degrees, and observe which way the needle is deflected. By one of these two methods, we can determine the direction of the magnet's field for once and for all, and draw it with a marking pen on the board.

Now we have to find the strength of the magnet's field. This is fairly easy. For instance, we can turn the magnet so that it's field is at a 90-degree angle with respect to the earth's, and measure the angle by which the compass deviates from magnetic north. We're now adding two field vectors that are at 90 degrees to one another, and this vector addition can be visualized in terms of a right triangle if we put the two vectors tip to tail. The tangent of the angle tells us the ratio of the two legs, which is the ratio of the magnet's field to the earth's.

24-9 All that really matters is the cross-sectional area, a, of the smaller solenoid, and their separate fields, b and B. As the small one penetrates the big one to a depth x, we replace a volume xa from each one's interior with a new, overlapped volume xa in which the field is $B + b$. (a) The difference in energy is

$$\frac{c^2}{8\pi k}\left[(B+b)^2 - B^2 - b^2\right]xa = \frac{c^2}{4\pi k}\cdot Bbxa \qquad .$$

(b) The energy is proportional to x, so for every millimeter deeper that we push the small one, the same amount of work is done. The force is $F = \mathrm{d}W/\,\mathrm{d}x = (c^2/4\pi k)Bba$.

24-10 The hard part is visualizing the directions of the four fields that are adding together. Looking along the length of the apparatus, two of the fields point at a 45-degree angle down and to the left, while the other two point at 45 degrees down and to the right. By vector addition, this means that the total field is $2\sqrt{2}$ times the individual ones. Each individual field is $2kI/c^2R = 2\sqrt{2}kI/c^2b$ so the total field is $8kI/c^2b$.

24-13 Building the model, you have to sight along each vector in turn, and check the remaining two vectors to see which one is clockwise compared to the other. The following combinations occur,

$$\begin{aligned}&\mathbf{F},\mathbf{B},\mathbf{v}\\&\mathbf{v},\mathbf{F},\mathbf{B}\\&\mathbf{B},\mathbf{v},\mathbf{F}\qquad,\end{aligned}$$

while these don't:

$$\begin{aligned}&\mathbf{B},\mathbf{F},\mathbf{v}\\&\mathbf{v},\mathbf{B},\mathbf{F}\\&\mathbf{F},\mathbf{v},\mathbf{B}\end{aligned}$$

The first three are made by repeatedly taking the symbol off of the right end and sticking it on at the beginning. Physically, relabeling the three rods on your model in this way simply corresponds to reorienting your model in space, like changing a letter p to a letter d by rotating it 180 degrees. The later ones would be like q and b: you can't get them by rotating p and d in the plane of the page. (This is an inexact analogy, because you can turn a p into a q by flipping the page over. The p-q-b-d example only works if you restrict yourself to two dimensions, while the $\mathbf{F}-\mathbf{B}-\mathbf{v}$ relationship is inherently three-dimensional.)

24-14 We can break approximate any planar shape by breaking it up into a grid of tiny squares. Suppose we make each of these tiny squares into a magnetic dipole by sending a current I around it; each of these currents I is of the same magnitude as the current I around the original loop, and they are either all clockwise or all counterclockwise, to match the direction of circulation of the current around the original loop. The torque on the whole shape is the same as the sum of the torques on the tiny squares, because wherever two adjoining squares have a side in common, their currents are in opposite directions, and the two torques cancel. That is, the only currents that actually contribute to the total torque are the ones around the outside of the whole shape. We already know that the torque on a single square is proportional to IA. Since the areas add, and the torques add, that means that the torque depends only on IA, regardless of the shape.

24-15 The field at any point on the axis can be found by adding the contributions to the field from the two loops. By symmetry, both these fields are along the axis, so we can add them without worrying about vector addition. The whole problem just deals with comparisons, so the only part of the equation in section 24.2 we need to worry about is $(b^2+z^2)^{-3/2}$. Also, we can just let b equal 1, and interpret z as z/b.

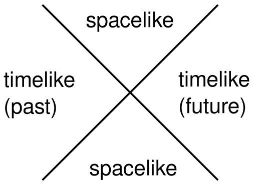
(a) Evaluating $(1+z^2)^{-3/2}$ at $z=0$ and 0.5, and 1, we get 1.0000, 0.7155, and 0.3536. The total field at the center ($z=0.5$ relative to both coils) is 0.7155×2, and the field in the center of one of the coils is $1.0000+0.3536$. The percentage drop is about 5.4% (expressed as a percentage of the stronger field).

(b) By experimenting with different values of h, we find a solution at about $h/b=1.2$, i.e., a little larger than the Helmholtz value of $h/b=1$.

24-20 At a given point in space, the magnitudes of the electric field and magnetic field are related by $E=cB$. The energy density of the magnetic field is $\frac{c^2}{8\pi k}B^2$, and the energy density of the electric field is $\frac{1}{8\pi k}E^2=\frac{1}{8\pi k}(cB)^2=\frac{c^2}{8\pi k}B^2$..

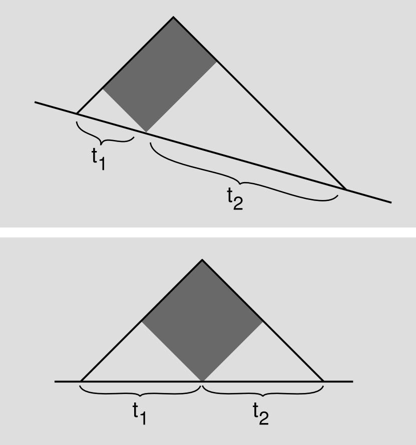
24-22 (a) They both have units of seconds squared.

(b)



(c) The original definition refers to the area of the light-rectangle. In the lightlike case, the light-rectangle squashes flat and has zero area. In Geroch's definition, O can emit the light ray from A and have it be received at B, so $t_1=0$, and therefore $\mathcal{I}=0$.

(d) The top of the following diagram shows the light-rectangle superimposed on the original diagram illustrating Geroch's definition. By switching to the frame in which O is at rest, we get the bottom diagram. Lorentz transformations preserve area, so the two light-rectangles have the same area A, but the second one's area is easier to calculate. In this frame, $t_1 = t_2$, and the light-rectangle is a square with sides of length $t_1/\sqrt{2}$, so that $-t_1 t_2 = -2A$. The left-hand side is Geroch's definition, and the right-hand side is the original definition.



(e) Suppose that observer O′ visits the same three events as O, but O′ insists on moving around noninertially in between. Then due to relativistic time dilation the clock times t'_1 and t'_2 measured by O′ will both be longer than the corresponding quantities measured by O, and $-t'_1 t'_2$ will be larger than $-t_1 t_2$ in absolute value.

24-23 In the figure, a field that was purely electrical in the original frame would lie on the bottom edge of the square. A field that was purely magnetic in the new frame would lie on the slanted left side of the parallelogram. The answer to the question is no, because a Lorentz transformation will never bring one of these lines on top of the other; Lorentz transformations squish and stretch the coordinate grid, but they never flip it over.

25-1 Solving $\omega \approx 1/\sqrt{LC}$ for C, we get $C = 1/L\omega^2 = 1/4\pi^2 L^2 f^2$. Looking at the nearest handy FM radio dial, the range of frequencies is about 88-108 MHz, so the capacitor needs to cover the range from about 2.2 to 3.3 pF.

25-2 (a) By definition, one henry is 1 J/A^2. The quantity $L\,\mathrm{d}I/\,\mathrm{d}t$ therefore has units of

$$\begin{aligned}\left(\frac{\mathrm{J}}{\mathrm{A}^2}\right)\left(\frac{\mathrm{C/s}}{\mathrm{s}}\right) &= \left(\frac{\mathrm{J}}{(\mathrm{C/s})^2}\right)\left(\frac{\mathrm{C/s}}{\mathrm{s}}\right)\\ &= \frac{\mathrm{J}}{\mathrm{C}}\\ &= \mathrm{V}\end{aligned}$$

(b) A farad is defined as 1 C^2/J, so RC has units of

$$\begin{aligned}\Omega\left(\frac{\mathrm{C}^2}{\mathrm{J}}\right) &= \left(\frac{\mathrm{V}}{\mathrm{A}}\right)\left(\frac{\mathrm{C}^2}{\mathrm{J}}\right)\\ &= \left(\frac{\mathrm{J/C}}{\mathrm{C/s}}\right)\left(\frac{\mathrm{C}^2}{\mathrm{J}}\right)\\ &= \mathrm{s}\end{aligned}$$

(c) L/R has units of

$$\begin{aligned}\left(\frac{\mathrm{J}}{\mathrm{A}^2}\right)\Omega^{-1} &= \left(\frac{\mathrm{J}}{\mathrm{A}^2}\right)\left(\frac{\mathrm{A}}{\mathrm{V}}\right)\\ &= \frac{\mathrm{J}}{\mathrm{A}\!\cdot\!\mathrm{V}}\\ &= \frac{\mathrm{J}}{\mathrm{W}}\\ &= \frac{\mathrm{J}}{\mathrm{J/s}}\\ &= \mathrm{s}\end{aligned}$$

25-3

$$\begin{aligned}U &= \frac{q^2}{2C}\\ &= \frac{(CV)^2}{2C}\\ &= \frac{1}{2}CV^2\end{aligned}$$

25-4 The definition of inductance is $U = LI^2/2$. If we think of the pair of inductors as one big inductor, then applying this definition, I has to be the total current that goes in. This gets split in half, so the energy stored in a single inductor is $U_{one} = L_{one}(I/2)^2/2$, and the total energy is $U = 2U_{one} = L_{one}I^2/4$. Equating this to $LI^2/2$, we get $L = L_{one}/2$. You get half the inductance by putting two in parallel.

25-5 We normally think of wires as having zero resistance. If the wire has some inductance, then the added impedance is higher at high frequencies, so stray inductance will be most important at high frquencies. The impedance of a capacitor, however, varies inversely with frequency, so stray capacitance will be most important at low frequencies.

25-6 (a) The capacitors have a total of four plates, and the two in the middle, which are connected, are isolated, so their total charge will always be zero. Therefore, the two middle plates must have the same amount of charge but with opposite signs. Thus the pattern of charges on the four plates must be $+q$, $-q$, $+q$, $-q$. If we're just connecting to them from the outside, we can think of them as one big capacitor, and we don't care about the middle plates; we just think about the charges $+q, \ldots -q$ on the ends. Each capacitor stores an energy $U_{one} = q^2/2C_{one}$, so the total energy is $U_{combined} = 2U_{one}$. Setting this equal to $q^2/2C_{combined}$, we get

$$C_{combined} = C_{one}/2 \qquad .$$

(b) Doubling the distance between the plates is like sandwiching together two capacitors in series, so it cuts the capacitance in half. The capacitance is evidently inversely proportional to the separation between the plates.

25-7 The energy of a spherical capacitor,

$$U = \frac{kq^2}{2}\left(\frac{1}{a} - \frac{1}{b}\right) \qquad ,$$

was derived in an example, and when b approaches infinity, we have simply $U = kq^2/2a$. The definition of capacitance is $U = q^2/2C$, so we have $C = a/k$. Plugging in the radius of the earth yields 710 μF. This is a surprisingly small value, considering how big the earth is!

25-8 The reasoning is very similar to the reasoning by which we derived the impedance of a capacitor. Suppose an inductor has a current $I = \tilde{I}\sin\omega t$ passing through it. The voltage difference across the inductor is

$$\begin{aligned} V &= L\,\frac{\mathrm{d}I}{\mathrm{d}t} \\ &= L\omega\tilde{I}\cos\omega t \qquad . \end{aligned}$$

The amplitude of this voltage is $L\omega\tilde{I}$. The impedance is defined as the amplitude of the voltage divided by the amplitude of the current, so we get $L\omega$.

26-1 A bowl of Cheerios with skim milk has about 150 nutritional "calories," which is actually 150 *kilocalories*, or 6×10^5 J. I'd guess it's about 0.2 kg worth of food, which would be something like 10^{25} atoms. The energy in one atom (rounded off to one sig fig) would be about 10^{-19} J.

26-3 [Note: An old version of the problem listed the parent isotope incorrectly as ^{234}Pu. It should be ^{241}Pu.]

Plutonium is atomic number 94. In alpha decay, we're removing two protons and two neutrons, so the mass number will be reduced by four, and the atomic number (number of protons) by 2. The nucleus left over after alpha decay therefore has atomic number 92 (uranium), and mass number 237; it's ^{237}U.

In electron decay, we're converting one neutron to a proton. The mass number is unchanged, but the atomic number is bumped up by one, to 95, americium. The product is ^{241}Am.

26-4 (a) Coulomb's law is directly analogous to Newton's law of gravity, except for the fact that there are two types of charge, and the forces can be either repulsive or attractive. Se we can translate directly from the gravitational case to the electrical case just by changing each symbol to the analogous electrical quantity, as long as we make sure to put in the plus and minus signs by hand. Regarding the sign, two positive masses (which is the only kind that exist!) attract, but two positive charges repel. This means that, at least in the case of two positive charges, the sign has to be reversed for the electrical equation: $PE = kq_1q_2/r$. To check that this makes sense, we can imagine two positive charges being released, and accelerating away from each other because of their mutual repulsion. As r gets bigger, dividing by r gives a smaller result for the potential energy, and that makes sense, because PE has to be converted to KE. Two negative charges would also accelerate away from each other and convert PE to KE as r increased, so the same equation still works ($q_1q_2 > 0$). In fact, the equation even works when one charge is positive and one is negative; here, the product q_1q_2 is negative, the charges would accelerate toward each other, and the PE would decrease (become more negative) as r got smaller.

(b) The equation only applies when the electron is outside the cookie, so we're estimating the energy difference between having the electron at the surface of the cookie and having the electron infinitely far away. The final potential energy is zero (dividing by $r = \infty$ gives 0). For the initial potential energy, we plug in the basic unit of charge, e, for each charge, and $r = 10^{-10}$ m, giving $PE = -2 \times 10^{-18}$ J. The change in PE is final − intial = 2×10^{-18} J, which is roughly of the right order of magnitude compared to the result of problem #2.

26-5 An electron in the tube is feeling repulsive forces from all the negatively charged particles in one end cap, and attractive forces from the positively charged ones at the opposite end. The total force on it is the vector sum of all these individual forces. According to Coulomb's law, each individual force is proportional to the charge of the electron. The situation is the same for the singly charged positive neon atom (ion). Since the magnitude of the neon atom's charge is the same as the magnitude of the electron's charge, all the forces on it are of the same magnitude, but opposite in direction. Therefore the magnitude of the total force on the neon atom is the same as the magnitude of the total force on the electron. $F = ma$, so equal forces will result in a much greater acceleration for the electron. The electron's mass is about 4×10^4 times less, so its acceleration is greater by a factor of 4×10^4.

26-6 There are four particles involved: two electrons and two positively charged spheres. (There are no protons in this model.) Atom A and atom B experience two repulsive forces (A's positive sphere on B's positive sphere and A's electron on B's electron) and two attractive forces (A'selectron on B's positive sphere, and A's positive sphere on B's electron). If the four forces were all of the same strength and along the same line, then they would cancel. However, Coulomb's law says that the strengths of the forces depend on the distances. (For the positive spheres, we'd measure the distance from the center.) The electrons are free to move around, so at any given moment in time, the distances involved for each of the four forces are different. Therefore the four forces will all be different in magnitude, and should not be expected to cancel. Also, the forces add like vectors, not scalars, and they are not all lined up along one line, and that's another reason why they should not cancel out exactly to zero.

Note that this does *not* explain why the net force is attractive; it merely explains why it doesn't necessarily have to be zero. Using the physics you know right now, there is no reason to think that attraction would be any more likely than repulsion.

26-8

26-9 Its kinetic energy is equal to its mass-energy minus its resting value,

$$\begin{aligned} K &= \mathcal{E} - mc^2 \\ &= m(\gamma - 1)c^2 \\ &= (8.0 \times 10^7 \text{ kg})(\frac{1}{\sqrt{1-(1/2)^2}} - 1)c^2 \\ &= 1.1 \times 10^{24} \text{ J} \end{aligned}$$

This is a thousand times greater than the total energy content of the world's nuclear arsenals. In other words, the Enterprise is the ultimate weapon of mass destruction. If it crashed into a planet (as it did, in one of the movies), it would destroy all life on the planet's surface.

26-10 (a) In units of 10^{-27} kg, the amount of mass lost is

$$1.67495 - 1.67265 - 0.00091 = 0.00139 \qquad ,$$

i.e., 0.00139×10^{-27} kg. This mass has been converted into energy:

$$\begin{aligned} E &= mc^2 \\ &= (0.00139 \times 10^{-27})(3 \times 10^8)^2 \\ &= 1.25 \times 10^{-13} \text{ joules} \qquad . \end{aligned}$$

(In the meter-kilogram-second system of units, kilograms multiplied velocity squared give units of joules.)

(b) For this process, the total mass changes from 1.67265 to 1.67586 (again in units of 10^{-27} kg). This is an *increase* in mass, which means that we would have to have a source of energy to make the reaction happen. A free proton has no such source of energy; it's all by itself, so there's nothing for it to get energy from.

26-11 (a)

$$\begin{aligned} p &= m\gamma v \\ &= \frac{mv}{\sqrt{1-v^2}} \\ p^2\left(1-v^2\right) &= m^2v^2 \\ p^2 &= \left(m^2+p^2\right)v^2 \\ v &= \frac{p}{\sqrt{m^2+p^2}} \end{aligned}$$

(b) At low velocities, the second term inside the square root is negligible, and we have

$$\begin{aligned} v &\approx \frac{p}{\sqrt{m^2}} \\ &= \frac{p}{m} \qquad , \end{aligned}$$

which is the nonrelativistic result.

(c) For very large momenta, the m^2 term is negligible compared to the p^2 term, so

$$\begin{aligned} v &= \frac{p}{\sqrt{p^2}} \\ &= \pm 1, \end{aligned}$$

which is the speed of light in natural units.

(d) Converting to SI units, we have

$$v = \frac{p}{\sqrt{m^2 + p^2/c^2}}.$$

26-12 (a) If v is very close to c then the denominator in $\gamma = 1/\sqrt{1-v^2/c^2}$ gets very close to zero. Dividing by a very small number gives a very big result. Ultrarelativistic particles have very large values of γ.

(b) The energy due to its mass is mc^2, which is a big number because c is big. However, the energy due to its motion equals $m(\gamma-1)c^2$, which is even bigger, because not only does it have the c^2 factor, it also has $\gamma - 1$, which is big. The particle has much more energy due to its motion than due to its mass.

(c) $\mathcal{E}/p = m\gamma c^2/m\gamma v = c^2/v$.

(d) For $v = c$, this becomes $\mathcal{E}/p = c^2/c = c$.

(e) The beam of light was emitted with a power of 1 watt, i.e., 1 joule per second. Since it lasted for one second, its energy is one joule.

$$\begin{aligned} p &= \mathcal{E}/c \\ &= (1 \text{ J})/(3 \times 10^8 \text{ m/s}) \\ &= (1 \text{ kg}\cdot\text{m}^2/\text{s}^2)/(3 \times 10^8 \text{ m/s}) \\ &= 3 \times 10^{-9} \text{ kg}\cdot\text{m/s} \end{aligned}$$

It has only a miniscule amount of momentum, which fits the correspondence principle: until Einstein came along, nobody even knew light had momentum at all.

26-13 (a) A piece of rope with length ℓ has volume $A\ell$ and mass $m = \rho A\ell$, so $\mu = m/\ell = \rho A$. Setting $v = \sqrt{T/\mu}$ equal to c and solving for T, we find $T = c^2\rho A$.

(b) Plugging in $\rho = 1 \text{ g/cm}^3 = 10^3 \text{ kg/m}^3$ gives a maximum tensile strength of $T/A = 10^{20}$ Pa, which is about a thousand times greater than that hypothesized for the strongest carbon nanotubes.

(c) Most likely the rope would break. The existence of a theoretical maximum strength for the rope also suggests that even the strongest rope theoretically possible would break.

26-16 (a) In units with $c = 1$, we have

$$\begin{aligned} v &= L/t \\ K &= m(\gamma - 1) \\ \tau &= t/\gamma. \end{aligned}$$

The solution of these equations is

$$K = m\left[\sqrt{1+\left(\frac{L}{\tau}\right)^2} - 1\right],$$

or, reinserting factors of c,

$$K = mc^2\left[\sqrt{1+\left(\frac{L}{c\tau}\right)^2} - 1\right].$$

(b) For the given data, the result is something like 10^{12} megatons of TNT. Your ultrarelativistic friend's body has so much kinetic energy that if he collided with the earth, it would be the end of the world, so I think Congress should pass a law prohibiting him from doing this.

27-2 As in figure f in section 27.1, the easy test is to figure out whether a piece of paper can be formed into the relevant shape without tearing or crumpling. We can form a piece of paper into a cone without tearing or crumpling it, so the answer is no, the creature could not detect any curvature. The only exception would be at the tip of the cone, because, for example, we can't extend a line beyond the tip, violating Euclid's second postulate.

27-3 (a) The number is supposed to represent the ratio of two times or the ratio of two frequencies, so it has to be unitless. The 1 is unitless, which makes sense. It remains to calculate the units of gh/c^2. These are $(\text{m/s}^2)(\text{m})/(\text{m/s})^2$, which does come out to be unitless.

(b) Yes. If the time dilation factor had been very different from 1 under ordinary conditions, then it would have been noticed hundreds of years ago, and nobody would have ever believed in the Newtonian concept of absolute time. But in reality, c is huge in metric units, while g and h in everyday life are not, so the time dilation factor will be very close to 1.

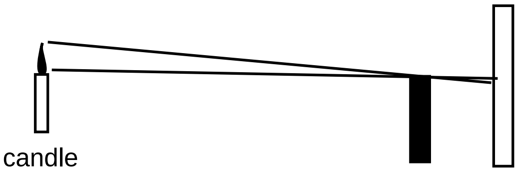
27-4 (a) The Doppler shift is a factor of $1 - gh/c^2$, which corresponds to a fractional change of $gh/c^2 = 2.5 \times 10^{-15}$. (b) The predicted fractional effect is $(1340\ \text{m})g/c^2 = 1.5 \times 10^{-13}$. The observed result is $(22\ \text{ns})/(48\ \text{hr}) = 1.3 \times 10^{-13}$. These seem consistent to within the precision of the experiment, as inferred by the sizes of the random fluctuations in the graph.

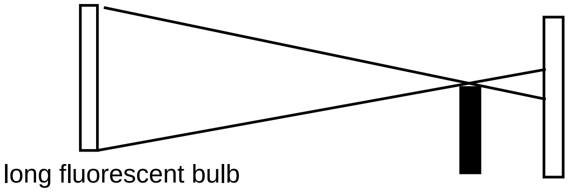
27-5 The kinematic effect slows down time on the ISS relative to time on the ground, while the gravitational one is in the opposite direction. Let's start by calculating the gravitational time dilation. The gravitational field g does fall off with altitude, but the decrease at an altitude of 350 km is not really all that much (see problem 10-22), so let's use the surface value of about 10 m/s^2, which is good enough to get a reasonable estimate. The time dilation factor is $1 - gh/c^2$, so the fractional change in the rate of time is $gh/c^2 \approx 4 \times 10^{-11}$. The kinematic time dilation factor is γ, and the corresponding fractional change is $\gamma - 1$. Plugging in the velocity, this comes out to be about 4×10^{-10}. We find that the kinematic time dilation is an order of magnitude greater than the gravitational one, so over all, time runs more slowly on the ISS than on the ground.

27-6 Equating the escape velocity $\sqrt{2GM/r}$ to c and solving for r we find that $r = 2GM/c^2$ comes out to be roughly a centimeter.

27-8 The easiest one to use is an octant, like the one bounded on the earth's surface by the equator, the Greenwich meridian, and the 90° – *east* meridian. It has an area of $4\pi R^2/8 = (1/2)\pi R^2$ and an angular defect of $\pi/2$, so we find that $d = A/R^2$.

28-1 The fuzzy part of the shadow is the part that is only illuminated by some of the rays from the light source. This is the area between the extreme rays shown in the figures. The fuzzy area is larger in the second figure.

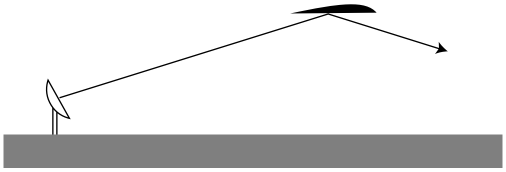




28-2 The time it takes light to travel a few meters, say 3 m, is (3 m)/(3×10^8 m/s) = 10^{-8} s.

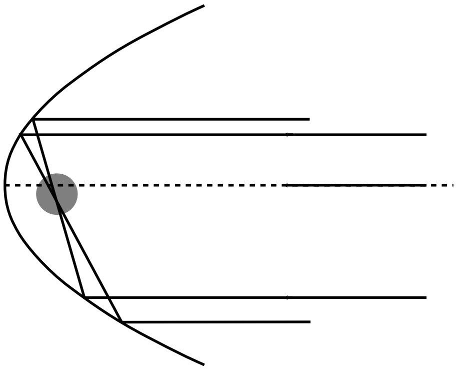
28-3 $f = c/\lambda = (3\times10^8\text{ m/s})/(10^{-9}\text{ m}) = 3\times10^{17}$ Hz, or call it 10^{17} Hz, since it's just an estimate. It would be UV or an x-ray.

28-4 Because the surfaces are flat, you get specular reflection. In specular reflection, all the reflected rays go in one direction. Unless the plane is directly overhead, that direction won't be the right direction to make the rays come back to the radar station.

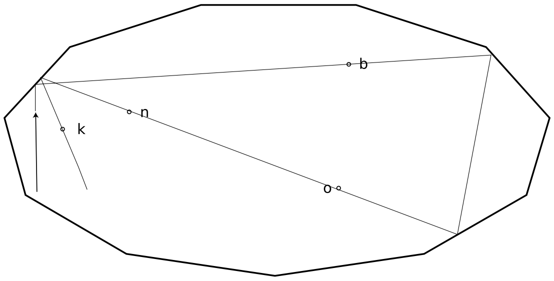


This is different from a normal plane, which has complicated, bumpy surfaces. These surfaces give diffuse reflection, which spreads the reflected rays randomly in more or less every possible direction.

28-5 (a) The rays all cross at pretty much the same place, given the accuracy with which we can draw them.
(b) It could be used to cook food, for instance. All the sunlight is concentrated in a small area.
(c) Put the lightbulb at the point where the rays cross. The outgoing rays will then form a parallel beam going out to the right.



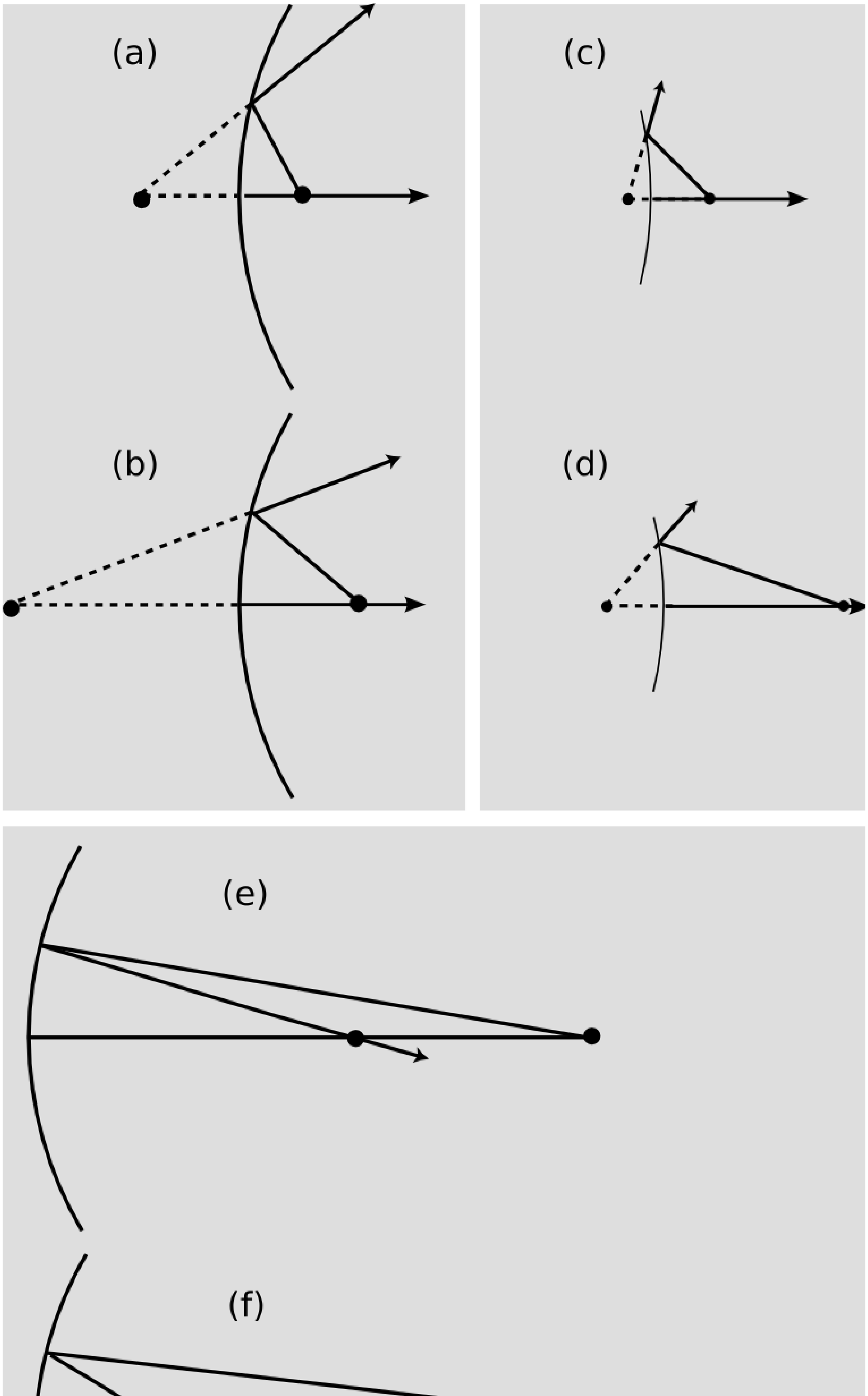
28-6 It spells "bonk."

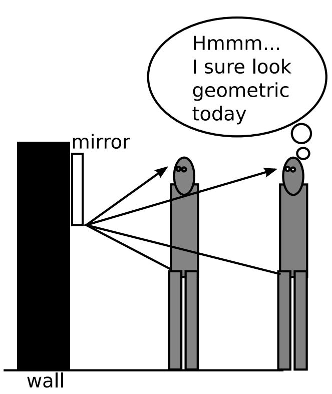


29-1 The image is at an equal distance behind the mirror, so the distance between the man and his image is twice his distance from the mirror, and his separation from his image is reducing at a rate of 2.0 m/s.

29-2 If you're going to see a particular part of your body, then rays from that part of your body have to be able to go to the mirror, be reflected, and go into your eye. If we're trying to find the lowest part you can see, then we're interested in the case where the ray hits the very bottom of the mirror. Backing up doesn't help, because the triangles are isoceles by the law of specular reflection, and the top of the base is fixed in height.

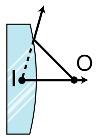
Another way of understanding the result is that if you go twice as far away, the mirror subtends half the angle, but your image also subtends half the angle, so the mirror still encompasses the same amount of your image.


(a)
(b)
(c)
(d)
(e)
(f)



29-3 (a) The image is closer to the surface of the mirror, so the magnification is less than one.

(b) It's virtual, because the rays only appear to have passed through point I. There is no way it could make a real image, because in order to make a real image it would have to bring the rays back together.



29-4 See figures a-f.

a and b: converging mirror, virtual image; increased object distance gives increased image distance

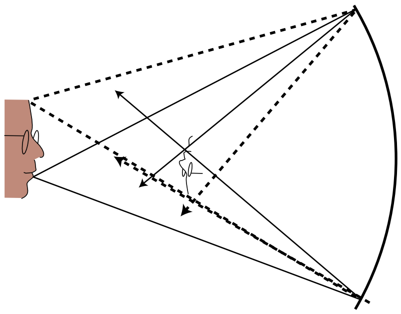
c and d: diverging mirror, virtual image: increased object distance gives increased image distance

e and f: converging mirror, real image: increased object distance gives decreased image distance

29-5 The magnification is the ratio of the image's size to the object's size. It has nothing to do with the person's location. The angular magnification, however, does depend on the person's location, because things farther away subtend smaller angles. The distance to the actual object is not changed significantly, since it's zillions of miles away in outer space, but the distance to the image does change if the observer's point of view changes. If you can get closer to the image, the angular magnification is greater.

29-6 Usually you need to draw in a normal in order to get an accurate drawing of the reflected rays. Here, however, we already have one set of rays, and it's pretty easy to do the new rays so that they fan in or fan out by the correct angle compared to the old ones.

As claimed, the image of the forehead (where the dashed rays cross) is below the image of the lips.



29-7 An image is where *all* the rays that started from a certain point come together for one big family reunion. Only 3 represents an image. Points like 1, 2, and 4 look special, but only because of the particular rays we chose to trace, out of the infinite number of rays fanning out from the point on the object. If we had chosen a different set of rays, they would all have crossed at 3 as before, but not at 1, 2, and 4.

29-8 The problem can be simplified conceptually by replacing the hypothetical square watch with a square hole in a piece of paper. This version is also easy to construct and test experimentally. The bundle of rays transmitted through the hole is identical to the bundle that would be reflected by a specular-reflecting square of the same size. Let A be the aperture size, i.e., a linear dimension of the hole, and let d be the distance from the aperture to the surface on which the spot is being projected. The results depend on how A and d compare. When d is small compared to A, the spot is square, but when d is large compared to A, we get a pinhole camera, which forms a circular spot shaped like the sun.

30-1 For a flat mirror, d_i and d_o are equal, so the magnification is 1, i.e., the image is the same size as the object.

30-3 (a) In problem #2 we found that the equation relating the object and image distances was of the form $1/f = -1/d_i + 1/d_o$. Let's make $f = 1.00$ m. To get a virtual image we need $d_o < f$, so let $d_o = 0.50$ m. Solving for d_i, we find $d_i = 1/(1/d_o - 1/f) = 1.00$ m. The magnification is $M = d_i/d_o = 2.00$. If we change d_o to 0.55 m, the magnification becomes 2.22. The magnification changes somewhat with distance, so the store's ad must be assuming you'll use the mirror at a certain distance. It can't have a magnification of 5 at all distances.

(b) Theoretically yes, but in practical terms no. If you go through a calculation similar to the one in part a, you'll find that the images of both planets are formed at almost exactly the same d_i, $d_i = f$, since $1/d_o$ is pretty close to zero for any astronomical object. The more distant planet has an image half as big ($M = d_i/d_o$, and d_o is doubled), but we're talking about *angular* magnification here, so what we care about is the angular size of the image compared to the angular size of the object. The more distant planet has half the angular size, but its image has half the angular size as well, so the angular magnification is the same. If you think about it, it wouldn't make much sense for the angular magnification to depend on the planet's distance — if it did, then determining astronomical distances would be much easier than it actually is!

30-4 (a) This occurs when the d_i is infinite. Let's say it's a converging mirror creating a virtual image, as in problems 2 and 3. Then we'd get an infinite d_i if we put $d_o = f$, i.e., the object is at the focal point of the mirror. The image is infinitely large, but it's also infinitely far away, so its angular size isn't infinite; an angular size can never be more than about 180° since you can't see in back of your head!.

(b) It's not possible to make the magnification infinite by having $d_o = 0$. The image location and object location are related by $1/f = 1/d_o - 1/d_i$, so $1/d_i = 1/d_o - 1/f$. If d_o is zero, then $1/d_o$ is infinite, $1/d_i$ is infinite, and d_i is zero as well. In other words, as d_o approaches zero, so does d_i, and d_i/d_o doesn't blow up. Physically, the mirror's curvature becomes irrelevant from the point of view of a tiny flea sitting on its surface: the mirror seems flat to the flea. So physically the magnification would be 1, not infinity, for very small values of d_o.

30-5 (a) Let the virtual image formed by the top mirror lie at a height b above the top mirror. Then

$$\frac{1}{f} = \frac{1}{h} - \frac{1}{b} \qquad .$$

Solving for b, we have

$$b = \left(\frac{1}{h} - \frac{1}{f}\right)^{-1} \qquad .$$

The second image is at a distance h from the bottom mirror, so

$$\begin{aligned} \frac{1}{f} &= \frac{1}{b+h} + \frac{1}{h} \\ &= \frac{1}{h + \left(\frac{1}{h} - \frac{1}{f}\right)^{-1}} + \frac{1}{h} \end{aligned}$$

(b) Multiplying both sides by h,

$$\begin{aligned} \frac{h}{f} &= \frac{h}{h + \left(\frac{1}{h} - \frac{1}{f}\right)^{-1}} + 1 \\ &= \frac{1}{1 + \left(1 - \frac{h}{f}\right)^{-1}} + 1 \\ x &= \frac{1}{1 + (1-x)^{-1}} + 1 \end{aligned}$$

This simplifies somewhat if we define a new variable $u = 1 - x$:

$$\begin{aligned} -u &= \frac{1}{1 + 1/u} \\ -u(1 + 1/u) &= 1 \\ -u - 1 &= 1 \\ u &= -2 \\ x &= 3 \\ h &= 3f \end{aligned}$$

Here's why this is a hard problem. The solution $h = 3f$ is unphysical. If h was greater than f, then the first image would have been real, not virtual, which would be inconsistent with the assumptions we made in the first place. The trick is in realizing that there was a hidden assumption involved in the step where we multiplied both sides by $1 + 1/u$, which was that $1 + 1/u$ was a real number. However, if $u = 0$, then $1 + 1/u$ is infinite, and we can't multiply both sides by it. That's the second solution referred to in the problem:

$$u = 0$$
$$x = 1$$
$$h = f$$

In other words, the first image is at infinity. Images at infinity are the crossing-over point between real and virtual images. That means it didn't actually matter that we assumed the first image would be virtual. We could have gotten the same answer by assuming it was real.

30-6 If the sound gets concentrated again at your ear, it must be a real image. You're at the center of the sphere formed by extending the surface. We've already found in the text that for a real image formed by a converging mirror, the equation for the image's location is $1/f = 1/d_\mathrm{o} + 1/d_i$. If the object and image distances are equal, then $1/f = 2/d_\mathrm{o}$, or $f = d_\mathrm{o}/2$. The focal length is half the distance from the surface to your head.

30-7 We can tell that the object in the drawing is infinitely far away, because the rays are parallel. The focal length is defined in terms of the image distance in the case where the object is infinitely far away, so the focal length equals the image distance. Tracing the rays, we find that they cross at about 11 mm from the mirror, so that's its focal length.

30-8 The diameters of the mirrors are irrelevant. The focal length simply relates to the curvature of the mirror, a more strongly curved mirror having a shorter focal length. From shortest focal length to longest, the ranking of the mirrors is C, B, A.

30-9 (a) For a virtual image formed by a converging mirror, the extreme possibilities are $d_o \to 0$, which gives $M \to 1$ because you can't see the curvature of the mirror from up close, and $d_o = f$, which gives parallel reflected rays, $M = \infty$. The result is that $1 < M \leq \infty$.

(b) For a real image formed by a converging mirror, the extremes are obtained when either $d_o = \infty$ or $d_i = \infty$. We have $0 \leq M \leq \infty$.

For a diverging mirror, $d_o \to 0$ gives $M \to 1$ for the same reason as in part a, while $d_o \to \infty$ gives $M \to 0$. The result is $0 < M < 1$.

30-11 (a) Initially, the object is so close to the mirror that the mirror's curvature is irrelevant — any curve looks flat if you're close enough to it. Therefore for $d_o << f$, we have $d_i \approx d_o$, and the image's velocity equals the object's. As the object's distance approaches infinity, on the other hand, the image approaches a position that is one focal length above the mirror. Therefore,the image must accelerate from rest, move upward a distance equal to the focal length, and decelerate again, approaching zero velocity.

(b) The only two variables occurring in the statement of the problem are f and g, and there is only one way to combine these to make something with units of velocuty, which is to form the expression $\sqrt{fg}$. The answer must equal $\beta\sqrt{fg}$, where β is a unitless constant.

(c) As shown in example 5, the relatonship between the image and object locations is $1/f = 1/d_i - 1/d_o$. Since we already know the form of the solution's dependence on f and g, we can arbitrarily set $f = 1$ and $g = 1$ in order to simplify the writing. We then have $d_i = 1/(1 + 1/d_o)$, where $d_o = t^2/2$, and differentiation gives $v_i = t/(t^2/2 + 1)^2$. To find the maximum velocity, we differentiate again, and set the derivative equal to zero. In other words, we're looking for the time when the acceleration equals zero. The acceleration is $a_i = 1/(t^2/2 + 1)^2 - 2t^2(t^2/2 + 1)^3$, which equals zero at $t = \sqrt{2/3}$. Plugging this back into the expression for v_i, we have a maximum velocity of $2^{-7/2}3^{3/2}$. This is the mysterious unitless constant β from part b. Recovering the general solution for $f \neq 1$ and $g \neq 1$, we have a maximum velocity of $2^{-7/2}3^{3/2}\sqrt{fg}$.

30-12 For simplicity, let's assume that the cylinder has unit radius, so that its focal length is 1/2, and the relationship between the object and image distances is $1/d_i - 1/d_o = 2$. To see that the standard Peaucellier linkage doesn't do the right thing, consider the case when the object point is at infinity. Then the image distance is 1/2, whereas inversion in a circle would give a point at radius zero.

The standard definition of object and image distances is relative to the surface of the mirror, whereas the geometrically relevant quantities in this problem would clearly be measured from the center of the cylinder, $u = d_o + 1$ and $v = 1 - d_i$. The relationship between these two variables is $1/u + 1/v = 2$, which is the equation of a hyperbola with perpendicular asymptotes at $u = 1/2$ and $v = 1/2$.

This gives us strong reason to hope that the a construction based on the Peaucellier can be done, since the Peaucellier linkage calculates the inverse of a number, and the graph of the function $y = 1/x$ is also a hyperbola with perpendicular asymptotes. To make the connection, let $p = u - 1/2$ and $q = v - 1/2$, so that $p = 1/4q$. This shows that we need to make a Peaucellier linkage with $k = 1/2$. Positioning it so that when the rhombus is flattened, points P and P' lie on the surface of the cylinder. The point O should not be located at the center of the cylinder; we need to modify the linkage by adding a seventh rod, with length 1/2, to tie O to the center of the cylinder. We also need to add constraints to keep the axis of the Peaucellier linkage lined up with the seventh rod.

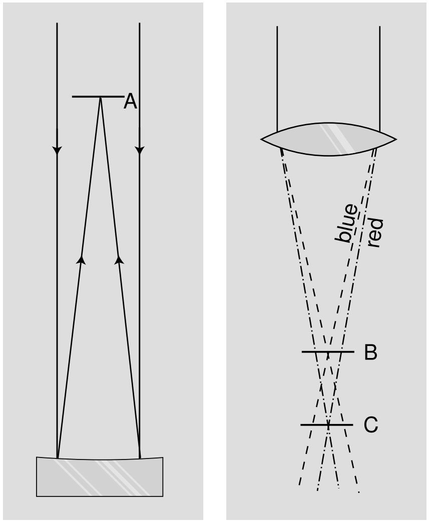
31-1 The refracted ray that was bent closer to the normal in the plastic when the plastic was in air will be bent farther from the normal in the plastic when the plastic is in water. It will become a diverging lens.

31-2 The figure shows the simplest possible reflecting and refracting telescopes, with no diagonal mirror or eyepiece.

In both cases, parallel rays enter from the star, which is infinitely far away. Each of these white rays is really a mixture of rays of all the different colors.

With the reflecting telescope, all the colors are reflected at the same angle and form a single image at one place. If we insert a piece of paper or a digital camera chip at A, only one spot is illuminated with the light from the star.

With the refracting telescope, it is never possible to get all the colors in focus at once. Each color has its own image, at a different distance from the lens. If the paper or chip is at B, where the image occurs for a specific wavelength of blue light, we get a blue dot surrounded by a fuzzy halo consisting of other colors such as red. If we move it to C, where the image occurs for a certain wavelength of red, we get a red dot surrounded by the other colors.



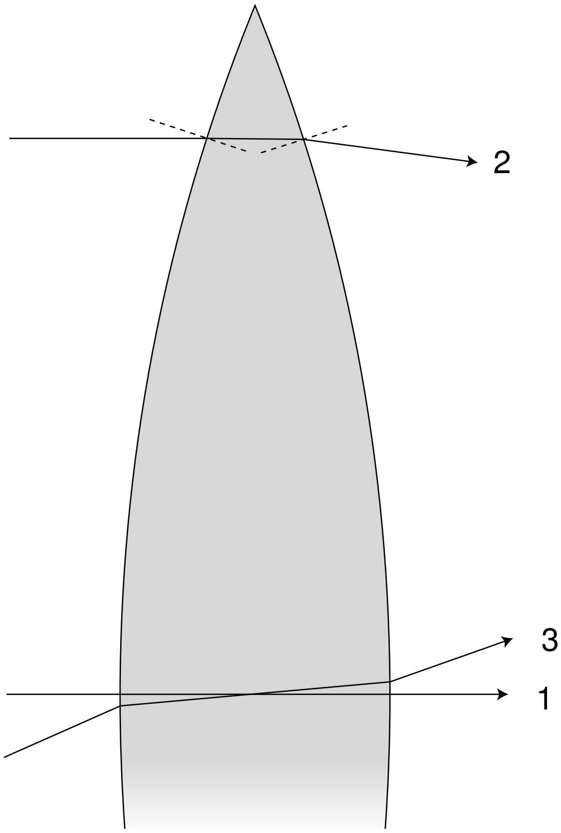
31-3 Refraction occurs only at the boundary between two substances, which in this case means the surface of the lens. Light doesn't get bent at all inside the lens, so the thickness of the lens isn't really what's important. What matters is the angles of the lens' surfaces at various points.

Ray 1 makes an angle of zero with respect to the normal as it enters the lens, so it doesn't get bent at all, and likewise at the back.

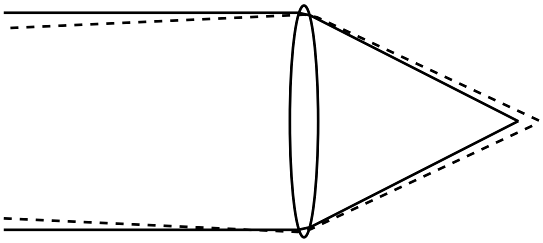
At the edge of the lens, 2, the front and back are not parallel, so a ray that traverses the lens at the edge ends up being bent quite a bit.

Although I drew both ray 1 and ray 2 coming in along the axis of the lens, it really doesn't matter. For instance, ray 3 bends on the way in, but bends an equal amount on the way out, so it still emerges from the lens moving in the same direction as the direction it originally had.

Summarizing and systematizing these observations, we can say that for a ray that enters the lens at the center, where the surfaces are parallel, the sum of the two deflection angles is zero. Since the total deflection is zero at the center, it must be larger away from the center.



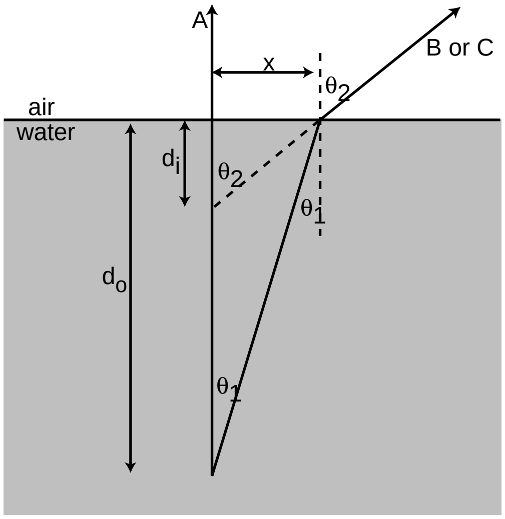
31-4 The rays from a point on a very distant object are parallel, like the solid lines in the figure. The rays from a point on a nearer object are slightly diverging, like the dashed lines. The lens cannot bend the rays from the nearer object strongly enough to make them converge as close to the lens, so the lens-to-film distance has to be greater when the object is nearer.



31-5 (a) Doing trig on the diagram below gives $\tan\theta_1 = x/d_o$, and $\tan\theta_2 = x/d_i$. Eliminating the unknown x and solving for d_i gives $d_i = d_o\tan\theta_1/\tan\theta_2$. Ray B has $\theta_1 = 1.0°$, and applying Snell's law with $n = 1.3$ gives $\theta_2 = 1.3°$, so rays A and B intersect at a depth $d_i = 0.769$ m. (Note that although we only know n to 2 significant figures, the point of the problem is to compare image depths for various rays, so we need to keep more sig figs.) Similarly, rays A and C intersect at $d_i = 0.733$ m.

(b) It might make it a little blurry if you were looking at it from close enough so that widely divergent rays like B and C could both get into your eye. Another effect is usually far more noticeable, however: if you try looking at an object on the bottom of a swimming pool from various angles, it appears to shift around.

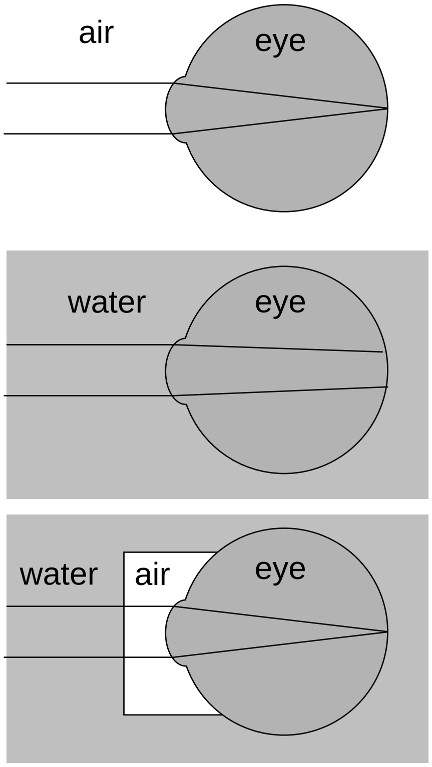
(c) For small angles, $\sin\theta = \tan\theta$, so $d_i = d_o\tan\theta_1/\tan\theta_2 = d_o\sin\theta_1/\sin\theta_2 = d_o/n$, which is the same number regardless of θ.



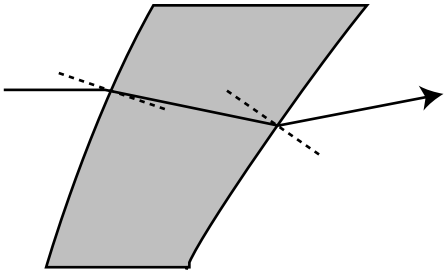
31-6 I tried various circular household objects and found one whose radius of 1.5 cm seemed to make it coincide with the cornea's curvature. Because the drawing is two times bigger than life, the radius of curvature of the cornea is 1.5 cm/2=0.75 cm. The lensmaker's formula is $f = \left[(n-1)\left|1/r_1 \pm 1/r_2\right|\right]^{-1}$, but the air-cornea interface is just a single surface, so we just need to use $f = \left[(n-1)\left|1/r_1\right|\right]^{-1} = |r_1|/(n-1) = 2.5$ cm.

31-7 Normally, in air, your eyes do most of their focusing at the air-eye boundary. When you swim without goggles, there is almost no difference in speed at the water-eye interface, so light is not strongly refracted there (see figure), and the image is far behind the retina.

Goggles fix this problem for the following reason. The light rays cross a water-air boundary as they enter the goggles, but they're coming in along the normal, so they don't get bent. At the air-eye boundary, they get bent the same amount they normally would when you weren't swimming.



31-8 (a) See the figure below. The first refraction clearly bends it inward. However, the back surface of the lens is more slanted, so the ray makes a bigger angle with respect to the normal at the back surface. The bending at the back surface is therefore greater than the bending at the front surface, and the ray ends up being bent *outward* more than inward.

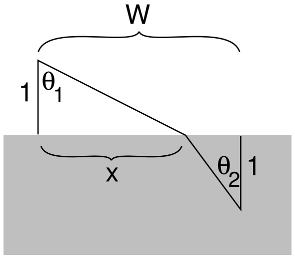


(b) Lens 2 must act the same as lens 1. It's diverging. One way of knowing this is time-reversal symmetry: if we flip the original figure over and then reverse the direction of the ray, it's still a valid diagram.

Lens 3 is diverging like lens 1 on top, and diverging like lens 2 on the bottom. It's a diverging lens.

As for lens 4, any close-up diagram we draw of a particular ray passing through it will look exactly like the corresponding close-up diagram for some part of lens 1. Lens 4 behaves the same as lens 1.

31-9 If the principle of least time is going to work at all, it must work between any two points, so without loss of generality, we assume the points to lie at unit distance from the boundary between the two media. We also simplify things by letting $c = 1$ (which can be achieved, for example, by the proper choice of units).



The time taken by the ray along the path shown in the figure is

$$T = n_1\sqrt{1+x^2} + n_2\sqrt{1+(W-x)^2} \qquad .$$

The minimum occurs when the derivative with respect to x equals zero:

$$0 = \frac{\mathrm{d}T}{\mathrm{d}x}$$

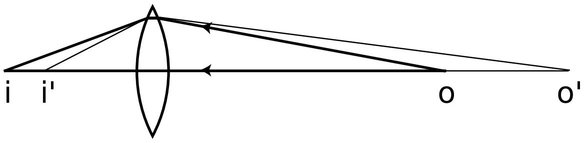
$$0 = n_1\left(1+x^2\right)^{-1/2}x - n_2\left[1+(W-x)^2\right]^{-1/2}(W-x)$$

$$0 = n_1\sin\theta_1 - n_2\sin\theta_2$$

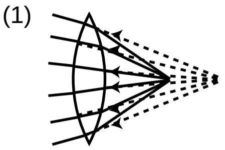
31-10 (a) See the ray diagram drawn with thick lines, showing object location o and image location i.

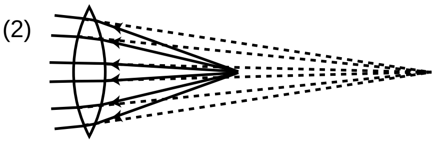
(b) If we move the object farther away, to o′ the cone of rays intercepted by the lens (thin lines) is less strongly diverging, and the lens is able to bring it to a closer focus, at i′. In the diagrams, we see that a smaller θ_o leads to a larger θ_i, so the signs in the equation $\pm\theta_o \pm \theta_i = \theta_f$ must be the same, and therefore both positive, since θ_f is positive by definition. The equation relating the image and object locations must be $1/f = 1/d_o + 1/d_i$.

(c) Solve the equation for $d_i = (1/f - 1/d_o)^{-1} = 32\text{cm}$.



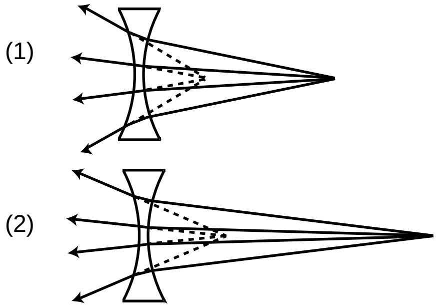
31-11 (a) See figure (1).
(b) If we move the object farther from the lens, as in (2), the cone of rays intercepted by the lens is less strongly diverging, so the lens is able to bring them closer together, resulting in an image that is farther from the lens. Since a smaller θ_o leads to a smaller θ_i, the signs in the equation $\pm\theta_o \pm \theta_i = \theta_f$ must be opposite. Since θ_o is greater than θ_i, we must have $\theta_o - \theta_i = \theta_f$ in order to get a positive focal angle. The equation relating the image and object locations is therefore $1/f = 1/d_o - 1/d_i$.
(c) Solving for f and plugging in, we get 18 cm.





31-12 (a) See figure (1).
(b) Decreasing θ_o, as in figure (2), makes a cone of rays that is less strongly diverging, which the lens cannot bend outward as far, so θ_i becomes smaller. Since decreasing θ_o makes θ_i smaller as well, the signs in the equation equation $\pm\theta_o \pm \theta_i = \theta_f$ must be opposite. Since θ_i is greater than θ_o, we must have $-\theta_o + \theta_i = \theta_f$ in order to get a positive θ_f. The equation relating the image and

object locations must be $1/f = -1/d_o + 1/d_i$. Solving for d_i and plugging in, we get 30.8 cm.



31-15 Snell's law is

$$n_a \sin\theta_a = n_d \sin\theta_d \qquad ,$$

where a refers to air and d to diamond, and $n_a \approx 1$. Total internal reflection occurs when there is no solution for θ_a, which occurs when the value required for $\sin\theta_a$ is greater than 1. The critical angle is $\theta_d = \sin^{-1}(1/n_d) = 24.4$ deg. It's measured from the normal, since that's how the angles in Snell's law are always measured. It's a minimum, because making θ_d bigger would require $\sin\theta_a$ to be more than 1, which is impossible. In a substance with a smaller index of refraction, $1/n$ would be bigger, and $\sin^{-1}(1/n)$ would also get bigger, since the inverse sine increases as you make its input bigger; the minimum angle for total internal reflection would be bigger, meaning that it would be harder to get total internal reflection.

31-21 (Note that versions of the book before 2012 have a scale that is too small by a factor of 7.8.) I looked around the house for circular objects that I could compare against the diagram in order to match its radii of curvature. At the air-lens interface, I found the lid from a jar of yeast with a diameter of 56 mm, and for the n_2-n_3 interface a big three-pound jar of peanuts with a diameter of 158 mm. Taking into account the scale of the figure, this results in radii of curvature $r_{12} = 220\ \mu$m and $r_{23} = 610\ \mu$m. In the lensmaker's equation, the factor $n-1$ represents the difference in index of refraction between the air (1) and the lens (n). For our present purposes, we need to modify this to

$$f^{-1} = \left(\frac{n_2}{n_1} - 1\right)\frac{1}{r_{12}} + \left(\frac{n_2}{n_3} - 1\right)\frac{1}{r_{23}} \qquad ,$$

which results in $f = 510\ \mu$m. The the 2-3 term is less than a tenth the size of the 1-2 term. For an object at infinity, this focal length equals the image distance. The distance from the middle of the light-sensitive parts of the retinal cells to the average location of the 1-2 surface is about 550 μm, which is in pretty good agreement with the estimated image distance. In reality, the image position will be somewhat different, both because the object may not be at infinity and because the indices of refraction depend on wavelength.

32-1 You don't want the wave properties of light to cause all kinds of funny-looking diffraction effects. You want to see the thing you're looking at in the same way you'd see a big object. Diffraction effects are most pronounced when the wavelength of the light is relatively large compared to the size of the object the light is interacting with, so red would be the worst. Blue light is near the short-wavelength end of the visible spectrum, which would be the best.

32-2 The center-to-center spacing of patterns 2 and 3 are the same. The one with the small center-to-center spacing, pattern 1, corresponds to the grating with the large slit spacing, C. Grating A has more repetition, so its pattern is the one with narrower lines, 3. To summarize, 1=C, 2=B, 3=A.

32-3 (a) The grating's spacing is the number of centimeters per line, which is one over the number of lines per centimeter, $d = (1/2000\ \text{lines/cm}) = 5 \times 10^{-4}$ cm/line. "Lines" are a fake unit that we can get rid of at will. Converting to meters, we have $d = 5 \times 10^{-6}$ m. The angles of the $m = -1$ and $m = 1$ spots are given by $\theta = \sin^{-1}(m\lambda/d) = +8.0$ degrees. The spacing of the spots on the screen is related to this by simple trig: $(2.0\ \text{m})(\tan\theta) = 28$ cm.

(b) Note that the approximation $\sin\theta \approx \theta$ only works if we use units of radians. With it, we get

$\theta = m\lambda/d = .139$ radians, which works out to be almost exactly the same as the answer we got without the approximation.

32-4 There is not actually a sharp edge to the visible rainbow; there is a range of wavelengths at each end of the spectrum over which your eye's sensitivity drops off. However, as stated in the problem, the range is roughly 400 to 700 nm, which is slightly less than a factor of two. You get a series of rainbows, each broader than the previous one, but with a constant center-to-center angular spacing. The angle of a given color within the rainbow of order m is determined by the equation $m\lambda = d\sin\theta$, or $\theta = \sin^{-1}(m\lambda/d)$. Since the red end is very nearly twice the wavelength of the violet end, the $m = 1$ red has very nearly the same value of θ as the $m = 2$ violet, so the first and second order spectra almost overlap, but not quite. The $m = 2$ and $m = 3$ spectra will definitely overlap, because the quantity $m\lambda/d$ is greater for $m = 2$ red than for $m = 3$ violet.

32-6 For the size of the diffraction blob, we have:

$$\begin{aligned} \frac{\lambda}{d} &\sim \sin\theta \\ &\approx \theta \\ \theta &\sim \frac{700\ \text{nm}}{10\ \text{m}} \\ &\approx 10^{-7}\ \text{radians} \end{aligned}$$

For the actual angular size of the star, the small-angle approximation gives

$$\begin{aligned} \theta &\sim \frac{10^{9}\ \text{m}}{10^{17}\ \text{m}} \\ &= 10^{-8}\ \text{radians} \end{aligned}$$

The diffraction blob is ten times bigger than the actual disk of the star, so we can never make an image of the star itself in this way.

32-8 High frequencies have short wavelengths, which leads to small diffraction angles. This is a good thing, because we want the image to be sharply defined. Ultimately, you can't pick out any details that are smaller than one wavelength. (This only explains why they would use the highest possible frequency for imaging, not why they would use a lower one for heating. The most likely explanation is that it's just more expensive to get a speaker to vibrate really fast.)

32-9 (a) You can tell it's a single slit because of the double-width central fringe.

(b) Four fringes on the top pattern are about 23.5 mm, while five fringes on the bottom one are about 14.5 mm. The spacings are 5.88 and 2.90 mm, with a ratio of 2.03. A smaller d leads to larger diffraction angles, so the width of the slit used to make the bottom pattern was almost exactly twice as wide as the one used to make the top one.

32-10 Red light is at the long-wavelength end of the spectrum. Longer wavelengths give larger diffraction angles, so if the slits had been identical, the red pattern would have been the more widely spaced one. We conclude that different slits were used.

32-11 (a) The patterns have two structures, a coarse one and a fine one. You can look up in the book which corresponds to w and which to d, or just use the fact that smaller features make bigger diffraction angles. The top and middle patterns have the same coarse spacing, so they have the same w. The fine structure in the top pattern has 7 fringes in 12.5 mm, for a spacing of 1.79 mm, while the middle pattern has 11 fringes in 41.5 mm, giving a spacing of 3.77 mm. The value of d for the middle pattern is therefore $(0.50\ \text{mm})(1.79/3.77) = 0.23$ mm.

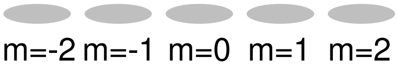
(b) This one has about the same d as the top one (it's difficult to measure accurately because each group has only a small number of fringes), but the coarse spacing is different, indicating a different value of w. It has two coarse groupings in 23 mm, i.e., a spacing of 12.5 mm. The coarse groupings in the original pattern were about 23 mm apart, so there is a factor of two between the $w = 0.04$ mm of the top pattern and the $w = 0.08$ mm of the bottom one.

32-12 Let's write x for the distance from the slits to the wall, and y for the spacing between fringes. Three fringes cover about 5.1 cm in the photo, so the spacing is about $y = 1.70$ cm. An example in the text worked out the case where the angles are small, and we want to relate the wavelength to the angular spacing. The result (using radians, not degrees!) is

$$\begin{aligned}\lambda &\approx d\Delta\theta \\ &= d\tan^{-1}(y/x) \\ &\approx d\cdot(y/x) \\ &= 580\text{ nm}\end{aligned}$$

As a check on the precision of my result, I measured across five fringes and got 8.3 cm, which gives $y = 1.66$ cm, and $\lambda = 570$ nm. Typically, measuring across more fringes would reduce the relative error in the measurement and give a better result. However, the picture only shows five maxima, so measuring across them required me to estimate the position of the minima on the ends without being able to see the next bright fringe. Roughly, then, I'd say the two results of 580 and 570 nm are equally trustworthy, and it might make sense to average them to get about 575 nm as my best value, with error bars of five or ten nm (one or two percent).

32-13 The natural variable to use for labeling the fringes is m, as shown in the figure.



The angle θ is also different for the different fringes. The wavelength λ is a property of all the light being used, not a property of a particular fringe. For instance, if it's blue light, it might have a wavelength of 300 nm or so, and that would be the wavelength of all the light in every fringe. The variable d refers to a dimension of the physical object the light is diffracting around, e.g., the distance between two slits, so it's also not different for the different fringes. In particular, the light and dark pattern of fringes is *not* just a shadow of the slits — you could have hundreds of fringes made by only one or two slits.

32-14 (Note that versions of the book before 2012 have a scale that is too small by a factor of 7.8.) From the figure, the width of the light-sensing tip of one of the retinal cells appears to be somewhere around 4 μm. Comparing this with the $\sim$ 500 μm distance from the lens to the retina, and applying the definition of radian measure $\theta = s/r$, we find $\theta \approx 30$ minutes of arc. The diffraction limit is an angular resolution of $\sim \lambda/d$, which for $d = 240$ μm and $\lambda = 550$ nm (the middle of the visible spectrum for humans) comes out to be about 8 minutes of arc. The two figures are comparable, which makes sense in evolutionary terms.

33-1 After two years, the substance will be 75% decayed, not 100%.

33-2 The probabilities are independent, so $(0.014) \times (0.014) = 2.0 \times 10^{-4}$.

33-3 (a) In the first figure, we can just apply the definition of half-life directly: the time required for 50% of the muons to decay looks like about 1.5 μs. In the second figure, we haven't gotten down to 50% even after 8 microseconds. Instead of applying the definition directly, we have to take a point off of the graph and use it to solve the exponential decay equation for the unknown half-life.

$$\begin{aligned}P_{surv}(t) &= (0.5)^{\left(t/t_{1/2}\right)} \\ \ln P_{surv}(t) &= \ln\left[(0.5)^{\left(t/t_{1/2}\right)}\right] \\ \ln P_{surv}(t) &= \frac{t}{t_{1/2}}\ln 0.5 \\ t_{1/2} &= \frac{t\ln 0.5}{\ln P_{surv}(t)} \\ t_{1/2} &= -\frac{t\ln 2}{\ln P_{surv}(t)}\end{aligned}$$

Let's check this equation before we plug in numbers. The units check out, because the left side has units of seconds, and so does the right-hand side. (Probabilities are unitless, so t is the only thing on the right that has units.) We can also check whether the dependence on the survival probability makes sense. The survival probability is less than one, so its logarithm is negative.

If we make the survival probability bigger and bigger, it gets closer and closer to one, and its logarithm gets closer and closer to zero. That means we're dividing by something that's small, which gives a big result for the half-life. That makes sense, because a high survival probability should correspond to a long half-life. Plugging in data from the right-hand edge of the graph, we get

$$\begin{aligned} t_{1/2} &= -\frac{t \ln 2}{\ln P_{surv}(t)} \\ &= -\frac{(8\ \mu\text{s}) \ln 2}{\ln 0.7} \\ &= 15\ \mu\text{s} \end{aligned}$$

(b) $(15\ \mu\text{s})/(1.5\ \mu\text{s}) = 10$

33-4 Using $a = 0.5$ and $b = 0.001$, the approximation gives 0.9993067, which is very close to the correct answer of 0.9993069. Using $a = 0.5$ and $b = 0.2$, the approximate result of 0.861 is a much poorer approximation to the true answer of 0.871.

33-5 I made up this problem:

A sample of a radioactive substance initially contains 1.0×10^{18} atoms. Its half-life is 5.5 hours. How many decays will occur per second at $t = 27$ hours?

Using the approximate method gives 1.165136×10^{12}, and using the exact equation gives 1.165116×10^{12}. To within the 2-sig-fig accuracy of the data I made up, these are identical.

33-6 Suppose we record the results of a long string of games played one after another. One method of testing for independence would be to use the data to estimate the probability of winning, $P(W)$, and also break down the list into sequential pairs (1st and 2nd game, 3rd and 4th, etc.) and use these pairs to measure the probability of winning twice in a row, $P(W \text{ and } W)$. The definition of independence then gives $P(W \text{ and } W) = P(W)^2$. We can test whether the left-hand and right-hand sides of this equation are in fact equal, to within the statistical sampling error imposed by the number of games observed.

Another approach is to look for extremely long runs of wins. If independence holds, then the probability of n wins in a row is $P(W)^n$, which falls off exponentially. Any observation of a very large n would therefore prove that the rolls were not statistically independent. For a statistical analysis of data from players who ran up world-record winning streaks, see Ethier and Hoppe, arxiv.org/pdf/0906.1545. The results are described in terms of the number of rolls, which is greater than the number of games.

33-7 (a) Each rectangle has a width of 5 cm and a height of 0.005 cm^{-1}, so its area represents a probability of 0.025. The total number of rectangles under the curve is about 38, so the total area under the curve represents a probability of about $38 \times .025 = 0.95$, which is close enough to 1, considering that I had to estimate fractions of squares.

(b) The number of rectangles under the curve from 140 cm to 150 cm is about 6, so the the probability represented is about $6 \times .025 = .15$.

33-8 (a) The activity falls off by a factor of two as you go from $t = 5$ min to $t = 32$ min, so its half-life is about 27 min. It looks like it's ^{214}Pb.

(b) If it's a single isotope with a single well-defined half-life, then the ratio of the activity at 20 min to the activity at 10 min should be the same as the ratio of the activity at 30 min to the activity at 20 min. Only the stuff produced by the second ion speed gives ratios equal to three significant figures, which is the accuracy of the data.

33-9 (a) To a good approximation, the mass of a nucleus is proportional to the number of protons plus the number of neutrons, which is the mass number. The total change in mass number from 238 to 206 is 32, so if mass is to be conserved, the number of alpha particles, with mass number four, emitted must be 32/4=8.

(b) The charge of the original nucleus was 92 (in units of the fundamental charge), and the nuclei produced have a total charge of $82 + 8 \times 2 = 98$. To conserve charge, 6 electrons must have been produced, so 6 beta decays must have occurred.

(c) The mass of a uranium atom is about 238 times the mass of a neutron or proton, so the number of surviving U atoms is

$$\frac{1230\times10^{-6}\ \text{kg}}{238m_p}=3.09\times10^{21}\qquad .$$

(The masses of protons and neutrons are nearly the same, so we don't have to add their masses separately.) Similarly, the number of helium atoms is

$$\frac{2.3\times10^{-6}\ \text{kg}}{4m_p}=3.44\times10^{20}\qquad .$$

Each of the decayed U atoms produced 8 helium atoms, so the number of decayed U atoms is $3.44\times10^{20}/8=4.30\times10^{19}$. The probability of decay since the rock was formed equals (number that decayed)/(number that decayed + number that survived) = 0.0137, so the probability of survival was 0.9863, and $0.9863=0.5^{t/t_{1/2}}$. Taking natural logarithms of both sides, $\ln 0.9863=(t/t_{1/2})\ln 0.5$, yielding $t=90$ million years.

33-10 (a) We lose two protons and two neutrons. Losing the two neutrons makes bismuth, atomic number 83, into thallium, 81. The total number of protons and neutrons lost is 4, so the new isotope is ^{205}Tl.

(b) It's about a billion times longer.

(c) One over the rate of decay is

$$\begin{aligned}\left[\frac{(\ln 2)N(0)}{t_{1/2}}0.5^{0/t_{1/2}}\right]^{-1}&=\frac{t_{1/2}}{(\ln 2)N(0)}\\&=0.07\ \text{years}\\&=25\ \text{days}\end{aligned}$$

33-11 (a) The variable r varies continuously from 0 to b — it's not as though we have rings of finite width. However, suppose we imagine an extremely narrow ring extending from r to $r+\mathrm{d}r$. Since the person is blindfolded, the probability of hitting a given region depends only on its area. This ring has a circumference of $2\pi r$, so its area is $2\pi r\,\mathrm{d}r$. This is proportional to r, so the probability of getting a result between r and $r+\mathrm{d}r$ is proportional to r.

(b)

$$\begin{aligned}1&=\int_0^b D(r)\,\mathrm{d}r\\1&=\int_0^b kr\,\mathrm{d}r\\1&=\frac{1}{2}kb^2\\k&=\frac{2}{b^2}\\D(r)&=\frac{2r}{b^2}\end{aligned}$$

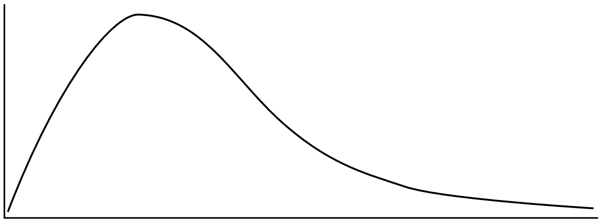
(c)

$$\begin{aligned}\bar{r}&=\int_0^b rD(r)\,\mathrm{d}r\\&=\int_0^b\frac{2r^2}{b^2}\,\mathrm{d}r\\&=\frac{2}{3}b\end{aligned}$$

(d) The average value of r is greater than $b/2$. This makes sense, because the area between $r=0$ and $r=b/2$ is only one fourth of the total area.

33-12

33-13 (a) Without the factor of v in front, it would have the form of the classic bell curve. Since it's only defined for positive v, we chop off the left half of the bell. Then we multiply by v, which makes the curve drop down to zero at the origin. The result looks like this:



(b) We want to integrate D from 0 to ∞. The subsitution $u=cv^2$ gives, $\mathrm{d}u=2cv\,\mathrm{d}v$, turning the integrand into $(b/2c)e^{-u}\,\mathrm{d}u$. The integral of $e^{-u}\,\mathrm{d}u$ from 0 to ∞ is 1, so the result is $1=(b/2c)\cdot1$, or $b=2c$.

(c) I warned you not to try it by hand! (It requires a trick.) Using computer software, the result is $\int_0^\infty 2cv^2 e^{-cv^2}\,\mathrm{d}v = (1/2)\sqrt{\pi/c}$.

34-1 In both case, we're talking about 100 joules worth of light. Consulting the chart of the electromagnetiv spectrum, we find that the frequency of visible light is greater than that of radio waves by a factor of about $(10^{14}\ \text{Hz})/(10^8\ \text{Hz}) = 10^6$. (The radio spectrum covers a lot of ground, but near the middle of it we have FM radio at about 100 MHz $= 10^8$ Hz. The visible spectrum only spans about a factor of two in frequency, so that's less of an issue.) Since a photon's energy is given by $E = hf$, that means a visible photon has a million times more energy than a radio photon. There are a million times more photons in the radio signal than in the visible light. If you assumed a different value for the frequency of the radio waves, you might have gotten an answer that differed from this one by a factor of 10 or 100.

34-2 The 400-nm wavelength is shorter, which means a higher frequency and higher energy per photon. It takes 1.5 times fewer of these economy-sized photons to make the same amount of energy, so the number of photons in the 400-nm flash is $3.0 \times 10^{18}/1.5 = 2 \times 10^{18}$.

34-3 $E = hf$, and frequency corresponds to color, so if each photon was losing 20% of its energy you'd notice a pronounced change in color.

34-4 The amount of energy lost by the electrons as they leave the surface, E_s, depends on the material. With a material with a high E_s, you wouldn't be sensitive to the red end of the spectrum, which has low-energy photons.

34-5 Hertz noticed that the cutoff for producing the photoelectric effect in metals was somewhere in the visible-light spectrum. We're assuming that the energy required to ionize DNA is similar to the energy required to rip an electron out of a metal, so again, the cutoff should at a frequency that's of roughly the same order of magnitude as the frequency of visible light. The types of waves are listed in order of increasing frequency, so the only ones that can cause cancer are the ones farther down the list than the cutoff. Ultraviolet and x-rays are certainly capable of doing it. It might be possible to get cancer from visible light — our analysis is too crude to tell exactly where the cutoff is. You are not going to get cancer from power lines, FM radio, cell phones, or microwave ovens.

34-6 Visible light has a wavelength on the order of 10^{-6} m, so a typical photon of visible light is about $E = hf = hc/\lambda \approx 2 \times 10^{-19}$ J, so the number of photons emitted per second is about $(100\ \text{J/s})/(2 \times 10^{-19}\ \text{J/photon}) = 5 \times 10^{20}$ photons/s. The time each photon spends on its voyage is $(3\ \text{m})/c \approx 10^{-8}$ s. The number of photons in the beam equals the number of photons emitted during the time required for a photon to get across, $(5\times10^{20}\ \text{photons/s})\times(10^{-8}\ \text{s}) = 5\times10^{12}$ photons. Given the crudeness of the initial assumptions, this is only an order-of-magnitude result, so let's call it 10^{13} photons.

34-7 There's a grid superimposed on the patterns, but I just took a ruler and measured the distance between the $m = -1$ and $m = 1$ fringes. The ratio of these was $292/176 = 1.66$, so the photons in B have a wavelength that is shorter by a factor of 1.66, and an energy that is higher by a factor of 1.66. There are about 101 photons in the $m = -1$, 0, and 1 fringes in A, and 140 in B. B is going to have more energy, both because there are more photons in it and because its photons are individually more energetic. The ratio is $1.66 \times (140/101) \approx 2.3$.

34-8 The square of the wave represents probability per unit volume. In the case of this one-dimensional representation of the waves, we should interpret it as probability per unit length. The really long and laborious way to do this problem would be to draw the squared waves as four new graphs, and then count squares under the curves. But since they're all sine waves, we can dispense with some of the details about their shapes, and just compare their squared amplitudes, and the amounts of space they cover.

A	A^2	space	(space) × (A^2)
4	16	12	192
5.5	30	6	180
4	16	12	192
8	64	6	354

The final column of the table represents relative probability, and we see that it comes out about the same for the first three, so with the correct calibration of the axes on the graph, these could all come out to have a total probability of 1. The fourth one is the one that's different.

Interpretation: The first and third ones have the same amplitude, and cover the same amount of space, so naturally they have the same total probability. The fourth one has twice the amplitude and covers half the space, which you might think would compensate for each other, but probability depends on the square of the amplitude, so we get a factor of 4 due to amplitude, multiplied by the factor of 1/2 for space, which gives a total probability of twice as much as the others. That one's not properly normalized. We could say that it represented *two* photons.

34-9 frequency: Fred's is greater because $E = hf$, and Fred's energy is greater.
speed: the same, because all photons travel at the speed of light
wavelength: Fred's is shorter because $c = f\lambda$, and Fred's frequency is higher.
period: Fred's is shorter because $T = 1/f$.
The field strengths are impossible to determine. The fields could be weaker or stronger depending on how much space each wave is spread out over.

34-10 In figure h/3 in section 3.3, we get a double-slit interference pattern, even though, as discussed later in the section, there was never more than one photon in flight at an one time. This proves that a single photon can go through both slits, which would be impossible if the photon was a particle traveling along a well-defined trajectory like a zigzag.

34-11 (a)

$$\begin{aligned}\text{power} &= (1\ \text{mW/m}^2)(\text{cross-sectional area})\\ &= (1\ \text{mW/m}^2)(\pi r^2)\\ &\sim 10^{-32}\ \text{W}\end{aligned}$$

(rounding to a power of 10).
(b) power $= \Delta E/\Delta t$, so

$$\begin{aligned}\Delta t &= \Delta E/\text{power}\\ &\sim 10^{13}\ \text{s}\end{aligned}$$

The classical theory says it should take thousands of years for an electron to soak up enough energy to be ejected, which contradicts experiment, so the classical theory must be wrong.

34-12 Let p be the probability that it's emitted to the right. The two pieces of the wave have the same wavelength, but their amplitudes differ. Probability is proportional to amplitude squared, so $p/(1-p) = (5/1.5)^2$, giving $p = 0.92$.

34-13 (a) The surface area of the spherical surface is $A = 4\pi r^2$. During a time interval t, the energy from the transmitter travels a distance $w = ct$, and by the time this energy reaches the distance r, it has been spread out over a spherical shell of volume Aw. The energy emitted during this time is Pt, so the energy density is $E/V = PT/(Aw) = P/4\pi cr^2$.
(b) The number of photons n in a volume $V = \lambda^3$ is

$$\begin{aligned}n &= \frac{E}{hf}\\ &= \frac{(E/V)\lambda^3}{hc/\lambda}\\ &= \frac{P\lambda^4}{4\pi hc^2r^2}\end{aligned}$$

(c) Let's take $P = 10$ kW and $r = 10$ km. The result then comes out to be about 10^{21}. This huge number is consistent with the correspondence principle.

35-1 Momentum is related to kinetic energy by $E = p^2/2m$, so

$$\begin{aligned}\lambda &= h/p\\ &= h/\sqrt{2mE}\\ &= h/\sqrt{2mq\Delta V}\\ &= 1\times10^{-11}\ \text{m}\end{aligned}$$

35-2 It's conceivable that the Δx is actually *less* than the diameter of the nucleus, but it can't be *more*, since the proton stays inside the nucleus. The maximum possible value of Δx is therefore roughly the same as the diameter of the nucleus. A maximum value of Δx leads to a minimum value of Δp, since they're inversely related: $\Delta p \gtrapprox h/\Delta x$. Plugging in, we get $\Delta v = \Delta p/m \gtrapprox h/m\Delta x \gtrapprox 3 \times 10^7$ m/s. Note the two inequalities in a row: one because the maximum uncertainty is the diameter of the nucleus, and one because the uncertainty principle is an inequality. It's perfectly possible that they're moving much faster than this; what we've found is only a minimum.

Our result is 10% of the speed of light, so the nonrelativistic equations are good enough for this kind of estimate.

35-3 (a) $\lambda = h/p = h/mv = 10^{-8}$ m $= 10$ nm.
(b) The area of one circuit would be 10^{-8} cm^2, which might be a square 10^{-4} cm on a side, i.e. 1000 nm.
(c) No. Diffraction is only an important effect when the size of the object the wave is diffracting around is comparable to the wavelength.
(d) If the linear size of a circuit was of the same order of magnitude as the wavelength, then we'd definitely have a lot of diffraction. A 10 nm × 10 nm circuit would have an area of 100 nm^2 = 10^{-12} cm^2, so you could fit 10^{12} of them on the chip.

35-4 (a) $p = mv = m(c/100) \sim 10^{-24}$ kg·m/s
(b) 0.01 nm $= 10^{-11}$ m is the wavelength of a gamma ray.
(c) $p = h/\lambda \approx h/(10^{-11}\text{ m}) = 10^{-22}$ kg·m/s
(d) The momentum of the photon is orders of magnitude greater than that of the electron, so the electron's momentum would be completely altered by the first photon that hit it — in fact, it would be knocked out of the atom completely. That means you can't use this technique to map out the orbit of an electron.

35-5 The wavelength equals $2L/n$, so we have

$$\begin{aligned} KE &= \frac{p^2}{2m} \\ &= \frac{(h/\lambda)^2}{2m} \\ &= \frac{h^2n^2}{8mL^2} \end{aligned}$$

35-6 (a) The lowest-energy state is half of a sine wave, as shown in the $n = 1$ diagram in figure f in section 4.3. The maximum is in the middle, where the thing we're taking the sine of is 90 degrees. Squaring this function we get something with a shape of $\sin^2\theta$, and the half-max points would be where $\sin^2\theta = 1/2$, i.e., $\theta = \sin^{-1}(1/\sqrt{2}) = 45$ degrees and 135 degrees. In other words, the maximum is in the middle, and the half-maximum points are at 1/4 and 3/4 of its length. That means the FWHM is $L/2$.
(b) $\Delta x \Delta p = (L/2)(2h/L) = h$.

35-7 (a) The lowest-energy state is half of a sine wave, as shown in the $n = 1$ diagram in figure f in section 4.3. Since the definition of the standard deviation was written under the assumption that the average would be zero, we need to write the equation of a sinusoidal wave that has a peak at $x = 0$ and nodes at $x = -L/2$ and $L/2$:

$$\Psi = A\cos((\pi/L)x) \qquad ,$$

where the amplitude A has to be determined from the condition that the wave is properly normalized:

$$\begin{aligned} 1 &= \int_{-L/2}^{L/2} \Psi^2\,\mathrm{d}x \\ &= A^2 \int_{-L/2}^{L/2} \cos^2((\pi/L)x)\,\mathrm{d}x \\ &= A^2 \int_{-\pi/2}^{\pi/2} \cos^2 u\,\mathrm{d}x \\ &= A^2 \int_{-\pi/2}^{\pi/2} \cos^2 u(L/\pi)\,\mathrm{d}u \\ &= LA^2/2 \\ A &= \sqrt{2/L} \end{aligned}$$

Therefore

$$\begin{aligned}
\Psi &= \sqrt{2/L}\cos((\pi/L)x)\\
D &= (2/L)\cos^2((\pi/L)x)\\
\sigma^2 &= \int_{-L/2}^{L/2} D(x)x^2\,\mathrm{d}x\\
&= (2/L)\int_{-L/2}^{L/2}\cos^2((\pi/L)x)x^2\,\mathrm{d}x\\
&= (2/L)\int_{-\pi/2}^{\pi/2} x^2\cos^2 u\,\mathrm{d}x\\
&= (2/L)(L/\pi)^3\int_{-\pi/2}^{\pi/2} u^2\cos^2 u\,\mathrm{d}u\\
&= L^2(2/\pi^3)\int_{-\pi/2}^{\pi/2} u^2\cos^2 u\,\mathrm{d}u
\end{aligned}$$

This is the kind of integral that you should look up in a table or do using computer software. The definite integral equals $(\pi^3-6\pi)/24$, so we have

$$\begin{aligned}
\sigma &= \sqrt{\frac{1}{12}-\frac{1}{2\pi^2}}L\\
&\approx (0.18)L
\end{aligned}$$

(b) $\Delta x\Delta p = (\sigma)(2h/L) \approx (0.36)h$.

35-8 The original equation, for a single rectangular barrier, was

$$P = \exp\left(-\frac{2w}{\hbar}\sqrt{2m(PE-E)}\right) \qquad .$$

Suppose we had two different barriers, of heights PE_1 and PE_2 and widths w_1 and w_2. Although we might get complicated patterns of reflections back and forth, these are presumably unimportant in the approximation that the probability of penetration is low, because in that approximation, the intensity of the waves is dying out so rapidly that the ones that got through after some reflections would be negligible. The probability of penetrating the second barrier is then independent of the probability of penetrating the first one. When probablities are independent, the probability of the "and" is found by multiplying them:

$$P = \exp\left[-\frac{2\sqrt{2m}}{\hbar}\left(w_1\sqrt{PE_1-E}+w_2\sqrt{PE_2-E}\right)\right]$$

For a barrier that isn't just a series of rectangles, we can still approximate it as such a series — this is exactly the idea of the usual definition of the integral in terms of a limit. The widths w_1, …are then simply dx, and we have

$$P = \exp\left[-\frac{2\sqrt{2m}}{\hbar}\int_a^b\sqrt{PE_1(x)-E}\,\mathrm{d}x\right] \qquad ,$$

where a and b represent the beginning and end of the region that classical physics forbids the particle to enter. This is known as the WKB approximation, after the initials of the three physicists who first published it.

35-9 (a) The uncertainty in position is on the order of magnitude of the diameter of the nucleus, so the uncertainty principle gives $\Delta p \sim h/\Delta x \sim 10^{-18}$ kg·m/s. (b) $K = (1/2)mv^2 = p^2/(2m) \sim 10^{-7}$ J. (c) $m = E/c^2 \sim 10^{-24}$ kg, which is a thousand times greater than the actual mass of the neutron.

36-1 (a) The sizes of the realistic wavefunctions are on the order of 0.1 nm. Plugging in $n=1$ to the approximation $r \sim h^2n^2/mke^2$, we get 2 nm. The estimate was too high by about a factor of 20, which is reasonable considering the crude assumptions we made.

(b) The proton is about 2000 times more massive than the electron, so the center of mass lies about 1/2000 of the way from the center of the proton to the electron. A typical separation between the proton and electron is 0.1 nm, so the center of the proton would typically be about (0.1 nm)/2000 $\sim 10^{-13}$ m from the center of mass. The size of a proton is about 10^{-15} m, so the center of mass is normally outside the proton.

36-2 (a) The energy of a photon is proportional to its frequency, and therefore inversely proportional to its wavelength. Of the visible-light transitions, F has the greatest energy, and thus the shortest wavelength, i.e. it is near the violet end. Transition D would be closest to the red end.

(b) A, B, and C have high energies, which correspond to short wavelengths. We're told they're not visible, so they must be in the ultraviolet. (They can't be in the x-ray or gamma regions, because the energies and wavelengths are not orders of magnitude different from those of visible light.) Similar reasoning places G and H in the infrared.

(c) A long photon wavelength corresponds to a low photon frequency, a low photon energy, and a small difference between energies of atomic levels. There is no upper limit to the wavelengths, since the states get infinitely close together near the top of the hydrogen atom's energy-level diagram. On the other hand, there is a lower limit to the photon wavelength, since the bound states of the hydrogen atom span a limited range of energies, imposing a maximum on the difference between any two levels. Physically, if you put too much energy into a hydrogen atom, you strip off the electron entirely.

36-3 The energies are inversely proportional to the wavelengths, so in terms of energies the equation would be $E_1 = E_2 + E_3$. This would happen for sets of transitions like G, D, and E in the previous problem, where the atom can get from one state to another either by a single hop or by two shorter hops.

36-4

$$\begin{aligned}
E_i &= E_f \\
E_{atom,i} &= E_{atom,f} + E_{photon} \\
E_{photon} &= E_{atom,i} - E_{atom,f} \\
&= \frac{mk^2e^4}{2\hbar^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)
\end{aligned}$$

But

$$\begin{aligned}
E_{photon} &= hf \\
&= hc/\lambda \qquad ,
\end{aligned}$$

so

$$\begin{aligned}
hc/\lambda &= \frac{mk^2e^4}{2\hbar^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \\
\lambda &= \frac{2\hbar^2 hc}{mk^2e^4}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)^{-1} \\
&= \frac{4\pi\hbar^3 c}{mk^2e^4}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)^{-1}
\end{aligned}$$

36-5 Say the basketball has a mass of 1 kg, and in general its mass is about 0.1 m from the axis of rotation. If it's spinning at a speed of 1 m/s, then its angular momentum is on the order of 0.1 J·s. This is roughly $10^{32}\hbar$. The fact that this number is so huge is an example of the correspondence principle. Angular momentum is quantized in units of $\hbar$, so the basketball can't continuously speed up or slow down its rotation; it has to do it in quantum leaps from one value of L to another. But because L is so huge when expressed in units of $\hbar$, the process appears smooth and continuous. Therefore quantum mechanics doesn't contradict the results of the centuries of previous experiments in which rotation appeared to be smoothly variable.

36-6

36-7 (a) The size of the diagram is comparable to a, but b is so small that you wouldn't be able to see it on that scale.

(b) The stuff inside the exponential is $-r/a$, which is essentially zero for all values of r between 0 and b. In other words, the proton is super-tiny compared to the size of the electron cloud, and the electron's probability density therefore won't change appreciably from one part of the proton to the other. We don't need to do an integral at all. We can just multiply:

$$\begin{aligned}P &= \Psi^2 V \\ &= \left(\pi^{-1/2}a^{-3/2}e^{-r/a}\right)^2\left(\frac{4\pi}{3}b^3\right) \\ &\approx \left(\pi^{-1/2}a^{-3/2}\right)^2\left(\frac{4\pi}{3}b^3\right) \\ &= \frac{4b^3}{3a^3}\end{aligned}$$

(c) 1×10^{-15}
(d) The probability density never dies down to zero, but it gets very very close to zero. The atom is fuzzy. It doesn't have a sharply defined boundary. There is theoretically a finite probability for the electron to be a centimeter away from the proton, but it's extremely small.

36-8 In the approximate derivation in subsection 12.4.4, the only place charge came in was in the use of the expression $-ke^2/r$ for the electrical energy. If the charges were Ze and $-e$ rather than e and $-e$, we'd simply have Ze^2 instead of e^2. The result contains e^4, so we need to replace $e^4 = (e^2)^2$ with $(Ze^2)^2 = Z^2e^4$. The result would be

$$E_n = -\frac{mk^2Z^2e^4}{2\hbar^2} \cdot \frac{1}{n^2}.$$

36-9 (a) The relation

$$r \sim \frac{h^2n^2}{mke^2}$$

has the mass of the electron in the denominator. Increasing the mass by a factor of 207 scales the atom down by a factor of 207.
(b) The equation

$$E_n = -\frac{mk^2e^4}{2\hbar^2} \cdot \frac{1}{n^2}$$

would scale up all the energies by a factor of 207. If the atom's energies are scaled up, then so are the energies of its absorption and emission lines. They would be ultraviolet or x-ray lines.

36-10 Let T be the time required for one complete orbit. The current is $-e/T$, so the magnetic dipole moment is $IA = (-e/T)(\pi r^2)$. The angular momentum is $L = mvr = m(2\pi r/T)r = \pi mr^2/T$. Dividing the two quantities gives the result claimed in the problem.

36-11 All of the constants cancel out, so we only need the n-dependent part of the expressions. The result is

$$\frac{\lambda_E}{\lambda_F} = \frac{E_F}{E_E} = \frac{2^{-2}-5^{-2}}{2^{-2}-4^{-2}} = \frac{28}{25}$$

2 Simple Nature

0-1 (a) If your calculator has parentheses, you have to use them around the whole denominator. On a calculator without parentheses, you'd add up the denominator, put the sum in memory, and then do the division. The answer is 0.4951584.
(b) $\$3.5 \times 10^5$ equals \$350,000, the price of a big house, and $\$3.5^5$ equals \$525.21, the price of a big TV.

0-2 (a) 8 cm; (b) 1.33 m; (c) doesn't make sense; (d) 60 miles/hour

0-3 $23.4\text{ m} + 2 \times 0.294\text{ m} = 23.988\text{ m}$, which should be rounded to 24.0 m (3 significant figures)

0-4

$$\begin{aligned}&3.0 \times 10^8\text{ m/s}\\ &\times \left(\frac{100\text{ cm}}{1\text{ m}}\right)\left(\frac{1\text{ inch}}{2.54\text{ cm}}\right)\left(\frac{1\text{ foot}}{12\text{ inches}}\right)\\ &\times \left(\frac{1\text{ yard}}{3\text{ feet}}\right)\left(\frac{1\text{ furlong}}{220\text{ yards}}\right)\\ &\times \left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right)\left(\frac{24\text{ hr}}{1\text{ day}}\right)\left(\frac{14\text{ days}}{1\text{ forthnight}}\right)\\ &= 1.8 \times 10^{12}\text{ furlongs/fortnight}\end{aligned}$$

0-5 (a) 10^4 μg; (b) 10^{10} μg; (c) 10^{10} μg; (d) 10^{11} μg; (e) 1 μg

0-7 (a)

$$\begin{aligned}\frac{0.5\text{ mg}}{0.2\text{ ng/hamburger}} &= \frac{0.5 \times 10^{-3}\text{ g}}{0.2 \times 10^{-9}\text{ g}}\text{ hamburgers}\\ &= 2\text{ million hamburgers!}\end{aligned}$$

(Note how the fake units of "hamburgers" help to reassure us that dividing the first number by the second one gives the right result.)
(b) (0.5 mg)/(300 ng/serving)=2000 servings

0-9 The range of uncertainty amounts to more than a factor of 2. That means that each of the two figures has been quoted with some completely meaningless sig figs. A better way to write the numbers would be 6 days and 14 days.

0-10 The weight is given in U.S. units with not much more than one sig fig of precision, but the metric conversion is given with three sig figs. It's silly to try to give tenths of a gram here; there must be an order of magnitude more variation than that from one bag to the next, since one pretzel is probably several grams.

0-11 Both r and h have units of meters. The product rh has units of m^2, and taking its square root gives units of meters again. That makes sense, because the result is supposed to be a distance.

0-13 The units don't work out: $\text{m/s}^2 \times \text{s} = \text{m/s}$, not m.

0-14 $s = \frac{1}{2}at^2$, and $a = g = 10\text{ m/s}^2$, so $s = 45$ m, or call it 40 m (1 sig fig).

0-15 (a) The trip consists of four parts of equal duration. The duration, $T/4$, of each part is related to half the distance, d, to Proxima Centauri, $d/2 = \frac{1}{2}a(T/4)^2$, so $T = 4\sqrt{d/a}$.

(b) Plugging in to the formula from (a), we get 2.6×10^8 s = 7 years.

(c) The maximum speed is $(a)(T/4) = 6 \times 10^8$ m/s. This is more than twice the speed of light, which is not actually possible.

0-16 We have $v^2 = 2a\Delta x$, and the factor of $2a$ is the same in both cases, so we can rewrite this as a proportionality $v^2 \propto \Delta x$. Solving for v gives $v \propto \sqrt{\Delta x}$, and converting this into the language of ratios results in $v_1/v_2 = \sqrt{\Delta x_1/\Delta x_2} = \sqrt{2}$.

0-18 (a) b: meters, c: meters/s (because you multiply it by t to get something with units of meters), k: units of s (because it doesn't make sense to take $e^{-t/k}$ unless the units of $-t/k$ cancel out)

(b) $v = \mathrm{d}y/\,\mathrm{d}t = \mathrm{d}(-ct)/\,\mathrm{d}t + \mathrm{d}(-cke^{-t/k})/\,\mathrm{d}t = -c + ce^{-t/k}$

(c) For large values of t, $e^{-t/k}$ is nearly zero, so v approaches c. Thus, c can be interpreted as the maximum velocity she will ever reach (known as her terminal velocity).

(d) Differentiating again, we get $a = \mathrm{d}v/\,\mathrm{d}t = \mathrm{d}(-c + ce^{-t/k})/\,\mathrm{d}t = -(c/k)e^{-t/k}$.

(e) For large t, the exponential becomes very small, so her acceleration is nearly zero.

0-20 (a)

$$v_f^2 = v_o^2 + 2a\Delta x$$

The initial velocity v_o, is the velocity of the spinning earth's surface, which at the equator is $(2\pi r)/(1 \text{ day}) \approx 500$ m/s. This is an order of magnitude less than the orbital velocity given in this problem. Since v_o is much less than v_f, v_o^2 is even more negligible compared to v_f^2, and we're justified in ignoring it for the purposes of this approximate calculation, whose accuracy is really limited by the assumed acceleration.

$$\begin{aligned} v_f^2 &= 2a\Delta x \\ \Delta x &= \frac{v_f^2}{2a} \\ &= 1\times 10^6 \text{ m} \\ &= 1000 \text{ km} \end{aligned}$$

(b) It's not very practical to build a railgun that would stretch halfway across a continent. The railgun idea is definitely not going to be practical for crewed launches, but it might work for lofting payloads that could handle more violent accelerations, such as raw materials. For instance, a payload that could handle $300g$ of acceleration would only need a 10-kilometer railgun.

0-21 We're given distance and acceleration, and we want to find time. Of the constant acceleration equations, the one that has the right variables in it is $\Delta x = v_o\Delta t + \frac{1}{2}a\Delta t^2$. The initial velocity is zero, and solving for Δt, we have $\Delta t = \sqrt{2\Delta x/a}$. Converting the given distance to SI (meter-kilogram-second) units, we have 4000 mi $= 6.4\times 10^6$ m, which results in $\Delta t = 1100$ s, or about 20 minutes (1 sig fig).

0-22

$$(1 \text{ ft}^3)\times(12 \text{ in}/1 \text{ ft})^3 = 1728 \text{ in}^3$$

0-23 Volume scales like the third power of length, so the dog has 8 times more brain cells.

0-24 Solving the proportionality $A \propto L^2$ for L, we have $L \propto \sqrt{A}$. Converting this into the language of ratios gives $L_{LA}/L_{SF} = \sqrt{A_{LA}/A_{SF}} =$ 1.2.

0-25 Area and volume are different units. You can't compare 10 in^2 to 1 in^3. 10 in^2 is not "greater than" 1 in^3. (The dog's nose membrane is crumpled up like a crumpled piece of paper.)

0-26 I don't even know the exact width of a football field, but let's guess the dimensions are 100 yd × 30 yd, which is close enough to 100 m × 30 m, for a total area of 3000 m^2. If we assume each blade of grass takes up on the order of 1 mm^2, then the number of blades equals 3000 m^2/1 mm^2 $= 3\times 10^9$ mm^2/1 mm^2 $= 3\times 10^9$. Call it 10^9 blades.

0-27 Let's say the chip is 1 cm × 1 cm. It contains about 10^8 bits (64×10^6 bytes$\times$8 bits/byte), so the area of each circuit is about 10^{-8} cm^2. Assuming the circuits are little squares, their dimensions would be about 10^{-4} cm on a side, since 10^{-4} cm $\times\ 10^{-4}$ cm $= 10^{-8}$ cm^2.

0-28 The world's population is about 5 billion, or call it 10^{10}. Suppose each person gets an apartment that's 5 m x 5 m x 2 m, for a volume of 50 m^3, or round off to 100 m^3. The total volume of the building needs to be about 10^{12} m^3. Its area at the base is 10^8 m^2, so its height needs to be 10^{12} m^3/10^8 m^2 $= 10^4$ m = 10 km.

0-29 Suppose one bongo burger is 1/4 lb, which is about 0.1 kg. If the usable meat on a cow amounts to 100 kg, then one cow makes 1000 burgers. Clearly the cow has to eat much more over the course of its life than its own adult weight. (After all, you could not raise a human to adulthood on 100-200 lb of food.) Let's say 10000 kg of feed are needed to raise a several-hundred-kg cow. Then 10000 kg of feed makes 1000 burgers, or 10 kg of feed makes 1 burger. That means $\sim 10^{11}$ kg of feed were needed to make 10^{10} burgers.

0-30 Apply two of the general rules for making order-of-magnitude estimates: don't try to estimate volume directly, and approximate complicated shapes as simple ones. Suppose a person is sort of a rectangle, 2 m tall, 0.5 m wide, and 0.2 m from front to back. The volume would then be 0.2 m^3 $= 2\times 10^5$ cm^3, which we can round off to 10^5 cm^3.

0-34 (a) You can get two A5 sheets by cutting an A4 in half, so the area of an A5 is half that of an A4. The two types are the same shape, so the area and linear dimensions are related by $A \propto L^2$, or $L \propto \sqrt{A}$. Cutting the area in half means reducing all the linear dimensions by a factor of $1/\sqrt{2}$.

(b) Let the dimensions of the B0 box, in units of meters, be a, b, and c ($a > b > c$). If you can pack two B1 boxes inside a B0 box, two of the dimensions of a B1 must be the same as the two smallest dimensions of the B0, b and c, while the third is $a/2$. Since $L \propto V^{1/3}$, the dimensions of the B1 must be smaller than those of the B0 by a factor of $1/2^{1/3}$. If the series is to continue with B2, B3, ..., the three sides of a particular box must be related by this same factor:

$$\begin{aligned} b &= \frac{a}{2^{1/3}} \\ c &= \frac{b}{2^{1/3}} = \frac{a}{2^{2/3}} \end{aligned}$$

Since the volume is one cubic meter, we have

$$\begin{aligned} 1\ \text{m}^3 &= abc \\ &= (a)\left(\frac{a}{2^{1/3}}\right)\left(\frac{a}{2^{2/3}}\right) \\ &= \frac{a^3}{2} \\ a &= 2^{1/3}\ \text{m} \\ b &= 1\ \text{m} \\ c &= 1/2^{1/3}\ \text{m} \end{aligned}$$

0-35 We never try to estimate mass or volume directly; we estimate linear dimensions first. Let's say the hair has a diameter of 0.1 mm, and a length of 1 cm. There's no real point in trying to compute its volume as a cylinder, since this is just an estimate. Let's just consider it to be a long, skinny box, with dimensions 0.1 mm × 0.1 mm × 1 cm. We'll assume it has the same density as water, 1 g/cm^3, so let's get all the dimensions in terms of centimeters before computing the volume and mass: $0.01\ \text{cm} \times 0.01\ \text{cm} \times 1\ \text{cm} = 10^{-4}\ \text{cm}^3$. Using the density, we get a mass of $10^{-4}\ \text{g} = 10^{-7}\ \text{kg}$.

0-36 First let's estimate the total volume of the Earth's atmosphere. We know that by going to the top of a mountain that's a few miles high, we can get above most of the atmosphere, so let's say the atmosphere is roughly 10 km, or 10^4 m thick. The diameter of the earth is about 12000 km, or about 10^7 m, so its surface area is on the order of $10^{14}\ \text{m}^2$, and multiplying by the thickness of the atmosphere gives about $10^{18}\ \text{m}^3$. A human breath is probably on the order of a liter, or $10^{-3}\ \text{m}^3$, so Caesar's last breath has been diluted by a factor of something like $10^{18}/10^{-3} \sim 10^{21}$.

Now the question is whether your breath has a lot more than, or a lot less than, 10^{21} atoms in it. If you've taken chemistry, you know that a mole is something like 10^{24} atoms, and a mole of air would only be on the order of 10 grams. That means your breath has a lot more than 10^{21} atoms in it, so it's probably true that it contains many of the atoms from Caesar's last breath. If you haven't taken chemistry, you might not have as good a feel for this, but you could simply look up the density of air and the mass of an oxygen molecule on the internet or at the library.

(One implicit assumption that could be wrong here is that air stays in the atmosphere indefinitely. Some air could spend a lot of its time in plants, or dissolved in the oceans or in rocks.)

0-37 Area scales like the square of the linear dimensions, so Earth's total surface area is four times that of Mars. Since only 30 percent of Earth's area is land, the ratio is 1.2 to 1 (Earth to Mars).

0-39 Area scales like the square of the linear dimensions, so 0.35 of the linear dimensions corresponds to $0.35^2 = 0.11$, or 11% of the area.

0-42 (a) Living things typically have about the same density as water, $1\text{g/cm}^3 = 1000\ \text{kg/m}^3$. The linear dimensions are about 1 μm, so the volume is about $10^{-18}\ \text{m}^3$, giving a mass of 10^{-15} kg.

(b) Estimating the mass of the feces to be 1 kg (some of us are more full of it than others), we get about 10^{15} bacteria in the body. This is two orders of magnitude greater than the number of

human cells.
(c) Since the human cells are much less numerous, but take up much more space, they must be many orders of magnitude bigger than bacterial cells.

0-43 (a) The variables involved are distance, acceleration, and time, so we want the constant-acceleration equation that relates them, which is $\Delta x = \frac{1}{2}g\Delta t^2$. The balls are two separate objects, so we need to have two separate equations of this form.

$$\begin{aligned}
\Delta x_1 &= \frac{1}{2}g\Delta t_1^2 \\
\Delta x_2 &= \frac{1}{2}g\Delta t_2^2 \\
\Delta t_1 &= \sqrt{\frac{2\Delta x_1}{g}} \\
\Delta t_2 &= \sqrt{\frac{2\Delta x_2}{g}} \\
T &= \Delta t_2 - \Delta t_1 \\
&= \sqrt{\frac{2\Delta x_2}{g}} - \sqrt{\frac{2\Delta x_1}{g}} \\
g &= \left(\frac{\sqrt{2\Delta x_2} - \sqrt{2\Delta x_1}}{T}\right)^2
\end{aligned}$$

(b) The units inside the parentheses are $\text{m}^{1/2}/\text{s}$, so squaring this quantity, we have m/s^2.

(c) When $\Delta x_1 = 0$, T is the same as Δt_2, and we have simply $g = (\sqrt{2\Delta x_2}/\Delta t_2)^2$, which is what we would have gotten if we had just solved $\Delta x_2 = \frac{1}{2}g\Delta t_2^2$ for g. This case reassures us that the result is correct, but it is useless in practical terms: we have to make Δx_1 great enough so that the ball will make a sound we can record.

(d) Mathematically, we get zero over zero, which is undefined. Physically, we cannot measure all the quantities in the equation with perfect precision, so we will not get exactly zero on the top or on the bottom; the result will be not an error on our calculator but rather a wildly unreliable answer! Nature is giving us a hint that the most accurate results will be obtained when the two heights are as different as possible, and will get less and less reliable as the heights are made closer to one another.

0-46 The estimate of 4,480 calories is stated with three significant figures, which is absurd. Individuals often differ by a factor of two in their food consumption, and in any case this is a rough estimate for a species that has been extinct for tens of thousands of years.

1-1 No, it doesn't violate conservation of mass. What's being measured is a change in volume. If the fluid becomes less dense with rising temperature, then it can increase its volume without increasing its mass.

1-2 Conservation of mass states that

$$\begin{aligned}
m_i &= m_f \\
m_{NaCl} &= m_{Na} + m_{Cl} \qquad ,
\end{aligned}$$

so

$$\begin{aligned}
\frac{m_{NaCl}}{m_{Na}} &= 1 + \frac{m_{Cl}}{m_{Na}} \\
\frac{m_{Cl}}{m_{Na}} &= \frac{m_{NaCl}}{m_{Na}} - 1 \\
&= 1.5421
\end{aligned}$$

(with five sig figs, since the limiting factor is the five-sig-fig value given for the mass of the remaining sodium). As a check, let's see if it makes sense that this number is greater than one. The fact that it's greater than one means that a chlorine atom should have a mass greater than that of a sodium atom. That makes sense, because boiling off the chlorine caused more than half of the mass to be lost.

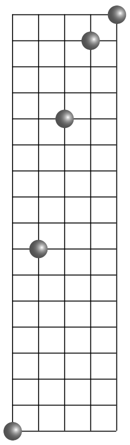
1-3 Conservation of mass would require

$$\begin{aligned}
m_i &= m_f \\
m_U &= m_{Th} + m_{He} \\
3.95292849 \times 10^{-25}\ \text{kg} &= 3.95285229 \times 10^{-25}\ \text{kg} \qquad .
\end{aligned}$$

This equality fails, and the discrepancy, although small compared to the atomic masses, is nevertheless many orders of magnitude greater than the precision of the data. Mass is not being conserved. As discussed in section 1.4, what's really conserved is not m but $E + mc^2$. In ordinary chemical reactions, the huge factor of c^2 makes the mc^2 term many, many orders of magnitude greater than the E term, and therefore we don't have to worry about the E term, and it's an excellent approximation to say that it's just mass that's conserved. This is a nuclear reaction, however, so the energy scale is much greater — like an atomic bomb compared to a stick of dynamite. The effect of the E term is orders of magnitude greater than it would be for a chemical reaction, great enough that it can be measured fairly easily.

1-8 This is closely analogous to example 1 in section 1.1.1. Since all three channels have the same width and depth, they all have the same cross-sectional area. If the speed of the current was the same in all three channels, then we'd have nonconservation of mass, because the rate at which mass was leaving the fork would be double that at which it entered. We'd be creating water out of nothing. The speed of the currents leaving the fork can't both be as high as the speed of the current entering. Based on the symmetry of the situation, it's probably a good approximation to expect that each of the two channels below the fork will have a current with half the speed seen above the fork.

1-11 I made the new observer move to the right relative to the original one. In the new frame of reference we get a parabola.



2-4 We're testing conservation of energy,

$$E_{total,i} \quad =? \quad E_{total,f}.$$

The only form of energy we can determine from the photo is kinetic energy, so this amounts to testing whether

$$KE_{1,i} \quad =? \quad KE_{1,f} + KE_{2,f},$$

where the initial time is before the collision and the final time is after the collision is over. Using $KE = (1/2)mv^2$, and taking advantage of the fact that the masses are equal, this becomes

$$v_{1,i}^2 \quad =? \quad v_{1,f}^2 + v_{2,f}^2.$$

Now $v = d/t$, where t is the time between flashes, and the common factor of t^{-2} can be divided out on both sides, so we end up with

$$d_{1,i}^2 \quad =? \quad d_{1,f}^2 + d_{2,f}^2,$$

where d is the distance between one position and the next. That is, the square of the distance is our measure of energy.

But rather than measuring directly from one position to the next, it's more accurate to measure the distance over several of the time intervals in succession. In four flashes, the incoming ball moves 6.0 cm. In the same amount of time after the collision, it moves 2.8 cm, and the target ball moves 5.0 cm.

Since we're leaving out lots of factors that have units, I'll write the numbers below without units.

$$\begin{aligned} 6.0^2 &\quad =? \quad 2.8^2 + 5.0^2 \\ 36.0 &\quad =? \quad 7.8 + 25.0 \\ 36.0 &\quad =? \quad 32.8 \end{aligned}$$

There appears to have been a slight loss of energy, but it's really at the limit of our ability to measure things from the figure. We also know that there will be some extra energy after the collision in the form of heat and sound. The only thing that would really disprove conservation of energy in this collision would be if the final KE was *greater* than the initial KE, and if the increase was significant given the limited accuracy of the measurements.

2-6 Kinetic energy depends on how fast the ball is moving. It's moving fastest at the bottom, so that's where it has the maximum kinetic energy. Energy is conserved, so it's trading KE and gravitational energy back and forth, and it has the most gravitational energy when it has the least KE, which is at the top when it's momentarily at rest.

2-7 (a) Magnetic energy plus kinetic energy stays the same, so if it's speeding up, the KE is increasing and the magnetic energy must be decreasing. (b) By similar reasoning, magnetic energy must be increasing as they approach each other.

2-8 If she can run the 100 m dash in about 10 s, then she's going about 10 m/s. If her mass is 60 kg, then her kinetic energy is about 3000 J (one sig fig).

2-9 Converting the speeds to mks, we have 18 m/s and 36 m/s. (a) $(1/2)mv^2 = 2.4 \times 10^5$ J; (b) Four times greater, 9.6×10^5 J. In this sense, driving twice as fast is four times more dangerous, not twice as dangerous as you might think.

2-10 The total amount of energy required to kill her is

$$(60\,\text{kg})\left(\frac{1\text{ J}}{0.24\text{ g}\cdot{}^\circ\text{C}}\right)\left(\frac{10^3\text{ g}}{1\text{ kg}}\right)(6^\circ\text{C}) = 1.5\times 10^6\text{ J}$$

Solving the equation $P = \Delta E/\Delta t$ for $\Delta t = \Delta E/P$, we find that she can survive for 7500 s=120 min.

2-11 (a) We have to heat it by 20 degrees, melt it, heat it by 100 degrees, boil it, and then heat it by another 37 degrees. The result is

$$\begin{aligned}&(20)(2.05\text{ J})\\ &+333\text{ J}\\ &+(100)(4.19\text{ J})\\ &+2500\text{ J}\\ &+(37)(2.01\text{ J})\\ &=3.4\times 10^3\text{ J}\end{aligned}\quad.$$

Surprisingly, the majority of the energy is used in a single step of the process, the boiling.

(b) To minimize the amount required, we want the initial temperature of the water to be the maximum, 100°C, and similarly the ice should be at 0°C. At the end of the process, we'll just have one big sample of water at some intermediate temperature, and to minimize the amount of water, we want this temperature to be as low as possible, zero degrees. Conservation of energy gives

$$\begin{aligned}\Delta E &= 0\\ 0 &= -m(100^\circ\text{C})(4.19\text{ J/g/}^\circ\text{C}) + (1\text{ g})(333\text{ J/g})\\ m &= 0.795\text{ g}\end{aligned}$$

2-12 Energy is conserved. Anya's penny starts out with double the gravitational energy (taking the ground to be the zero level of gravitational energy). The gravitational energy is converted into kinetic energy as the pennies fall. (Although there may be some energy turned into heat by air friction, we expect that to be a negligible effect for something as dense and compact as a penny, dropped from this relatively small height.)

Anya's penny must therefore have twice the kinetic energy of Ivan's on impact. Since kinetic energy is proportional to v^2, we have $v \propto \sqrt{KE}$, so the ratio of the velocities is not 2 but rather $\sqrt{2}$.

2-13 Kinetic energy doesn't depend on the direction of the motion, so since both pennies start out at the same speed, they both start out with the same kinetic energy. They also start out at the same height, so they have the same gravitational energy and the same total energy. Energy is conserved, so if they start out with a certain amount of energy, that's the same amount of energy they'll always have; their total energies will always be equal to each other. Although the impacts with the ground will happen at different times, we don't care about that. On impact they have the same gravitational energy and the same total energy, so they must have the same kinetic energy on impact as well. They hit the ground with equal kinetic energies and equal speeds.

2-14 (a) The distance traveled by each atom in one revolution is $2\pi r$, so every atom is traveling at a speed of $v = (2\pi r)/T = \omega r$. Substituting this into $K = (1/2)mv^2$ gives the claimed result.
(b) The quantity ω has units of s^{-1}, inverse seconds, so $m\omega^2 r^2$ has units of kg·s^{-2}·m^2, equivalent to joules.
(c) Let's use lowercase letters for the hoop, uppercase for the block. After descending by a vertical distance h, conservation of energy gives

$$mgh = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}mv^2$$

and

$$Mgh = \frac{1}{2}MV^2 \qquad ,$$

and dividing out the masses and substituting $\omega = v/r$ simplifies these to

$$gh = v^2$$

and

$$gh = \frac{1}{2}V^2 \qquad .$$

These are similar in form to the kinematic equation $v_f^2 = 2ax$ for the final velocity of an object that accelerates from rest, traveling a distance x. Although one equation has h, the vertical distance, and the other has x, the distance along the slope, the relationship between these is the same for the block and the hoop, so it's clear that inserting a factor of 2 on the v^2 side of the equation corresponds to a change in acceleration by a factor of 2.

2-15

$$\begin{aligned}
v &= \int a\,\mathrm{d}t \\
&= at + v_o \\
y &= \int v\,\mathrm{d}t \\
&= \frac{1}{2}at^2 + v_o t + y_o \\
E &= K + U \\
&= \frac{1}{2}m\left(at + v_o\right)^2 + mg\left(\frac{1}{2}at^2 + v_o t + y_o\right) \\
&= \left(\frac{1}{2}ma^2 + \frac{1}{2}mga\right)t^2 + \left(mav_o + mgv_o\right)t + \ldots \qquad ,
\end{aligned}$$

where ... denotes constant terms. Conservation of energy requires that the entire expression be constant, which means that the t^2 and t coefficients have to vanish. In both cases, this happens if and only if $a = -g$.

2-16 First we set up an equation that states conservation of energy:

$$\begin{aligned}
&E_{total,i} = E_{total,f} \\
&K_{1,i} + U_{1,i} + K_{2,i} + U_{2,i} = K_{1,f} + U_{1,f} + K_{2,f} + U_{2,f}
\end{aligned}$$

Since gravitational energy is only defined up to an over-all additive constant, it's more convenient to rearrange things so that everything is expressed in terms of differences in gravitational energy.

$$K_{1,i} + K_{2,i} = K_{1,f} + K_{2,f} + \Delta U_1 + \Delta U_2$$

Let's also simplify things by assuming that the masses start from rest.

$$0 = K_{1,f} + K_{2,f} + \Delta U_1 + \Delta U_2$$

If the acceleration turns out to be constant, then it won't matter what we assumed for the initial state of motion. If it turns instead out to depend in some complicated way on the initial conditions, then this will have been a blunder, and we'll have to go back and redo it without the simplifying assumption. We no longer have any variables that occur in the equation with both i and f subscripts, so we can stop writing them without any risk of confusion.

$$0 = K_1 + K_2 + \Delta U_1 + \Delta U_2$$
$$0 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + m_1 g\Delta y_1 + m_2 g\Delta y_2$$

As suggested in the hint, if one weight goes up by a height $\mathrm{d}y$, then the other weight has to come down by a height $-\mathrm{d}y$. This means we have $v_1 = -v_2$, and $\Delta y_1 = -\Delta y_2$, so we can eliminate v_2 and y_2, reducing the number of variables from four to two:

$$0 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_1^2 + m_1 g\Delta y_1 - m_2 g\Delta y_1$$

We no longer need 1 and 2 subscripts on v and y to keep things straight, so let's stop writing them. We can also write y instead of Δy. Simplifying a little, we get

$$0 = \frac{1}{2}(m_1 + m_2)v^2 + (m_1 - m_2)gy$$

Of the two techniques suggested in the hint, I don't think either one is necessarily any easier than the other. Let's try the chain rule technique from section 2.1. Even though there are two objects here, we've reduced the problem to a form in which it looks like there's only one object, with kinetic energy $K = (1/2)(m_1 + m_2)v^2$ and gravitational energy $U = (m_1 - m_2)gy$.

$$\begin{aligned}\frac{\mathrm{d}v}{\mathrm{d}t} &= \left(\frac{\mathrm{d}v}{\mathrm{d}K}\right)\left(\frac{\mathrm{d}K}{\mathrm{d}U}\right)\left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)\\ &= \frac{1}{(m_1+m_2)v}\,(-1)\,[(m_1-m_2)g]\,(v)\\ &= -\frac{m_1-m_2}{m_1+m_2}g\end{aligned}$$

The interpretation of the negative sign is that v is actually v_1, so this is actually the acceleration of mass 1, which will be downward if $m_1 > m_2$. This expression is constant, so the acceleration doesn't depend on the initial conditions, and it turned out okay that we assumed the masses started from rest.

Let's do some routine checks. The units work out, because the units of kilograms cancel out, leaving us with only the units associated with g. Although g is defined as having units of J/kg/m, we've already seen that this is equivalent to $\mathrm{m/s^2}$. We can also look for a special case to try out and check for the right physical behavior. In the case of $m_1 = m_2$, the equation gives zero. That makes sense physically, because in this situation there's no way to release any gravitational energy by moving the weights: whatever gravitational energy one weight gains, the other loses, so there's no way to transform gravitational energy into kinetic energy, and the weights can't speed up or slow down. If they're released at rest, they'll stay at rest. If they're initially set in motion, they'll continue at the same speed until the rope is used up.

2-17 (a) The piece of string connected to B has three vertical parts: one connecting to the ceiling, one between the two pulleys, and one going down to B. If the part connecting to B lengthens by, say 2 cm, then this extra 2 cm has to come out of the other two segments. In other words, 1 cm of string is lost from the part connected to the ceiling, and 1 cm is lost from the part between the two pulleys. Therefore, if B goes down 2 cm, A rides up by only 1 cm.
(b) If they move, one will lose gravitational energy while the other gains gravitational energy. If A was a boulder and B was a feather, then A could release more gravitational energy by dropping than B would consume by rising. Likewise if B was ridiculously heavy compared to A, you'd release gravitational energy by letting B drop. We want to figure out how to arrange it so that there can't be any release of gravitational energy: even if one rises and the other falls, no gravitational energy will be transformed into kinetic energy. Since B travels twice as much as A, we want B to have half the mass. Then if B drops 2 cm, it will lose the same amount of gravitational energy that A gains by rising 1 cm.

2-18 (a) As suggested in the hint, let's start by finding the heights of the masses in terms of θ. If we really wanted to, we could find their heights above the floor, but that would bring in lots of irrelevant variables such as the length of the rope and the height at which the whole apparatus has been placed. Physically it's intuitively obvious that none of this can matter. Mathematically, the condition for equilibrium is going to be that the total gravitational energy have a zero derivative, and taking the derivative will get rid of any constant terms, so we're free to express each mass's height relative to some initial height that's convenient for the purposes of calculation. For the middle mass, it's clear that the natural reference height to choose is the one at which the central part of the rope is flat. Letting the distance between the two pulleys be 2ℓ, the height of the central weight is $-\ell\tan\theta$. If we release the central weight at height zero and then let it sag down to this position, then the segment of rope on each side of the central "V" has increased its length from ℓ to $\ell/\cos\theta$, for a change of $\ell(1/\cos\theta - 1)$. This is the amount of rope fed into the center from each side, so each of the weights on the side must have risen by this amount. The total gravitational energy is therefore

$$U = -mg\ell\tan\theta + 2Mg\ell(1/\cos\theta - 1) \qquad .$$

Since we're going to set this expression's derivative equal to zero, constant factors like m and g don't matter, and we're better off dividing them out. It's also a good idea to divide through by one of the masses, because then we can express everything in terms of the unitless ratio of the two masses. Physically it's clear that θ can only depend on this ratio, so it's reassuring to see this happening mathematically, and it's nicer to have the result expressed in a way that makes it manifestly obvious. The quantity we're going to work with is then

$$\frac{U}{mg\ell} = -\tan\theta + 2\alpha(1/\cos\theta - 1) \qquad ,$$

where $\alpha = M/m$. Setting the derivative equal to zero, we have

$$\begin{aligned} 0 &= -\sec^2\theta + 2\alpha\frac{\sin\theta}{\cos^2\theta} \\ \sin\theta &= \frac{1}{2\alpha} \\ \theta &= \sin^{-1}\frac{m}{2M} \end{aligned}$$

(b) For $M \gg m$, we're taking the arcsine of a very small number, which will itself be a very small number. That makes sense physically, because the very large value of M will pull down hard on the sides, making the rope very taut and bringing the central weight up higher.

(c) Mathematically, you can't take the arcsine of a number that's greater than 1, which would happen for $m > 2M$. Physically, this is the condition under which the central weight would be heavier than the combination of the two weights on the sides, so it would just drop straight down and flip them over their pulleys.

2-19 If the weight on the left rises by an infinitesimally small height $\mathrm{d}y$, the central pulley will move $\mathrm{d}y$ to the right, shortening both of the horizontal segments of rope on the right by $\mathrm{d}y$, and paying out $2\,\mathrm{d}y$ worth of rope to the weight on the right. The change in gravitational energy is $+mg\,\mathrm{d}y - 2mg\,\mathrm{d}y = -mg\,\mathrm{d}y$. Using the chain-rule technique from section 2.1.5, we have

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \left(\frac{\mathrm{d}v}{\mathrm{d}K}\right)\left(\frac{\mathrm{d}K}{\mathrm{d}U}\right)\left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) \qquad ,$$

where y, v, and a refer to the left-hand mass, but K and U are totals for both masses. The velocity of the right-hand mass is $-2v$, so the total kinetic energy is $K = (1/2)mv^2 + (1/2)m(-2v)^2 = (5/2)mv^2$. We'll see that this extra factor of 5 is the main thing that makes the problem different from the case of a free-falling mass. Instead of the usual $\mathrm{d}v/\mathrm{d}K = 1/mv$, we have $\mathrm{d}v/\mathrm{d}K = 1/5mv$. The $\mathrm{d}K/\mathrm{d}U$ factor is still -1, because both K and U are totals, and it's the total energy that's conserved. The $\mathrm{d}U/\mathrm{d}y$ factor is $-mg$ by our calculation above, differing only in sign from the free-fall result in which the mass would go down rather than up. The $\mathrm{d}y/\mathrm{d}t$

factor is still equal to v, since v and y refer only to the left-hand weight. The result is that the acceleration of the left-hand mass differs from the free-fall case only in sign and in a factor of 1/5: it accelerates upward at $g/5$.

2-20

(a) The equilibrium occurs where the interaction energy has a local minimum. Setting $a = 1$ as suggested, we have

$$\begin{aligned}\frac{\mathrm{d}U}{\mathrm{d}r} &= \frac{\mathrm{d}}{\mathrm{d}r}k\left(r^{-12}-2r^{-6}\right)\\ &= k\left(-12r^{-13}+12r^{-7}\right)\end{aligned} \qquad .$$

Setting this equal to zero,

$$\begin{aligned}12r^{-13} &= 12r^{-7}\\ r^{-13} &= r^{-7}\\ r &= 1\end{aligned} \qquad .$$

Since $a = 1$, this is the same as saying $r = a$.
(b) Making a graph shows that this is a minimum of U rather than a maximum. (You could also use the second derivative test.)
(c) At $r = \infty$, the equation gives $U = 0$. At $r = a$, we have $U = k(1 - 2) = -k$. The difference in energy is k.

2-21 It's tempting to say that the gravity on the space station is zero, since we see the astronauts floating around on TV. But that can't be true, because Newton's law of gravity implies $g \propto 1/r^2$, which could only be zero if r were infinite. (By the shell theorem, this holds as long as we're on or above the surface of the Earth.) The astronauts just seem weightless because they're orbiting the earth in the same orbit as the space station that surrounds them, so they move along with it. Let r be the radius of the Earth, which, by the shell theorem, is also the distance that appears in Newton's law of gravity for people living on the Earth's surface. Let primed variables refer to the space station, so that in particular r' is its distance from the center of the earth.

$$\begin{aligned}g' &= g\left(\frac{r}{r'}\right)^2\\ &= g\left(\frac{r}{r+h}\right)^2\\ &= g\left(\frac{1}{1+h/r}\right)^2\end{aligned} \qquad ,$$

where h is the space station's height above the earth's surface. Since h/r is small, the quantity in parentheses won't be too different from one, and g' won't be very different from g. Looking up r, we find g' to be about 8.7 or 8.8 $\mathrm{m/s^2}$ for the possible range of h values.

2-22 (a) We're given the period, T, and we know that the condition for a circular orbit is

$$v = \sqrt{gr} \qquad .$$

The gravitational field is

$$g = GM/r^2 \qquad .$$

To get the given period, the speed of the satellite must be related to the radius of its orbit by $v =$ circumference/T, or

$$v = \frac{2\pi r}{T} \qquad .$$

We have three equations in three unknowns, v, g, and r. We don't care about v or g, so let's eliminate them first. Plugging the first equation into the third one, we eliminate v. The result is two equations in two unknowns:

$$\begin{aligned}g &= GM/r^2\\ \sqrt{gr} &= \frac{2\pi r}{T}\end{aligned}$$

Now we substitute the first equation into the second one, giving one equation in one unknown:

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

Solving this for r, the result is

$$\begin{aligned} r &= \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} \\ &= 4.2\times10^4\ \text{km} \end{aligned}$$

This is the distance from the center of the earth, not the altitude. Subtracting the radius of the earth, we get an altitude of 3.6×10^4 km.

(b) The gravitational field at this altitude can either be calculated directly from $g = GM/r^2$ or, more easily, by using the inverse-square nature of the field and computing it by comparison with the field at the earth's surface:

$$\begin{aligned} \frac{g}{g_{surface}} &= \left(\frac{\text{radius of earth}}{\text{radius of orbit}}\right)^2 \\ &= \left(\frac{6.4\times10^3\ \text{km}}{4.2\times10^4\ \text{km}}\right)^2 \\ &= 0.023 \\ &= 2.3\% \end{aligned}$$

2-23 The sun's gravitational field is

$$g = \frac{GM}{r^2} \qquad ,$$

where M is the sun's mass. For the Earth's circular orbit, we have

$$v = \sqrt{gr} = \frac{2\pi r}{T} \qquad ,$$

where r is one astronomical unit, and T is one year. Eliminating g and solving for G, we find

$$G = \frac{4\pi^2 r^3}{MT^2} \qquad ,$$

which equals $4\pi^2$ in this system of units, since r, M, and T are all defined as 1.

2-24 Kepler's law of periods says $T \propto R^{3/2}$, and solving for R gives $R \propto T^{2/3}$. Converting this into a statement about ratios gives $R_1/R_2 = (T_1/T_2)^{2/3} = 2.9$, or 3 if we round off to one sig fig.

2-25 (a) If it's moving at exactly escape velocity, then its kinetic energy will approach zero as r approaches infinity. Therefore, the total energy is $U + K = 0$. We then have

$$\begin{aligned} 0 &= U + K \\ &= -\frac{GMm}{r} + \frac{1}{2}mv^2 \\ &= -\frac{GMm}{r} + \frac{1}{2}m\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 \end{aligned}$$

Now we need to separate the variables r and t.

$$\begin{aligned} \frac{GMm}{r} &= \frac{1}{2}m\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 \\ \frac{2GM}{r} &= \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 \\ \sqrt{\frac{2GM}{r}} &= \frac{\mathrm{d}r}{\mathrm{d}t} \\ \mathrm{d}t &= \sqrt{\frac{r}{2GM}}\ \mathrm{d}r \end{aligned}$$

Integrating both sides, we get

$$t + \text{constant} = \sqrt{\frac{2}{9GM}}\, r^{3/2} \qquad .$$

The constant of integration just indicates when we start our clock, so we're free to pick it to be zero. Solving for r, we then have

$$r = \left(\frac{9GM}{2}\right)^{1/3} t^{2/3} \qquad .$$

The velocity is

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \left(\frac{4GM}{3}\right)^{1/3} t^{-1/3}$$

(b) For $r(t)$, we have

$$\begin{aligned} \left(\frac{\text{J}\cdot\text{m}}{\text{kg}^2}\cdot\text{kg}\right)^{1/3}\text{s}^{2/3} &= \left(\frac{(\text{kg}\cdot\text{m}^2/\text{s}^2)\cdot\text{m}}{\text{kg}^2}\cdot\text{kg}\right)^{1/3}\text{s}^{2/3} \\ &= \text{m} \qquad . \end{aligned}$$

The units of the expression for $v(t)$ differ only by a factor of 1/s, so they work out as well.

(c) As t gets large, the velocity approaches zero, which makes sense.

2-27 Your path to Mars is half of an elliptical orbit. Kepler's third law says that that the time required for an elliptical orbit is proportional to its long axis to the power 3/2. It is convenient to work the problem in units of astronomical units (a.u.) and years, where 1 a.u. is the average distance from the earth to the sun. The long axis of our ellipse equals the sum of the radii of the earth's and Mars' orbits, which is 1.00 a.u.+1.53 a.u.=2.53 a.u. Comparing with the earth's orbit, which takes one year, we have

$$\frac{\text{time for half of your orbit}}{\text{time for half the earth's orbit}} = \left(\frac{\text{long axis of your orbit}}{\text{diameter of earth's orbit}}\right)^{3/2}$$

The quantities in the denominators are 0.5 years and 2.0 a.u., and solving for the unknown we find that the outward leg of the trip takes 0.70 years.

2-28 At the atomic level, the helium's heat energy is the random motion of the helium atoms. Cylinder B's atoms are moving at the same average speed as A's, since they're at the same temperature, but B has twice as many atoms, so its total KE is twice as much.

2-29 When the sweat evaporates, electrical potential energy is increasing, because as the sweat changes from a liquid to a gas, its molecules, which are attracting each other electrically, get farther away. By conservation of energy, some other form of energy must be decreasing: heat.

If you wipe the sweat off with a towel, you lose the cooling effect. The water evaporates after it's out of contact with you, so it doesn't cool you off. As a method of cooling yourself, it's no more effective than spitting!

2-31 This is only an order-of-magnitude estimate, so let's make some simplifying approximations. We'll assume the surface area of the oceans is the same as the surface area of the planet ($4\pi R^2$), and that the density of ice is about the same as that of water, 1000 kg/m^3. Then the amount of energy required is

$$E = (1000\ \text{kg/m}^3)(10\ \text{m})(4\pi R^2)(3\times 10^3\ \text{J/kg}) \sim 10^{23}\ \text{J}$$

Dividing by 10 years (about 3×10^8 seconds), we find that the power is on the order of 1% of the sun's light output. This is a surprisingly small amount. The equilibrium is extremely delicate.

2-32 Sixteen periods take about 8.8 ms, so one period is 0.55 ms. The frequency is 1/0.55 ms=1.82 kHz. The angular frequency is $(1.82\ \text{kHz})(2\pi) = 1.1\times 10^4$ rad/s.

2-33 (a) Adding the interaction energy due to gravity and that due to the interatomic forces inside the spring, we have

$$U = \frac{1}{2}ky^2 + mgy \qquad ,$$

and we can find the minimum by setting the derivative equal to zero:

$$\begin{aligned} 0 &= \frac{\mathrm{d}U}{\mathrm{d}x} \\ &= ky + mg \\ y &= -\frac{mg}{k} \end{aligned}$$

(b) Just carry out the experiment, and calculate $k = -mg/y$.

2-34 The result from problem 33 was $h = mg/k$. The constant-acceleration equation $h = (1/2)gT_{fall}^2$ results in

$$T_{fall} = \sqrt{\frac{2h}{g}} \qquad .$$

The total interaction energy is $(1/2)ky^2 + mgy$, and its second derivative is k, just as it would have been without the gravitational term. The interpretation is that hanging the mass vertically from the spring merely shifts the equilibrium position, without changing the period of the oscillations, which is still

$$T_{osc} = 2\pi\sqrt{\frac{m}{k}} \qquad .$$

The ratio of the two times is

$$\begin{aligned}\frac{T_{osc}}{T_{fall}} &= 2\pi\sqrt{\frac{m}{k}\cdot\frac{g}{2h}}\\ &= 2\pi\sqrt{\frac{m}{k}\cdot\frac{g}{2(mg/k)}}\\ &= \pi\sqrt{2}\\ &\approx 4.45 \qquad .\end{aligned}$$

Is this reasonable? In the case where the mass is falling, the distance h traveled is the same as the distance that it would have traveled in 1/4 of a cycle of oscillation. Roughly, then, we might expect the result to be about 4, and that is about what it is. The reason it's a little more than 4 is that in the free-fall case, the force acting on the mass is constant, whereas in the oscillating case, the force starts out being just as strong as gravity, but weakens and becomes zero at the end of a quarter cycle. Since the force in the oscillating case is always less than or equal to the force in the free-falling case, this lengthens the time for the oscillation, making the ratio slightly higher than the crude estimate of 4.

2-35 Let L be the amount by which the spring has been stretched or compressed with respect to equilibrium, so that its energy is $U_{spring} = (1/2)kL^2$. Let y be the height of the mass, and for convenience, let's define $y = 0$ to be the height at which the spring is relaxed, so that $L = 0$ coincides with $y = 0$. If the mass goes down by a certain amount, the spring is stretched by half that amount, so $L = y/2$. (We don't care about positive or negative signs of L, since only L^2 occurs in the spring's energy.) The total interaction energy of the system is then

$$U = mgy + \frac{1}{8}ky^2 \qquad .$$

The effective spring constant experienced by the mass is not really k, but rather

$$\begin{aligned}k' &= \frac{d^2}{dy^2}U\\ &= \frac{d^2}{dy^2}\left(mgy + \frac{1}{8}ky^2\right)\\ &= \frac{1}{4}k\end{aligned}$$

In other words, this arrangement makes the spring appear to be four times less stiff than if the weight had just been hung directly from it. The period is $T = 2\pi\sqrt{m/k'} = 4\pi\sqrt{m/k}$.

2-36 (a)

$$\begin{aligned}U &= 2U_{one\ spring}\\ &= (2)(\frac{1}{2}k(L-b)^2)\\ &= k(L-b)^2\\ &= k(\sqrt{b^2+y^2}-b)^2\end{aligned}$$

(b) The effective spring constant is the second derivative of this quantity with respect to y, evaluated at $y = 0$.

$$\begin{aligned}\frac{d^2U}{dy^2} &= \frac{d^2}{dy^2}k\left(\sqrt{b^2+y^2}-b\right)^2\\ &= \frac{d}{dy}\left[2k\left(\sqrt{b^2+y^2}-b\right)\frac{1}{2}\left(b^2+y^2\right)^{-1/2}(2y)\right]\\ &= \frac{d}{dy}2ky\left[1-b(b^2+y^2)^{-1/2}\right]\\ &= 2k\left[1-b(b^2+y^2)^{-1/2}\right]+2ky\,[\ldots] \qquad ,\end{aligned}$$

where the ... indicates stuff we don't need to evaluate, because it won't contribute to the derivative evaluated at $y = 0$. The result is zero, which implies that the period of small oscillations is infinite. This is a loophole in the text's statement that small oscillations are *always* simple harmonic. If you were to start this system doing large oscillations, and then let them die out slowly, their period would get longer and longer.

2-37 The total energy is

$$U = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2 \qquad .$$

Eliminating x_2, we have

$$U = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 (b - x_1)^2 \qquad .$$

For a given value of b, the springs will adjust themselves to minimize this, so

$$\begin{aligned} 0 &= \frac{\mathrm{d}U}{\mathrm{d}x_1} \\ &= k_1 x_1 - k_2(b - x_1) \\ &= (k_1 + k_2)x_1 - k_2 b \\ x_1 &= \frac{k_2 b}{k_1 + k_2} \qquad . \end{aligned}$$

We can now plug this back into the equation for the total energy,

$$\begin{aligned} U &= \frac{1}{2}k_1 \left(\frac{k_2 b}{k_1 + k_2}\right)^2 + \frac{1}{2}k_2 \left(b - \frac{k_2 b}{k_1 + k_2}\right)^2 \\ &= \frac{1}{2}\left[k_1 \left(\frac{k_2}{k_1 + k_2}\right)^2 + k_2 \left(1 - \frac{k_2}{k_1 + k_2}\right)^2\right] b^2 \end{aligned}$$

The quantity in square brackets is the spring constant of the combined spring, so we could just stop here, but that would be pathetic, because it can be simplified a great deal.

$$\begin{aligned} K &= k_1 \left(\frac{k_2}{k_1 + k_2}\right)^2 + k_2 \left(\frac{k_1}{k_1 + k_2}\right)^2 \\ &= k_1 \left(\frac{k_2}{k_1 + k_2}\right)^2 + k_2 \left(\frac{k_1}{k_1 + k_2}\right)^2 \\ &= \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1} \end{aligned}$$

2-38 This just requires some straightforward tinkering with the program listing from section 2.5. Instead of a function $U(k,x)$, we now have $U(k,b,x)$. The function `osc` likewise needs both a k and a b, and to make it easier to keep track of what was going on, I added a `print` statement on the first line of the function to spit back the values of A and b.

```
import math
def u(k,b,x):
  return .5*k*x**2+b*x**4

def osc(m,k,b,a,n):
  print("A=",a,"  b=",b)
  x=a
  v=0
  dx = -2.*a/n
  t=0
  e = u(k,b,x)+.5*m*v**2
  for i in range(n):
    x_old = x
    v_old = v
    x = x+dx
    kinetic = e-u(k,b,x)
    if kinetic<0. :
      v=0.
      print("warning, K=",kinetic,"<0")
    else :
      v = -math.sqrt(2.*kinetic/m)
    v_avg = (v+v_old)/2.
    dt=dx/v_avg
    t=t+dt
  return 2.*t
```

Now let's try it:

```
1   $ python -i simulate-anharmonic.py
2   >>> osc(1.,1.,0.,1.,1000)
3   A= 1.0   b= 0.0
4   warning, K= -1.55431223448e-15 <0
5   6.2832905354078958
6   >>> osc(1.,1.,0.01,1.,1000)
7   A= 1.0   b= 0.01
8   warning, K= -1.55431223448e-15 <0
9   6.1912268107728945
10  >>> osc(1.,1.,0.01,10.,1000)
11  A= 10.0   b= 0.01
12  3.1798441947471718
```

On lines 2-5, we run the program with $b = 0$, and get the same result of exactly 2π that we had in section 2.5. On lines 6-9, we try a small value of b, and we observe that there is a slight decrease in the period, which makes sense, because the spring is stronger. On lines 10-12, a bigger amplitude produces a much smaller period. This is not the simple (and counterintuitive) behavior we observed when the spring's energy only had an x^2 term. In the pure x^2 case, the spring's greater energy at large amplitudes was exactly enough to compensate for the greater distance the mass needed to travel, causing it to cover that distance in the same time. The x^4 function, however, has a more severe dependence on x: it starts off flat, but then blows up like crazy when x gets big. The increased stiffness at large amplitudes does more than just compensate for the extra distance of travel, it actually makes the mass travel the *greater* distance in a *smaller* time.

2-40 Without air resistance, we had

$$E_i = E_f$$

$$mgy = \frac{1}{2}mv^2 + mgy$$

$$v = -\sqrt{2g(h-y)}$$

(with the negative root, since the original program was written in a coordinate system in which up was positive). If the total amount of energy dissipated as heat so far is Q, then this becomes

$$mgy = \frac{1}{2}mv^2 + mgy + Q$$

$$v = -\sqrt{2g(h-y) - 2Q/m}$$

```
import math
def time2(h,n,air): # air=density in kg/m3
  g=9.8             # gravitational field
  r = .01           # radius of ball, in m
  m = .021          # mass of ball, in kg
  y=h
  v=0
  dy = -h/n         # Split h into n parts
  t=0
  heat = 0

  for i in range(n):
    y = y+dy        # dy<0
    v = -math.sqrt(2.0*g*(h-y)-2.0*heat/m)
                    # from cons. of energy
    heat=heat+.25*math.pi*air*v*v*r*r*(-dy)
                    # dy<0, heat>0
    dt=dy/v         # dy<0, v<0, dt>0
    t=t+dt
  return t
```

To see the effect of air friction, we run the program with the density of air first set to zero, and then to its sea-level value:

```
>>> time2(1.0,100000,0.0)
0.45071197047404565
>>> time2(1.0,100000,1.2)
0.45104994977064927
```

The difference is 0.34 ms, as claimed.

2-41 The trickiest thing about this program is that in the python idiom `for i in range(30)`, `i` actually varies from 0 to 29, not from 1 to 30, so we have to add one to `i` to get the thing we're supposed to multiply by.

```
f=1
for i in range(30):
  f=f*(i+1)
print(f)
```

The result is 265252859812191058636308480000000.

2-42 As in the examples in ch. 1, we don't try to estimate the mass of the wing directly. Instead, we estimate linear dimensions and get mass from that. Let's approximate a fly's wing as a square 3 mm on a side. I don't know the thickness, but since flies' wings are translucent, it must be much less than the thickness of a piece of paper. I'm going to guess that the thickness is 0.1 mm. This gives a volume of about 1 mm^3, and if we assume the same density as water, we get a mass of 10^{-6} kg.

Let's say that a fly's wing beats 100 times a second, which would be of the right order of magnitude to match the fact that the buzzing is in the audible range of human hearing. Then the speed of a wing would be about (3 mm)/(10^{-2} s) $\approx$ 0.3 m/s.

The result for the kinetic energy is about 10^{-7} J.

2-43 As always (see sec. 1.4), we avoid estimating mass or volume directly and instead estimate linear dimensions. Let's say a blade of grass is 0.1 cm thick, 1 cm wide, and 10 cm tall. Then its volume is about 1 cm^3, and if it has the same density as water, then its mass is about 1 g, or 10^{-3} kg.

For the velocity, let's say it grows 1 cm in 10 days. This gives 10^{-8} m/s.

Putting these factors together, we have $KE = (1/2)mv^2 \sim 10^{-19}$ J.

3-1 $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$

3-2 The people plus the boat are, roughly speaking, a closed system, since the water does not exert much horizontal force on the boat. Momentum is conserved in a closed system. At the beginning, nothing is moving and there is zero momentum. If the two people's momenta didn't add up to zero, then the boat would have to move to conserve momentum. They should make sure their momenta add up to zero, i.e., they should move in opposite directions, and the heavier person has to move more slowly.

3-3 (a) $KE = \frac{1}{2}mv^2$, so $v = \sqrt{2KE/m} = 130$ m/s.
(b) $p = mv = 1.3$ kg·m/s.
(c) same as part b (conservation of momentum)
(d) $KE = \frac{p^2}{2m} = 0.2$ J. The recoiling gun is harmless, because it would take more energy than this to break bones, sever tissues, etc.

3-5 Let the big ball's initial and final velocities be V and V', and let the small ball's final velocity by v. Conservation of momentum and energy give, respectively,

$$2mV = 2mV' + mv$$
$$\frac{1}{2}(2m)V^2 = \frac{1}{2}(2m)V'^2 + \frac{1}{2}mv^2 \qquad ,$$

which can be simplified to

$$V = V' + \frac{1}{2}v$$
$$V^2 = V'^2 + \frac{1}{2}v^2 \qquad .$$

Solving the first equation for $v = 2(V - V')$ and substituting into the second equation, we find that it can be put in the form of a quadratic,

$$3V'^2 - 4VV' + V^2 = 0 \qquad ,$$

with solutions $V' = V/3$ and $V' = V$. The second solution is what would have happened if the balls hadn't interacted at all. The physically relevant equation is the first one. The ball is moving at one third of its former speed, and in the same direction, since V and V' have the same sign.

3-6 The simplest way to do this is by using the center-of-mass frame of reference. If the big object's mass is essentially infinite compared to the little one's ($M >> m$) then the center of mass of the whole system is always located at the big one's center of mass, so the center-of-mass frame is the frame fixed to the big object. (After the collision, the big object recoils a tiny bit from the collision, so you could say that it's no longer at rest in the c.m. frame, but because its mass is assumed to be infinite compared to the little one's mass, its recoil is negligible.)

In this frame, it's not the high-mass object with velocity v coming along and hitting the low-mass object, it's the other way around. The low-mass object, moving at velocity $-v$, hits the high-mass one. This is like hitting a brick wall: the recoil of the wall is negligible. The only thing moving before or after the collision is the small object, and since we're assuming no KE is converted into anything else, the small object must recoil at velocity $+v$, i.e., in the opposite direction but at the same speed. (Reasoning slightly more rigorously, it's not quite true that the big object doesn't recoil at all, but the momentum it absorbs equals $-2mv$, and its KE after the collision is $p^2/2M = 2m^2v^2/M$, which is extremely small because M is so big compared to m.)

In the center-of-mass frame, then, we've proved that the small object enters with velocity $-v$ and exits with $+v$. To convert back to the original frame of reference, we add v to all the velocities. In the original frame, the small object starts with velocity 0, and ends up with $2v$.

3-7 The elegant way to do this is to switch to the center of mass frame. In that frame, the balls are initially moving toward each other at velocities $v/2$ and $-v/2$. After the collision, we can maximize the amount of heat released if we simply make the two masses stick together and stop. That means that *in the center of mass frame*, it's possible to convert all the kinetic energy, $2(1/2)m(v/2)^2 = (1/4)mv^2$, into heat and sound. Back in the original frame of reference, the balls are still moving after the collision, so not all the kinetical energy has been lost, but the amount of heat and sound is still the same amount we found in the c.m. frame: $(1/4)mv^2$, or half the initial kinetic energy of the single moving ball.

As an alternative, a brute force technique would be to require conservation of momentum and energy and crank out the result. Let the final velocities be b and c. Then

$$\begin{aligned} mv &= mb + mc \\ \frac{1}{2}mv^2 &= \frac{1}{2}mb^2 + \frac{1}{2}mc^2 + Q \qquad , \end{aligned}$$

where Q is the amount of heat and sound that comes out. Solving the first equation for $b = v - c$ and substituting into the second, we find $Q = m(vc - c^2)$. Using calculus to find the maximum, we set $0 = \mathrm{d}Q/\,\mathrm{d}c = mv - 2mc$, giving $c = v/2$, and then the rest of the result carries through as in the center-of-mass approach.

3-8 (a) Suppose that at some time, the rocket has mass m remaining. Now an infinitesimal time passes, and the rocket's mass changes to $m + \mathrm{d}m$ (with $\mathrm{d}m < 0$) because it expels $-\,\mathrm{d}m$ worth of exhaust. This causes its velocity to increase by the infinitesimal amount $\mathrm{d}v$. We can now require conservation of momentum, and the algebra is reduced if we do this in the center of mass frame, in which the rocket is initially at rest, i.e., its velocity is increasing from 0 to $0+\mathrm{d}v$.

$$0 = (-\,\mathrm{d}m)(-u) + (m + \mathrm{d}m)\,\mathrm{d}v$$

Discarding the product of the two infinitesimals $\mathrm{d}m\,\mathrm{d}v$, and separating variables, this becomes

$$-\frac{\mathrm{d}m}{m} = \frac{\mathrm{d}v}{u} \qquad ,$$

and integration gives

$$-\ln m + c = \frac{v}{u} \qquad .$$

The interpretation of the minus sign is that as the mass gets smaller, the logarithmic term gets less negative, the left-hand side gets bigger, and the velocity goes up. If we now switch back to the original frame of reference, in which the velocity is not zero but v, the effect is only to add a constant onto v, and that can be absorbed into the constant of integration. Back in this frame, the only time when we have $v = 0$ is initially, when $m = m_i$, and this can be used to determine the constant of integration $c = \ln m_i$. For the final velocity, we have

$$v = u \ln \frac{m_f}{m_i}$$

(b) Solving for m_i/m_f, we have

$$\begin{aligned} m_i/m_f &= e^{v/u} \\ &= \exp(3 \times 10^7 \text{ m/s } / \text{ 4000 m/s}) \\ &= 2 \times 10^{434} \qquad . \end{aligned}$$

This is a ridiculously huge amount of fuel, vastly more than the mass of the entire galaxy.

3-9 No. Newton's laws only deal with the *total* force on the object. We can conclude that the total force is zero, but that doesn't mean no forces act. Forces could act, but cancel.

3-10 The mass is increased by a factor of 1.5, so the acceleration is reduced by a factor of 1.5, to 2 m/s^2.

3-11 (a) We are supposed to find an equation for mass based on information about forces and accelerations. Newton's second law, $a = F_{total}/m$, is what relates these three quantities. There are two forces acting on the elevator: a downward force from gravity and an upward force from the cable. Let's use positive numbers for upward forces and negative numbers for downward ones. The force of gravity will then be $-mg$, and the maximum force the cable can exert will be $+T$, giving $F_{total} = -mg + T$. Substituting this into Newton's second law gives

$$a = \frac{-mg + T}{m} \qquad .$$

Solving for m gives

$$m = \frac{T}{a+g} \quad .$$

(b) If $a = 0$, then the equation becomes $m = T/g$, i.e. $T = mg$. Interpretation: if the elevator is not accelerating, then the maximum mass is the mass that results in a gravitational force as great as the maximum tension in the cable. This is intuitively obvious if the elevator is at rest, but perhaps a little surprising for the case where the elevator is, say, moving upward at constant speed. Most people would not expect that the tension in the cable would be the same whether the car was at rest or moving upward at constant speed.

If the car is accelerating downward at 9.8 m/s^2, that would be $a = -g$, since we are using positive for up and negative for down. Plugging this in to the equation gives $m = \infty$ (division by zero). Interpretation: this is the case where the elevator is simply dropping like a rock. The cable does not need to exert any force in this situation, and so there is no limit on the mass of the car.

3-13 (a) Newton's second law says $F_{air} - F_{weight} = ma$, or $F_{air} - mg = ma$. Since the forces are constant, this will be motion with constant acceleration, so $v = at = (F_{air} - mg)t/m$. (b) 7.8 m/s

3-14 $F = ma = kt$, so $a = (k/m)t$. Integrating once gives its velocity, $v = (k/2m)t^2 + v_o$. The constant of integration, v_o, equals zero, because the initial speed is zero, so $v = (k/2m)t^2$. Integrating again gives $x = (k/6m)t^3 + x_o$.

3-16 Newton's third law says their forces on each other are equal. Since $a = F/m$, the less massive old lady has a greater acceleration.

3-17 When you drop an object, the earth does accelerate up to meet it, but the acceleration is very small because the earth's mass is so big.

3-18 Yes, they are equal and opposite, because otherwise the total force on you would not be zero, and you would accelerate vertically. No, Newton's third law does not apply, because there are three objects involved, not just two: the earth, your feet, and the floor. Newton's third law says that two objects' forces on each other are equal and opposite, so it never applies to two forces both acting on the same object.

3-19 (a) The two static frictional forces must together cancel out the elevator's weight $F_W = Mg$, so

$$F_s = \frac{Mg}{2}$$

The maximum amount of static friction is given by $F_s < \mu_s F_N$, so

$$F_N > \frac{Mg}{2\mu_s}$$

(b) If friction is weaker, we'd need to press more firmly. The equation demonstrates this behavior, since dividing by a smaller value of μ_s will give a greater result.

3-20 (a) (b) (c) Let's choose a coordinate system in which positive is forward. The forces acting on the tugboat give

$$a = \frac{F_f - T}{m}$$

and the ones acting on the ship result in

$$a = \frac{T}{m} \quad .$$

Note that we use the same symbol a in both cases: they're tied together, so their accelerations must be the same. To solve for T, we need to eliminate a, which we can do by setting the two equations equal:

$$\frac{F-T}{m} = \frac{T}{M}$$

Bringing all the T stuff to one side,

$$\frac{F}{m} = \left(\frac{1}{m} + \frac{1}{M}\right)T \quad ,$$

so

$$\begin{aligned} T &= \frac{F}{m(\frac{1}{m} + \frac{1}{M})} \\ &= \frac{F}{1 + m/M} \\ &= \frac{M}{M+m}F \quad . \end{aligned}$$

Problem ??:
Part a

force acting on tugboat			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
fluid fr.	forward	water	fluid fr.	backward	tugboat
tension	back	ship	tension	forward	tugboat

Problem ??:
Part b

force acting on ship			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
tension	forward	tugboat	tension	back	ship

(d) For $M = 0$, the tension is zero, which makes sense. For $M = \infty$, we have $M/(M+m) = 1$, since the bottom becomes essentially the same as M, so $T = F$. The interpretation is that the tugboat is so light that essentially none of the force from the water on its propellers is required in order to accelerate it.

3-21 If kinetic friction were stronger than static friction, then we could have, for example, a situation in which 4 N was the force necessary to break static friction and make an object slide across a horizontal surface, but 6 N was necessary in order to produce a force greater than kinetic friction in order to produce an acceleration. This wouldn't make sense, because what would happen if you applied 5 N? It would break static friction, but then there would be no acceleration because the force would be weaker than kinetic friction. The way you would actually measure a static frictional force experimentally would be to measure how much force you had to apply in order to get the object to accelerate, so there's no way you could have determined the 4 N force for static friction in the first place.

3-22 (a)

$$\begin{aligned} v_f^2 &= v_o^2 + 2a\Delta x \\ 0 &= v^2 + 2ad \\ a &= -\frac{v^2}{2d} \\ F &= ma \\ &= -\frac{mv^2}{2d} \end{aligned}$$

(b) The quantity $-\frac{mv^2}{2d}$ has units of

$$\frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{m}} = \text{kg}\cdot\text{m/s}^2 \qquad ,$$

which are the same as the units of force (mass multiplied by acceleration).

(c) If m gets bigger, F gets bigger. That makes sense, because more force would be required in order to stop a more massive object in the same distance. If v increases, so does F, and that also makes sense. Dividing by a bigger number gives a smaller result, so if d gets bigger, F gets smaller. That also makes sense, because a smaller force would take more distance to stop a moving object.

(d) The only thing to be careful about here is that if we want an answer in SI (meter-kilogram-second) units, we have to make sure to convert all the raw data to SI before plugging in:

$$\begin{aligned}F &= -\frac{mv^2}{2d}\\ &= -\frac{\left(6.7\times10^{-27}\ \text{kg}\right)\left(2.0\times10^{7}\ \text{m/s}\right)^2}{2\left(7.1\times10^{-4}\ \text{m}\right))}\\ &= -1.9\times10^{-9}\ \text{N}\end{aligned}$$

3-24 Let's use a coordinate system in which positive x is down. Then the downward force of gravity on Sally is positive, and the blaster's upward force on Sally is negative. (a)

$$\begin{aligned}a &= \frac{F_{total}}{m}\\ &= \frac{mg - F_B}{m}\\ &= g - F_B/m\\ h &= \frac{1}{2}at^2\\ t &= \sqrt{\frac{2h}{a}}\\ &= \sqrt{\frac{2h}{g-F_B/m}}\end{aligned}$$

(b) The units of g are m/s^2. F_B has units of newtons, and dividing newtons by kilograms gives m/s^2 (as in Newton's second law). In the denominator, we're subtracting m/s^2 from m/s^2. That's good, because you can't subtract things that have different units. Dividing h, with units of m, by m/s^2 gives units of s^2, and taking the square root of that gives units of seconds, which matches up correctly with the units of the variable t on the left side.

(c) For large values of F_B, the quantity inside the square root becomes negative, and the result for the time becomes an imaginary number. The point where this bogus behavior sets in is the point at which the denominator goes from positive to negative, i.e., when the denominator is zero: $g - F_B/m = 0$, or $F_B = mg$. In other words, if F_B is greater than the force of gravity, she'll fly up, and will never even hit the ground.

3-25 (a) Friction would be parallel to the ceiling, but all the motion is vertical. (b) If the rice were to stay stuck onto the ceiling of the cup, it would have to accelerate along with the cup. When you're banging on something like this the acceleration can be huge, perhaps thousands of meters per second squared. This is much more than anything gravity could supply. (c) The cup is the only object touching a grain of rice that could be exerting a force on it, and Newton's second law would then require that the this force equal ma. As the cup decelerates, we therefore need a large upward force from the cup on the rice if the rice is to decelerate along with it. This would be some kind of sticky force, and there's a limit on how strong such a force can be. If it isn't strong enough, then the grain's acceleration can't match that of the cup, and it will come loose.

3-26 (a) Let T be the tension in the rope at the corner of the table. For the bottom part of the rope with length x that hangs vertically, we let positive be down (in the direction of motion), so

$$\begin{aligned}F &= \frac{\mathrm{d}p}{\mathrm{d}t}\\ m_{bottom}g - T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(m_{bottom}\frac{\mathrm{d}x}{\mathrm{d}t}\right)\\ \frac{x}{L}mg - T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{x}{L}mg\frac{\mathrm{d}x}{\mathrm{d}t}\right)\\ xg - \frac{L}{m}T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(x\frac{\mathrm{d}x}{\mathrm{d}t}\right)\\ xg - \frac{L}{m}T &= \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + x\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\end{aligned} \quad .$$

For the top part, with length $L - x$, we let positive be toward the edge of the table (again, in the direction of motion).

$$\begin{aligned}
F &= \frac{\mathrm{d}p}{\mathrm{d}t} \\
T &= \frac{\mathrm{d}}{\mathrm{d}t}\left[\left(1-\frac{x}{L}\right)m\frac{\mathrm{d}x}{\mathrm{d}t}\right] \\
\frac{LT}{m} &= -\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + (L-x)\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \qquad .
\end{aligned}$$

Adding the results of the two calculations, we have $xg = La$, which is the result that was claimed.

(b) For $x = 0$, nothing is hanging over the edge, so the upward normal force of the table completely cancels the force of gravity. For $x = L$, the whole rope is hanging, and there is no upward normal force from the table at all; the result becomes $a = g$. In between these two extreme cases, the upward normal force of the table only partially cancels gravity.

(c)

$$\begin{aligned}
\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} &= \frac{g}{L}x \\
\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(be^{ct}\right) &= \frac{g}{L}be^{ct} \\
bc^2e^{ct} &= \frac{g}{L}be^{ct} \\
c^2 &= \frac{g}{L} \\
c &= \pm\sqrt{\frac{g}{L}}
\end{aligned}$$

(d) This is true because the derivative of a sum is the sum of the derivatives.

(e) The most general solution is of the form

$$x = qe^{ct} + re^{-ct} \qquad ,$$

where q and r are constants with units of meters, and we'll say c represents the positive root $\sqrt{g/L}$. At $t = 0$, both exponentials equal one, and we have

$$x_{\text{o}} = q + r$$

and

$$\begin{aligned}
0 &= \frac{\mathrm{d}x}{\mathrm{d}t} \\
&= qce^{ct} - rce^{-ct} \\
&= (q-r)c \\
q &= r
\end{aligned}$$

The solution is therefore $q = r = x_{\text{o}}/2$, or

$$x = x_{\text{o}}\left(e^{ct} + re^{-ct}\right)/2 \qquad ,$$

which if you like you can write using the hyperbolic cosine.

3-27

3-28

3-29

3-30

3-31

3-32 (a) The normal force at the top surface is Mg, and the one at the bottom surface is $(M+m)g$. Since there is slipping at both surfaces, the coefficient of kinetic friction is the one we care about, and the two frictional forces are $\mu_k Mg$ at the top surface (acting on both blocks) and $\mu_k(M+m)g$ at the bottom. The middle block decelerates, while the top one accelerates. The velocity of the middle block is $v - \mu_k(2M/m+1)gt$, and the velocity of the top block is $\mu_k gt$. Setting these equal to each other, we find

$$t = \frac{v}{2\mu_k g(M/m+1)} \qquad .$$

(b) The units of the right-hand side are $(\mathrm{m/s})/(\mathrm{m/s^2}) = \mathrm{s}$.

(c) If m gets bigger, M/m gets smaller, and t gets bigger. This makes sense physically. The acceleration of the top block doesn't depend on the mass of the middle block at all. The acceleration of the middle block depends on its own mass for two reasons: (1) if it has more mass it has more inertia; (2) if it has more mass then there will be more friction at the bottom surface. Although these two effects are in the opposite direction, they're not of equal strength. Doubling the mass of the middle block, for example, doubles its inertia, but doesn't double the frictional

Problem ??:

force acting on magnet			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
magnetism	up	car	magnetism	down	magnet
gravity	down	planet earth	gravity	up	magnet
normal	down	car	normal	up	magnet

Problem ??:
(a) A monkey climbing a palm tree.

force acting on monkey			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	right	tree	normal	left	monkey (right hand)
normal	left	tree	normal	right	monkey (left hand)
gravity	down	planet Earth	gravity	up	monkey
static friction	up	tree	static friction	down	monkey (right hand)
static friction	up	tree	static friction	down	monkey (left hand)

Problem ??:
(b) A piece of tape stuck on the ceiling.

force acting on tape			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	tape
sticky	up	ceiling	sticky	down	tape

Problem ??:

force acting on rower			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	↑	seat	normal	↓	rower
gravity	↓	Earth	gravity	↑	rower
normal	→	oar	normal	←	rower
static friction	←	seat	static friction	→	rower
normal	↖	footrest	normal	↘	rower

Problem ??:

force acting on farmer			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
static friction	up	cow	static friction	down	farmer
static friction	up	wall	static friction	down	farmer
gravity	down	planet earth	gravity	up	farmer
normal	left	cow	normal	right	farmer
normal	right	wall	normal	left	farmer

Problem ??:
(The "lift" force below is really not so different from a normal force.)

force acting on plane			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal?	east	air	normal?	west	plane (propeller)
friction	west	air	friction	east	plane
lift	up	air	lift	down	plane (wing)
gravity	down	earth	gravity	up	plane

force at the bottom surface, and has even less of an effect on the total force felt by the block.
(d) If M gets bigger, its own acceleration of $\mu_k g$ is unchanged, because the greater frictional force cancels out the effect of the greater inertia. On the other hand, the frictional force on the middle block is increased, so the time should get shorter. Mathematically, this is the behavior we see.
(e) Making g bigger makes the time shorter. That makes sense, because if gravity presses the surfaces together more firmly, there will be more friction.
3-33 (a) See table on last page.

(b) See table on last page.

(c) The vertical forces on Ginny cancel out, so the normal force between Ginny's pants and the sled has magnitude $N_2 = Mg$. Similarly, the normal force between the sled and the snow is $N_1 = (M+m)g$, since the snow supports the total mass of both Ginny and the sled. In the situation where she can just barely accomplish what she wants to do, the initial acceleration is vanishingly small; to find the limiting case, we set it to zero. That means there must be zero total horizontal force on her, and zero total horizontal force on the sled. There are two frictional forces on the sled, F_2 from Giny and F_1 from the snow. If they're canceling, their magnitudes must be equal, $|F_2| = |F_1|$. If the sled just barely starts slipping over the snow before Ginny's pants start slipping over the sled, then both these forces are at their maximum value for static friction,

$$\begin{aligned}\mu_2 N_2 &= \mu_1 N_1 \\ \mu_2 Mg &= \mu_1 (M+m)g \\ \mu_2 &= \mu_1(1+m/M) \\ M &= \frac{m}{\mu_2/\mu_1 - 1}\end{aligned}$$

(d) For $\mu_2 = \mu_1$, M becomes infinite. For smaller values of μ_2, we have a negative value for M, which is also impossible. In other words, for $\mu_2 \leq \mu_1$, it wouldn't matter if Ginny carried a backpack full of lead bricks; she'd still slip off the sled.

Problem ??:
Part a:

force acting on Ginny			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	Ginny
static friction	left	ropes	static friction	right	Ginny
normal	up	sled	normal	down	Ginny
static friction	right	sled	static friction	left	Ginny

Problem ??:
Part b:

force acting on sled			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	sled
normal	down	Ginny	normal	up	sled
static friction	left	Ginny	static friction	right	sled
normal	up	snow	normal	down	sled
friction	right	snow	friction	left	sled

3-35

$$\begin{aligned} v &= \sqrt{2aL} \\ &= \sqrt{2\frac{F_k}{m}L} \\ &= \sqrt{2\frac{\mu_k F_N}{m}L} \\ &= \sqrt{2\frac{\mu_k mg}{m}L} \\ &= \sqrt{2\mu_k gL} \end{aligned}$$

Note that the cop doesn't even need to know anything about the car other than the coefficient of kinetic friction between its tires and the asphalt μ_k, and μ_k probably doesn't vary all that much from one car to another.

3-36 (a) See table.

(b) The vertical forces cancel, so $F_N = mg$. Picking a coordinate system in which positive is forward, the horizontal force of friction produces an acceleration $a = F_k/m = -\mu_k F_N/m = -\mu_k g$. Applying the constant-acceleration equation $v_f^2 = v_i^2 + 2ax$, we find

$$x = \frac{v_i^2}{2\mu_k g} \quad .$$

(c) The coefficient of friction is unitless, because it's defined as the ratio of two forces. The units of v_i^2 are m^2/s^2, and dividing by m/s^2 gives meters.

(d) Mathematically, a higher v_i gives a higher result, and this makes sense physically, because you can go farther if you start out going faster. A greater value of μ_k gives a smaller final result, and this also makes sense; increasing friction (e.g., by roughening the ice) would make the skater stop sooner. Increasing g would also decrease the final result; this is because a stronger gravitational field would press the skater more firmly against the ice, increasing F_N, and thereby indirectly increasing friction.

(e) The result is 2.4 km.

(f) This is clearly unrealistic. Probably the biggest reason for the unrealistic result is that

Problem ??:

force acting on skater			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	Earth	gravity	up	skater
normal	up	ice	normal	down	skater
kin. fr.	back	ice	kin. fr.	forward	skater

we've neglected air resistance. Air resistance increases rapidly with speed, so that could be a reasonable approximation at lower speeds, but at these speeds (about 50% faster than an olympic sprinter), it's probably a poor approximation. When you see pictures of olympic-level speed skaters, they're always wearing special suits that are designed to cut down on air resistance; this shows that air resistance is a very important effect at the speeds they get up to.

3-39 Let the tension in the rope connected to the right-hand weight be T. By arguments similar to the ones in the example in section 5.6, we know that the tension in the rope connected to the left-hand weight is $2T$. The accelerations of the two weights are also related. For every centimeter that the weight on the left moves up, the pulley in the middle will move one centimeter to the right. This causes one centimeter of rope to come around the middle pulley, lengthening the part the goes over the right-hand pulley and down to the weight. The right-hand weight therefore drops two centimeters: one centimeter because an extra centimeter of rope is being fed around the middle pulley, and another centimeter because the middle pulley is moving a centimeter to the right. Therefore the acceleration of the right-hand weight is twice as much as the acceleration of the left-hand one, and is in the opposite direction, $a_2 = -2a_1$. Applying Newton's second law to each weight (with positive being up), we have the following set of three equations in three unknowns:

$$\begin{aligned} a_2 &= -2a_1 \\ a_1 &= \frac{2T - mg}{m} \\ a_2 &= \frac{T - mg}{m} \end{aligned}$$

Substituting the first equation into the third one reduces this to two equations in two unknowns:

$$\begin{aligned} a_1 &= 2T/m - g \\ -2a_1 &= T/m - g \end{aligned}$$

Now we solve the second equation for $T/m = -2a_1 + g$ and substitute into the first one, which makes it one equation in one unknown:

$$a_1 = 2(-2a_1 + g) - g$$

Solving this, we find $a_1 = g/5$. The positive sign indicates that this mass (the one on the left) goes up.

3-40 $FWHM = f_{res}/Q = 100$ Hz, so since the curve is approximately symmetric, the range will be ±50 Hz, i.e., 250-350 Hz.

3-41 The second derivative test tells us whether an extremum is a maximum or a minimum. The second derivative of the quantity inside the square root is $12\omega^2 - 2(2-1/Q^2)\omega_o^2$, and evaluating this at $\omega = 0$ gives $-2(2-1/Q^2)\omega_o^2$. For small values of Q, it's positive, indicating that the quantity inside the square root is at a minimum, and the amplitude is at a maximum. The value of Q at which the expression

$-2(2-1/Q^2)\omega_o^2$ becomes zero, on its way to becoming negative, is $Q=1/\sqrt{2}$.

3-42 Let's not worry about units, since this is just a comparison with the approximate answers. Setting all those variables equal to 1 for convenience, the expression for the amplitude becomes

$$A=\left[(\omega^2-1)^2+\omega^2/36\right]^{-1/2}$$

By trial and error, I found that the maximum amplitude was 6.02, at $\omega=0.993$. The FWHM is going to be the difference between the two frequencies where the square of the amplitude equals $(6.02^2)/2$. By trial and error, I found that this occurred at frequencies of 0.906 and 1.074, giving a FWHM of 0.168, which is pretty close to 1/6.

3-46 Horizontal momentum is conserved, since there are no external horizontal forces. Let's pick masses for the planes of 1 kg and 5 kg – although the real masses must be thousands of times bigger, it won't matter as long as the ratio is 5. Let the positive x axis point east and the positive y axis north, and measure angles counterclockwise from the x axis. The magnitudes of their initial momenta are $|p_L|=300\ \text{kg}\cdot\text{mi/hr}$ and $|p_J|=750\ \text{kg}\cdot\text{mi/hr}$. The x components are

$$\begin{aligned}p_{L,x}&=300\ \text{kg}\cdot\text{mi/hr} \qquad \text{and}\\ p_{J,x}&=(750\ \text{kg}\cdot\text{mi/hr})(\cos 225^\circ)\\ &=-530\ \text{kg}\cdot\text{mi/hr} \qquad ,\end{aligned}$$

giving

$$p_{total,x}=-230\ \text{kg}\cdot\text{mi/hr} \qquad .$$

Similarly,

$$\begin{aligned}p_{L,y}&=0 \qquad ,\\ p_{J,y}&=-530\ \text{kg}\cdot\text{mi/hr} \qquad \text{, and}\\ p_{total,y}&=-530\ \text{kg}\cdot\text{mi/hr} \qquad .\end{aligned}$$

Since momentum is conserved, their common direction of motion after the crash has to be the same as the direction of their total momentum vector before the crash, which was at an angle of

$$\tan^{-1}(-530/-230)=67^\circ \text{ or } 247^\circ \qquad .$$

(Both angles are arctangents of -530/-230.) Since the x and y components are both negative, 247° is the correct angle. That's the same as 23° west of south.

3-47 (a) For its motion from the hand to the top of its arc, it has a constant acceleration equal to g, so $v^2=2gy$, and $y=v^2/2g$. (b) Only the vertical component, $v_y=v\sin\theta$, of the initial velocity has any effect on its vertical motion, so the equation becomes $y=v^2\sin^2\theta/2g$.

3-48 In Miss Lettuce's frame of reference, the float is at rest and the sidewalk is moving. The bouquet is initially at rest, so when she drops it, it falls straight down and lands at her feet.

In her fans' frame, the float is moving and the sidewalk is at rest. The bouquet is initially moving to the right, and even after she drops it, it retains that horizontal motion, because there is no horizontal force that would change it. They see a parabolic arc going to the right.

3-49 (a) The horizontal and vertical motions are independent, so they hit simultaneously. (b) While she's in the air, the only force on her is gravity. There is no horizontal force on acting on her, so her horizontal acceleration is zero, meaning she keeps a constant speed. Her speed on impact is the same as when she went over the edge, 30 km/hr. (c) The same as when he went over the edge, 40 km/hr. (d) They have the same vertical velocities, but different horizontal velocities, so Bill's speed is greater.

3-50 (a) The horizontal and vertical motions are independent, so we can write down separate equations for them:

$$\begin{aligned}&x=v_xt \qquad \text{[motion with constant velocity]}\\ &y=y_o-\frac{1}{2}gt^2 \qquad \text{[motion with constant acceleration]}\end{aligned}$$

We're not given information about time, and we don't want to find it either, so we need to eliminate it. Solving the first equation for t and substituting into the second equation gives

$$y=y_o-\frac{1}{2}g\left(\frac{x}{v_x}\right)^2 \qquad .$$

We're interested in $x = L$, and the variable d corresponds to $y_\text{o} - y$, so we have

$$d = \frac{1}{2} g \left(\frac{L}{v_x}\right)^2 \qquad .$$

(b) Convert everything to mks to start with. 73.3 mi/hr=32.8 m/s, 60.0 ft=18.3 m. Plugging in, we get $d = 1.53$ m=5.00 ft. I calculated this using the two-sig-fig value $g = 9.8\ \text{m/s}^2$, so that ends up being the limiting factor for accuracy, and the result should be rounded off to two sig figs, $d = 5.0$ ft.

3-51 (a) The ball initially has $v_x = v\cos\theta$ and $v_y = v\sin\theta$. While it's in the air, it has $a = -g$, and its vertical velocity changes from v_y to $-v_y$, so it has $\Delta v = -2v_y$. The time it spends in the air is $\Delta t = -2v_y/a_y = (2v/g)\sin\theta$. The range is

$$R = \Delta x = v_x \Delta t = \frac{2v^2}{g}\sin\theta\cos\theta \qquad .$$

(b) It comes out to be zero for $\theta = 0°$, and that makes sense because it doesn't spend any time in the air. (Remember, we're neglecting the height above the ground at which the bat strikes the ball.) It also comes out to be zero for $\theta = 90°$, and that also makes sense, because the ball just pops up and falls back down to hit home plate.
(c) The angle at which the maximum occurs isn't affected by the constant factor out in front, so we just need to maximize $\sin\theta\cos\theta$. Setting its derivative equal to zero, we have $0 = \cos^2\theta - \sin^2\theta$, $\cos\theta = \pm\sin\theta$, which gives $\theta = \pm 45°$. The positive solution is the physically meaningful one.

3-52 Here's the program I came up with:

```
import math
def r(theta,k,dt):
  # theta = angle at which it's hit, in degs
  # k     = constant for air friction
  # dt    = amount of time for each step
  y0 = 1.  # height from which it's hit
  v = 45.  # speed of hit
  m = .146 # mass
  g=9.8 # gravitational field
  t=0
  # Built-in trig functions want radians:
  radians = theta*math.pi/180.
  vx = v*math.cos(radians)
  vy = v*math.sin(radians)
  x = 0
  y = y0
  for i in range(1000000):
    # ... actually we hit the ground
    #     before 1000000
    t = t + dt
    x = x + vx*dt
    y = y + vy*dt
    if y<0 :  # hit the ground
      return x
    v = math.sqrt(vx*vx+vy*vy)
    radians = math.atan(vy/vx)
    f_air = k*v*v
    f_grav = m*g
    fx = -f_air*math.cos(radians)
    fy = -f_air*math.sin(radians)-f_grav
    ax = fx/m
    ay = fy/m
    vx = vx + ax*dt
    vy = vy + ay*dt
```

Running the program:

```
1  >>> r(45,0,0.01)
2  208.101525703
3  >>> r(45,0.0007,.01)
4  122.955242954
5  >>> r(40,0.0007,.01)
6  123.93693697
7  >>> r(40,0.00057,.01)
8  133.058781684
```

Lines 1-2 approximately reproduce the result of problem 51, with air resistance turned off; the small disagreement is due to a combination of the errors in the approximation made by the numerical simulation and the use of a starting height of 1 meter.

Lines 3-4 show the effect of air resistance, while keeping the angle at the value of 45 that would have been optimal in vacuum. Air resistance decreases the range by almost a factor of 2!

Lines 5-6 show the best range, which is obtained at around 40 degrees. (Finding this optimal angle actually required some trial and error, not shown here.)

Lines 7-8 show that the ball flies about 9 m farther at Coors Stadium.

3-53 Since the two given angles are both measured counterclockwise from east, we can just use a coordinate system with positive x being east and positive y north, and the positive and negative signs of the components will come out right if we just use cosines for x components and sines for y components. We find the components of the total Δr vector by adding the components of the two individual vectors:

$$\begin{aligned}\Delta x &= \Delta x_1 + \Delta x_2\\ &= (35\ \text{km})\cos\ 25^\circ + (22\ \text{km})\cos 230^\circ\\ &= 17.6\ \text{km}\end{aligned}$$

(temporarily keeping 1 extra sig fig).

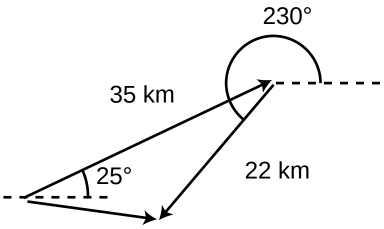
$$\begin{aligned}\Delta y &= \Delta y_1 + \Delta y_2\\ &= (35\ \text{km})\sin 25^\circ + (22\ \text{km})\sin 230^\circ\\ &= -2.1\ \text{km}\end{aligned}$$

(temporarily keeping 1 extra sig fig).

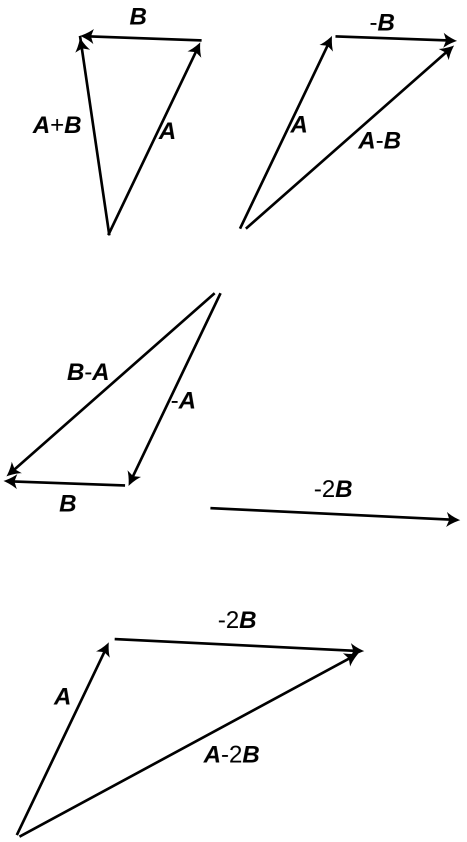
The magnitude of the total Δr vector is 18 km, and the direction is

$$\tan^{-1}(-2.1\ \text{km}/17.6\ \text{km}) = -7^\circ \text{ or } 173^\circ \qquad .$$

Checking against the diagram, the correct arctangent is -7°.



3-54



3-55 (a) Let's use a coordinate system with positive x being east and positive y being north. In these coordinates, the vector from Bangkok to Phnom Penh has

$$\Delta x = 470\ \text{km} \qquad \Delta y = -250\ \text{km}$$

and the one from Phnom Penh to Hanoi has

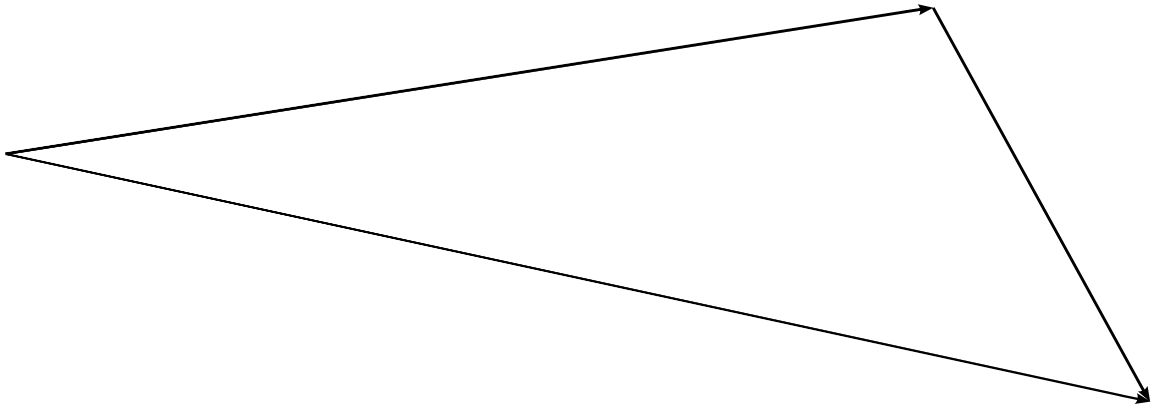
$$\Delta x = 60\ \text{km} \qquad \Delta y = 1030\ \text{km} \qquad .$$

(b) The components of the vector from Bangkok to Hanoi equal the sum of the components of the two vectors above,

$$\Delta x = 530\ \text{km} \qquad \Delta y = 780\ \text{km} \qquad .$$

3-56 Yes, it would be slowing down.

3-57 See the figure. I chose a scale of 0.5 cm $\leftrightarrow$ 10^{-7} m/s. The vector sum is 14.3 cm in length, which converts to 2.9×10^{-7} m/s. The direction,



Problem 3-57.

measured with a protractor, is about 12° south of east.

3-58 The original velocity vector had $v_x = 21.1$ m/s, $v_y = 0$. The later one has $v_x = (21.1 \text{ m/s}) \times \cos 7° = 20.94$ m/s and $v_y = (21.1 \text{ m/s}) \times \sin 7° = 2.57$ m/s. The change in the velocity vector is

$$\Delta v_x = -0.16 \text{ m/s} \qquad \text{and}$$
$$\Delta v_y = 2.57 \text{ m/s} \qquad ,$$

so the acceleration vector has components

$$a_x = \Delta v_x/\Delta t = -0.16 \text{ m/s}^2 \qquad \text{and}$$
$$a_y = \Delta v_y/\Delta t = 2.57 \text{ m/s}^2 \qquad ,$$

giving a magnitude of

$$\sqrt{(0.16 \text{ m/s}^2)^2 + (2.57 \text{ m/s}^2)^2} = 2.6 \text{ m/s}^2 \qquad .$$

Since the x component is negative and the y component is positive, it has to be pointing northwest, and the angle measured counterclockwise from east is $\tan^{-1}(2.57/-0.16) = 92°$. (Your calculator probably gives you the other arctangent, $-88°$, which is not the appropriate one.)

3-59 The main reason that this problem is difficult is that you have to realize there are two separate cases. Pressing the block and the wall together can result in either an upward or a downward frictional force on the block. If you're applying almost enough force to slide the block up the wall, then the frictional force is downward, but if you're applying just barely enough to keep it from sliding down, then the frictional force is upward. Because the vertical component of the frictional force has an opposite signs in the two cases, it is necessary to do a separate version of the algebra in each case.

Case A: upward friction on the block

Let the positive x axis be to the right and the positive y axis up. Then both the x and y components of the force on the block must equal zero. Let $\mathbf{F}_N$ be the normal force of the wall on the block. The maximum force of static friction is given by $|\mathbf{F}_{s,max}| = \mu_s|\mathbf{F}_N|$, and it is the maximum in which we're interested, since we're looking for the largest possible value of $|\mathbf{F}_H|$.

zero total force in the x direction:

$$|\mathbf{F}_H|\cos\theta - |\mathbf{F}_N| = 0$$

zero total force in the y direction:

$$|\mathbf{F}_H|\sin\theta - mg + \mu_s|\mathbf{F}_N| = 0$$

We solve the first equation for $|\mathbf{F}_N| = |\mathbf{F}_H|\cos\theta$, then substitute into the second equation and solve to find

$$|\mathbf{F}_H| = \frac{mg}{\sin\theta + \mu_s\cos\theta}$$

Case B: downward friction on the block

The solution of this part is exactly analogous, except that the total force in the y direction is $|\mathbf{F}_H|\sin\theta - mg - \mu_s|\mathbf{F}_N| = 0$, with a minus sign in front of the frictional force. The result is

$$|\mathbf{F}_H| = \frac{mg}{\sin\theta - \mu_s\cos\theta}$$

In the case of $\theta = 90°$, both the maximum and the minimum are mg, which makes sense. The maximum force blows up to infinity for $\theta = \tan^{-1}\mu_s$. With this angle, no amount of force will make the block slip, because the greater normal force leads to a greater frictional force, which is great enough to counteract the greater vertical force from the hand. For angles smaller than this, the equation for the maximum force gives a negative result, which is not physically meaningful because it was supposed to represent the magnitude of the force.

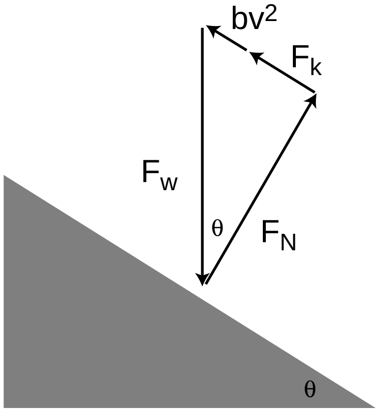
3-60 (a) There are four forces acting on the skier, a normal force F_N from the snow, a gravity force F_W, a kinetic friction force $F_k = \mu_k F_N$, and an air friction force bv^2. If the skier is to approach a constant velocity, the total force on her must become nearly zero. Laying the vectors tip-to-tail results in the right triangle shown in the figure. At first the bv^2 force will not be big enough to close the gap in the triagle, and the skier will be accelerating, but eventually as her velocity grows and the bv^2 force gets stronger the total force will approach zero and she will stop accelerating. Trigonometry gives

$$\begin{aligned} bv^2 + F_k &= F_W\sin\theta \qquad \text{and} \\ F_N &= F_W\cos\theta \qquad , \end{aligned}$$

and using these two equations plus $F_W = mg$ and $F_k = \mu_k F_N$ we find

$$v = \sqrt{\frac{mg}{b}\left(\sin\theta - \mu_k\cos\theta\right)} \qquad .$$

(b) For θ less than $\tan^{-1}\mu_k$, we'd be taking the square root of a negative number. These are angles so shallow that the skier would slow to a halt.



3-61 (a) The units of the first term are just the units of b, so b must have units of meters. The units of c have to be m/s, so that the seconds will cancel and give meters. Similarly d must have units of m/s^2, i.e., acceleration.

(b)

$$\begin{aligned} \mathbf{v} &= \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} \\ &= c\hat{\mathbf{y}} + 2dt\hat{\mathbf{z}} \\ \mathbf{a} &= \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \\ &= 2d\hat{\mathbf{z}} \end{aligned}$$

(c) The only force acting on the bullet is gravity, so its acceleration $2d$ equals g, and $\hat{\mathbf{z}}$ points down. The bullet's velocity at $t = 0$ is $c\hat{\mathbf{y}}$, so c is its initial velocity, and $\hat{\mathbf{y}}$ points to the west. If the coordinate system is right-handed, then $\hat{\mathbf{x}}$ must point south, and b is simply the bullet's location in the north-south direction, measured from some arbitrary origin.

3-66 (a) Only an inward force, i.e., a force perpendicular to the surface of contact, can have an effect on whether the batter separates from the beater. Friction is always parallel to the surface of contact.

(b) See table on last page.

(c) An inward force is required to keep the batter from following Newton's first law and flying off straight, on a tangent to the circle. The amount of force required is given by Newton's

Problem ??:

Part b:

force acting on tape			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
sticky	left	beater	sticky	right	tape

second law, $F = ma = mv^2/r$. If the speed increases, so does amount of inward sticky force needed to continue deflecting the batter off of its desired straight-line path. There's a maximum amount of force that the stickum can supply, beyond which it becomes unstuck.

3-67 $(\mathrm{m/s})^2/\mathrm{m} = \mathrm{m}^2/\mathrm{s}^2/\mathrm{m} = \mathrm{m/s}^2$

3-68 If neither the seat nor the seatbelt is applying a force to the pilot, then the only force on him is the earth's gravity, i.e. his weight. When the only force on an object is gravity, its acceleration is g. The speed needs to be such that the acceleration equals g. The acceleration is v^2/r, so $v^2/r = g$, or $v = \sqrt{gr} = 99$ m/s.

3-69

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= |\mathbf{a}||\mathbf{b}|\cos\theta_{ab}\\ \theta_{ab} &= \cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)\\ &= 12.9^\circ\end{aligned}$$

3-70 (a) Each block has three forces acting on it: gravity, a normal force from the ramp, and a force from the rope. The components perpendicular to the slope cancel out, while the ones parallel to it cause an acceleration a, which will be the same for both blocks provided that the rope doesn't go slack. We therefore have $ma = mg\sin\theta + T$ and $ma = mg\sin\phi - T$. Eliminating a and solving for T, we find $T = \frac{1}{2}mg(\sin\phi - \sin\theta)$.

(b) The units of mg are newtons, and the sines are unitless, so the result does have units of newtons, as expected.

(c) For $\phi = \theta$, the tension is zero, i.e., the rope goes slack. This makes sense. For $\phi < \theta$, the tension becomes negative; this is unphysical, but that makes sense, because the assumption we made (that the rope would stay taut) has failed. For $\theta = 0$ and $\phi = 90^\circ$, the result is $T = mg/2$, which we can readily verify is a correct solution. It accelerates the top block across its horizontal surface with $a = g/2$, and it cancels out half of the gravitational force on the bottom block, also causing $a = g/2$.

3-71 (a) The definition of Q says that when $n = Q$, the energy falls off by a factor of $e^{2\pi}$. Since energy is proportional to the square of the amplitude, this corresponds to a factor of $Z = e^{\pi}$ in amplitude. Now we need to generalize this to the case where the number of oscillations we actually observe isn't actually Q. If the amplitude drop in n oscillations is Z, then the drop in one oscillation is $f^{1/n}$. The drop in Q oscillations is then $(Z^{1/n})^Q$. Setting this equal to e^{π} and solving for Q, we find $Q = n\pi/\ln Z$.

(b) The figure is pretty small, so this will be a fairly rough estimate. After $n = 2$ cycles of oscillation, it looks like the amplitude falls off by a factor of $Z = 2$. Applying the result from part a, we have $Q \approx 9$.

3-72 (a) At position x, the fraction of his weight on the snow is $(L - x)/L$, and the fraction on the dirt x/L. Because the snow is frictionless, all that matters is the frictional force with the dirt. The normal force between the dirt and the skis is mgx/L, so the frictional force is $-\mu_k mgx/L$. The acceleration is $-\mu_k gx/L$.

(b) We want a function whose second derivative is basically the same function, but with a negative constant out in front. The functions that have this properties are the sine waves, and we can choose $f = \cos\omega t$ and $g = \sin\omega t$. In other words, the motion will be identical to part of one cycle of simple harmonic motion. Substituting into the differential equation, we find

$\omega = \sqrt{g\mu_k/L}$.

(c) The motion ends when the velocity is zero, but what's a little less obvious is at what point in the cycle the motion *starts*. At $x = 0$, the acceleration is zero, because the skis aren't on the dirt yet. For a sine wave, the zero of velocity occurs a quarter cycle after the zero of acceleration, so the motion is identical to one quarter of a period of simple harmonic motion. The time required to stop is $T/4 = \pi/2\omega = (\pi/2)\sqrt{L/g\mu_k}$. Ordinarily you would expect that it would take more time to stop if the initial speed was higher, but our answer doesn't depend on the intial speed at all. The reason is that if he's going faster, his skis will encounter more friction sooner, and the larger force will compensate for the larger speed. This also shows that the acceleration is higher if the initial velocity is higher, so running into a patch of dirt is a lot more dangerous at high speed.

3-73 In a solid, the molecules are packed in place, and aren't free to twist around very much. In a liquid, they can rotate more freely. The microwaves exert the same force F on a molecule, regardless of whether it's in a solid or a liquid, but it's not just the force that matters, since $W = Fd$. In the solid, d is small, so little energy is transferred.

3-74 Two forces act on the mass m: a downward gravitational force and an upward force from the rope. Similar forces act on the mass M. The rope makes an equal force at each end, equal to the tension T. Let up be positive, and let a be the (positive) acceleration of mass m. Newton's second law applied to mass m gives

$$a = \frac{T - mg}{m} \qquad ,$$

and for mass M,

$$-a = \frac{T - Mg}{M} \qquad .$$

We have two equations in two unknowns, T and a. To solve them, we need to eliminate one variable. Let's eliminate a.

$$\begin{aligned} \frac{T - mg}{m} &= -\frac{T - Mg}{M} \\ \frac{T}{m} - g &= -\frac{T}{M} + g \\ \left(\frac{1}{m} + \frac{1}{M}\right) T &= 2g \\ (M + m)T &= 2gMm \\ T &= \frac{2gMm}{M + m} \end{aligned}$$

Now we can find a.

$$\begin{aligned} a &= \frac{T - mg}{m} \\ &= \frac{T}{m} - g \\ &= \frac{2gM}{M + m} - g \\ &= \left(\frac{2M}{M + m} - 1\right) g \\ &= \left(\frac{2M}{M + m} - \frac{M + m}{M + m}\right) g \\ &= \left(\frac{M - m}{M + m}\right) g \end{aligned}$$

3-75 (a) As the bead speeds up, a greater force toward the axis is required to keep it from flying off straight. This means that the normal force has to get stronger, and therefore the kinetic friction force will get stronger.

(b) Define a coordinate system with axes t, r, and p. Axis r points radially outward away from the central axis; t downhill and tangent to the wire; and p perpendicular to t and r. The velocity has only a component $v_t = v$. The motion, projected into the horizontal plane, is circular motion with speed $v \cos\theta$, where θ is the angle of the downhill slope, $\tan\theta = \alpha$, where $\alpha = d/2\pi r$. The acceleration is purely in the r direction, with $a_r = -v^2 \cos^2\theta/r$.

The frictional force has only a t component. The normal force has both an r and a p component. The weight has both t and p components.

Newton's second law in the r direction gives $F_{n,r} = -mv^2/r \cos^2\theta$. Cancellation of the forces in the p direction gives $F_{n,p} = -F_{w,p} = mg\cos\theta$. We therefore have

$$F_{f,t} = -\mu_k F_n = -m\mu_k \cos\theta \sqrt{g^2 + \left(\frac{v^2\cos\theta}{r}\right)^2}$$

This has to cancel with $F_{w,t} = mg\sin\theta$, and solving for v we find

$$v = \sqrt{gr\sqrt{(1+\alpha^2)(1-\frac{\alpha^2}{\mu_k^2})}} \quad .$$

(c) In the limit of r approaching infinity, the problem is the same as that of an object sliding down an inclined plane. Under those circumstances, the only possible way of getting a constant velocity would be to have the slope chosen to have some particular value that would match properly with the coefficient of friction. A quick calculation shows that the necessary condition is $\mu_k = \tan\theta$. This matches up properly with our expression for v in the case of the helical wire, because the only way to get a finite limiting value for v as r approached infinity would be to let the factor $1 - \alpha^2/\mu_k^2$ approach zero.

3-76 Conservation of energy tells us that the initial energy equals the final energy. What times should we pick as initial and final? Our goal is to find out what height the bike has to start from, so the initial time should be when the bike is starting out, at the top of the ramp. The height is determined by the requirement that the bike not drop off the loop, even at the top, so to connect that to the other information, we should pick the final moment to be when the bike is at the top of the loop, at a height of $2r$.

First let's write conservation of energy in its most general form, and then make it more and more specific:

$$\begin{aligned}
E_{total,i} &= E_{total,f} \\
PE_i + KE_i &= PE_f + KE_f \\
mgh + 0 &= mg(2r) + \frac{1}{2}mv^2 \\
h &= 2r + \frac{v^2}{2g}
\end{aligned}$$

The height depends on the speed v the bike needs at the top of the loop. But how fast *does* the bike have to be moving when it's at the top? If it was just barely about to lose contact with the loop, then the only force on it would be gravity, so its downward acceleration would be $a = F_{grav}/m = (mg)/m = g$. For circular motion, the inward acceleration is v^2/r, so we have $g = v^2/r$, and $v = \sqrt{gr}$. Plugging this into the equation for h, we have

$$\begin{aligned}
h &= 2r + \frac{v^2}{2g} \\
&= 2r + \frac{r}{2} \\
&= \frac{5}{2}r
\end{aligned}$$

3-77 (a) The distance traveled in time Δt is $d = v\Delta t$, so the work done is $\Delta E = Fd = Fv\Delta t$, and $P = \Delta E/\Delta t = Fv$.

(b) The vertical forces cancel, so $F_N = mg$, and $F_{s,max} = \mu_s F_N = \mu_s mg$. To burn rubber, the engine has to supply $P > F_{s,max}v = \mu_s mgv$. The critical speed is $v = P/(\mu_s mg)$.

(c) W/N = J/N·s = N·m/N·s = m/s.

(d) Physically, a more powerful engine should be able to burn rubber at even higher speeds; mathematically, increasing P increases v. Physically, better traction makes it harder to burn rubber; mathematically, increasing μ_s decreases v. Increasing m or g also has the effect of increasing traction, so the dependence on these two variables also makes sense, for similar reasons.

(e) (Plug in.)

(f) The power consumed by friction is $Fv \propto v^3$. At top speed, 100% of the engine's power is being consumed by fighting air friction. At the lower speed, this is decreased by $(40/176)^3 \approx .01$.

3-78 Let m be the mass of the board, x the displacement of the board relative to the center, and N_1 and N_2 the two normal forces.

Suppose that (1) the board is not rotating, and (2) kinetic friction obtains on both sides. Then the total torque is zero, $mgx + r(N_1 - N_2) = 0$, and the total horizontal force on the board is $\mu_k(N_1 - N_2) = -mg\mu_k x/r$. This is a restoring force proportional to x, so we have simple harmonic motion, provided that the assumptions hold: no rotation, and purely kinetic friction. The frequency is $\omega = \sqrt{g\mu_k/r}$. This is a funny kind of simple harmonic motion, because the kinetic frictional forces are continually dissipating energy as heat, and yet the motion is undamped.

Under what conditions can assumptions 1 and 2 be violated? Let the amplitude of the simple harmonic motion be A.

For $A > r$, the board will tip as its center moves beyond the top of one of the wheels, violating assumption 1. The board might either fall on the ground or make a bobble and come back.

Assumption 2, purely kinetic friction, can also be violated. The maximum velocity in simple harmonic motion is $A\omega$. If this velocity ever becomes as high as the tangential velocity of the wheels, Ωr, then kinetic friction would be replaced by static friction on one side, and we would no longer have a self-consistent solution in simple harmonic motion. One possibility is that under these conditions, the board can accelerate up to velocity Ωr, then stick at that velocity for a while before its weight shifts onto the other wheel sufficiently, at which point it will start to decelerate again; the motion is periodic but not simple harmonic.

We can also have possibilities that combine the features of the tipping solutions with those of the static-frictional solutions.

3-79 (a) Let the positive x axis be to the right and positive y up. The x components of the forces cancel by symmetry, so they don't give us any useful information. The y components of the three forces are $S\cos\theta/2$, $S\cos\theta/2$, and $-L$. Setting the sum of these equal to zero, we find $S = L/(2\cos\theta/2)$. (b) For $\theta = 0$, we find $S = L/2$, which makes sense physically because two forces S are cooperating perfectly to cancel L. (c) For $\theta = 180°$, we have $S = \infty$, which is not physically possible, but does make sense. If we were to grab the two anchor lines in our hands and pull them to the right and left away from each other, it would take infinite force to make $\theta = 180°$ exactly. (c) This occurs when $2\cos\theta/2 = 1$, i.e., $\theta = 2\cos^{-1}(1/2) = 120°$.

3-80 (a) See the tables for an analysis of the forces involved.

Applying Newton's second law to the climber of mass M, and letting the positive direction be downward, we have

$$Mg - T = Ma \qquad .$$

The climber of mass m doesn't sink into the snow or fly up in the air, so

$$F_N - mg = 0 \qquad .$$

Newton's second law for m, with positive to the right, gives

$$T - \mu_k F_N = ma \qquad .$$

These are three equations in three unknowns, a, T, and F_N. Eliminating $F_N = mg$ reduces it two a system of two equations in two unknowns:

$$\begin{gathered} Mg - T = Ma \\ T - \mu_k mg = ma \end{gathered}$$

Next we solve the first equation for $T = M(g-a)$ and substitute into the other equation, giving one equation with one unknown:

$$M(g-a) - \mu_k mg = ma$$

Solving this for a, we find

$$a = \frac{M - \mu_k m}{M + m} g \qquad .$$

(b) The definition of μ_k is $F_k = \mu_k F_N$, so μ_k is unitless. That means that it makes sense to do the subtraction $M - \mu_k m$, since both terms have units of kg. The quotient $(M - \mu_k m)/(M + m)$ is unitless, and multiplying it by g gives units of acceleration, which checks out.

(c) Mathematically, increasing g increases a. That makes sense because more gravity should make them accelerate faster. Mathematically, increasing μ_k decreases a, and this checks out because increasing friction should keep them from accelerating as fast. Increasing m decreases the

Problem ??:

force acting on climber of mass m			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	climber
kinetic friction	left	snow	kinetic friction	right	climber
normal	up	snow	normal	down	climber

Problem ??:

force acting on climber of mass M			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	climber
normal	up	rope	normal	down	climber

numerator and increases the denominator, and both of these effects will reduce a. This makes sense because adding extra weight to m will help both by increasing friction and by increasing the inertia of the whole system, which is measured by $M+m$. M is the hardest variable to analyze mathematically. Increasing M increases both the numerator and the denominator, so there are two countervailing effects, and it's not obvious which is the greater. But since the numerator is smaller than the denominator, increasing M by a certain amount will cause a greater percent change in the numerator, so the net effect will be to increase a. This makes sense physically, because if M was miniature French poodle, we wouldn't expect the acceleration to be as fast.

3-81

$F_{F1,x}$	small, −	$-F_{F1}\sin\theta$
$F_{F1,y}$	large, +	$F_{F1}\cos\theta$
$F_{F2,x}$	small, −	$-F_{F2}\sin\theta$
$F_{F2,y}$	large, +	$F_{F2}\cos\theta$
$F_{N1,x}$	large, −	$-F_{N1}\cos\theta$
$F_{N1,y}$	small, −	$-F_{N1}\sin\theta$
$F_{N2,x}$	large, +	$F_{N2}\cos\theta$
$F_{N2,y}$	small, +	$F_{N2}\sin\theta$
$F_{W,x}$		0
$F_{W,y}$		$-F_W$

3-82 (a) Let positive x be inward and positive y up. Then $a_x = F_x/m$ gives $v^2/r = F_N/m$, and $a_y = F_y/m$ results in $0 = \mu_s F_N - mg$. Solving for r, we find $r = \mu_s v^2/g$. (b) $(\mathrm{m/s})^2/(\mathrm{m/s}^2) = \mathrm{m}$. (c) Mathematically, increasing v increases r; this makes sense because it should be harder to do if r is bigger, so you should have to run faster. Increasing g decreases r, and that makes sense because, e.g., if g were 0, you could do this with an unlimited r. Increasing μ_s increases r; this makes sense because, e.g., in the absence of friction, it should be impossible to do the stunt, and the maximum r should be 0, i.e., it shouldn't be possible for any positive r. (d) The result is 50 meters, which is counterintuitively huge.

3-83 (a) Let positive x be to the right and positive y up. Zero total force acts on the carabiner in the y direction,

$$-L + 2T\cos(\theta/2) = 0 \qquad .$$

Zero total force also acts on the left-hand anchor, so

$$\begin{aligned} S_y - T\cos(\theta/2) &= 0 \qquad \text{and} \\ S_x + T + T\sin(\theta/2) &= 0 \qquad . \end{aligned}$$

Algebra leads to the result

$$S = L\frac{\sqrt{(1+\sin(\theta/2))/2}}{\cos(\theta/2)} \qquad .$$

(b) When $\theta = 0$, we have $S = L/\sqrt{2}$, which makes sense because in this situation $T = L/2$, and each anchor experiences two forces equal in magnitude to T, acting at right angles. This is worse than the correct setup, in which $\theta = 0$ gives $S = L/2$.

3-84 Draining the can completely isn't optimal, because then its center of mass will be that of the empty can, at its own center. The center of mass is a kind of average. Any beer below the center will lower the average relative to the empty can, while any beer above it will raise it. You should drink half the beer before setting it down.

3-85 (a) To lift the bag at constant velocity, the person must cancel out the force of gravity mg. The work done is therefore about $mgd =$ 50 kJ. (b) The calculation comes out the same, and this is what we expect based on conservation of energy if the pulley is frictionless: the bag still gains the same amount of energy. (In reality there would be some friction in the pulley, so the force would be slightly more than the given value, the work would be slightly greater, and the extra energy would go into heating the pulley and the rope.)

3-86 (a) Let positive x be to the right, and positive y up. The horizontal motion is at constant velocity $u\cos\alpha$, so

$$x = ut\cos\alpha.$$

The vertical motion has initial velocity $u\sin\theta$ and acceleration $-g$, so

$$y = ut\sin\alpha - \frac{1}{2}gt^2.$$

Setting $x = \ell$ in the first equation and solving for t gives $t = \ell/u\cos\alpha$. Now we set $y = h$ in the second equation and substitute for t, which gives

$$h = \ell\tan\alpha - \frac{1}{2}g(\ell/u\cos\alpha)^2.$$

Solving this for u results in

$$u = \sqrt{\frac{gl}{2\cos^2\alpha(\tan\alpha - h/\ell)}}.$$

(b) The ratio h/ℓ is unitless, so it makes sense to subtract it from $\tan\alpha$. The units of the quantity inside the square root come solely from the factor gl, which has units of m^2/s^2. Taking the square root gives m/s, which makes sense.

(c) Physically, if gravity is stronger we'll need to roll the ball faster to get it up to the hole. Mathematically, g is on top, so increasing it increases u.

(d) Physically, if the height is increased we'll need to increase the speed. Mathematically, h/ℓ is subtracted on the bottom, so increasing h reduces the bottom, which increases the result.

(e) If $\alpha = 90°$, then u blows up to infinity. This makes sense, because if the ball is launched straight up, then it doesn't matter how fast we roll it, it will never cover the horizontal distance h.

(f) When $\tan\alpha = h/\ell$, the ramp is pointed straight at the hole. If we were playing the game in zero gravity, this would be just fine, but when there is gravity, the path of the ball can't be straight, so there is no way it can get to the hole when it's aimed this way. Mathematically, our result for u blows up under these conditions, which makes sense.

(g) Plugging in gives $u = 4.1$ m/s.

3-87 The acceleration is $a = \omega^2 r = (2\pi/T)^2 r = 4.7\times10^{-5}\ \text{m/s}^2$. If the astronaut stands on the inside of the outer bulkhead, the force pushing up on her feet is what produces her circular motion. This force is $F = ma = 2.8\times10^{-3}$ N. If she adopts the rotating frame, she interprets this "upward" force as canceling the "downward" fictituous force of apparent gravity. This fictitious force is roughly equivalent to the weight of a raisin.

3-89 For the limit $g_1 = \lim_{y\to0}\lim_{b\to0} g$, we know that the inside limit is proportional to $1/y$ from the result of the previous problem. Therefore $g_1 = \infty$.

In the case of $g_2 = \lim_{b\to0}\lim_{y\to0} g$, the inside limit is zero by symmetry for all nonzero values of b. Thus $g_2 = 0$.

We find that the two ways of defining the field on the filament do not agree, and therefore there is no unambiguous way of saying what is the truth of the matter. The notion of the field at a point on such a filament is an idealization, and the idealization does not cleanly map onto actual observables.

3-90 Let the filament be an arc of the unit circle, and let P be the point $(1, 0)$. Then the distance r between P and the point at angle θ on the unit circle is given by $r^2 = (1-\cos\theta)^2 + \sin^2\theta = 2(1-\cos\theta)$. The infinitesimal arc from θ to $\theta+\mathrm{d}\theta$ has mass proportional to $\mathrm{d}\theta$ and exerts a force $\mathrm{d}F$ on the mass element at P that is proportional to $\mathrm{d}\theta r^{-2}$. The x component of this force is $\sin(\theta/2)\,\mathrm{d}F$. The resulting force in the x direction is, ignoring positive constant factors,

$$\begin{aligned} F_x &= \int_{\theta_1}^{\theta_2} -\sin(\theta/2)\,\mathrm{d}F \\ &= \int_{\theta_1}^{\theta_2} -\frac{\sin(\theta/2)\,\mathrm{d}\theta}{1-\cos\theta}. \end{aligned}$$

Near $\theta = 0$, small-angle approximations give $\sin(\theta/2) \approx \theta/2$, and $1-\cos\theta \approx \theta^2/2$, so the integrand is proportional to $1/\theta$. Therefore the improper integral diverges logarithmically at $\theta = 0$. The integrand is negative everywhere, so the integral diverges to $F_x = -\infty$, i.e., the force acting on the element of mass at P is infinite and to the left (in the direction of concavity).

Based on this result, it would appear that such a filament would behave as if it had infinite stiffness with respect to being bent away from straightness. This result, however, is not necessarily physically realistic, for the reasons explored in the precednig problem.

3-91 (a) There are three forms of energy present: gravitational PE, electric PE in the rope, and the climber's KE. The climber's KE is zero at both the initial time and the final time, so we can ignore it. We then have

$$PE_{g,i} + PE_{e,i} = PE_{g,f} + PE_{e,f},$$

or

$$\begin{aligned} PE_{e,i} &= \Delta PE_g + PE_{e,f} \\ 0 &= -mgx\sin\theta + \frac{1}{2}kx^2. \end{aligned}$$

The result is $k = (2mg/x)\sin\theta$.

(b)

$$\frac{\mathrm{N}}{\mathrm{m}} = \frac{\mathrm{kg{\cdot}m/s^2}}{\mathrm{m}}$$

(c) The result increases with m, g, and θ, and decreases with x. A heavier climber, greater gravity, or higher angle will all make it harder to stop the climber in time, so the dependence on these variables makes sense. A greater x makes it easier to stop the climber in time, so this also makes sense.

4-2 You can get the maximum torque by setting everything up with θ=90°, giving

$$\begin{aligned} \tau &= rF\sin\theta \\ &= rmg \\ &= 270\ \mathrm{N{\cdot}m} \qquad . \end{aligned}$$

4-3 The total torque required is the torque needed to lift his arm plus the torque needed to lift the weight. Since his arm is not changing, she needs to tell him to set things up so that the torque needed to lift the weight is also the same as before. Torque is $rF\sin\theta$, and since he is decreasing r by a factor of 17/33, she should tell him to increase the mass by a factor of 33/17 so that the increased weight force will compensate for the decreased r. The new mass should be 2.1 kg × 33/17 = 4.1 kg.

4-4

No. It is momentarily at rest, but not at equilibrium, because there is a force on it, the force of gravity, which is not canceled out by any other force. Another way of putting it is that it has $p = 0$ at that moment, but not $\mathrm{d}p/\,\mathrm{d}t = 0$, which is what equilibrium means.

4-5 No, it just means that any torques that do act are adding up to produce zero *total* torque. For instance, a merry-go-round rotating with constant angular velocity has torques acting on it, but the motor's torque and the torque due to friction are just canceling out to produce zero total torque. Of course it is also *possible* that no torques at all are acting (e.g., the earth has no torques on it at all), but it would be incorrect to infer that the merry-go-round's motion at constant angular velocity implied that there were no torques on it.

4-6 Basically we have a situation with two equations in two unknowns. There is an equation that says the total torque on the foot is zero, and another equation that says the total force on the foot is zero. The two unknowns are the tension in the calf muscle, T, and the force exerted by the shinbone, F_s. We can however simplify the solution by choosing the axis to be the place where the shinbone attaches to the foot. With this choice, the shinbone makes no torque on the foot, and the torque equation thus has only the tension in the calf muscle as an unknown. There are two torques on the foot, a clockwise torque exerted by the muscle and a counterclockwise torque from the floor. The floor's upward force equals mg, so if we use plus signs for clockwise torques the torque equation is

$$Ta - mgb = 0 \qquad ,$$

which is easily solved to give

$$T = mgb/a \qquad .$$

Letting positive forces be upward, the force equation is

$$T - F_s + mg = 0 \qquad ,$$

so

$$\begin{aligned} F_s &= T + mg \\ &= mgb/a + mg \\ &= \left(1 + \frac{b}{a}\right) mg \qquad . \end{aligned}$$

4-7 No. The angular momentum of a particle is $mv_\perp r$, so if the particles differ in the distance, r, from the axis or in their direction of motion, they can have different angular momentum. Furthermore, they could even have the same r and p, but if their directions of motion were different they could have different angular momenta. As an example of the effect of the direction, a particle has zero angular momentum if it is moving straight in towards the axis or directly away from it ($\theta = 0$ or $180°$).

4-8 (a) Let's use subscripts r for the rope, g for gravity, and n for the floor's normal force. In the case where the block topples, it rotates and its center of mass rises as it tips up and over. In the case where it slides without tipping, neither of these is supposed to happen, so the vertical forces must cancel, and the torques must cancel. Cancellation of the vertical forces gives $F_n = mg$. The torques are most easily calculated using $\tau = r_\perp F$, with the axis taken to be at the center of the box. Note that in the condition where the box is not tipping, the normal force is actually being distributed over the whole bottom surface of the block, so it doesn't actually have a single, well-defined value of $r_\perp$; however, as soon as the tipping starts, the only point of contact is at $r_\perp = a$. The torques are therefore $\tau_r = bF_r$ and $\tau_n = amg$. If the box is just about to tip, i.e., the force from the rope has its maximum value, then these torques cancel, and $F_r = (a/b)mg$. The dependence on a/b makes sense, because a squat box with a large value of a would be more stable, allowing F_r to be greater. (What we've actually calculated here is the value of F_r for which the box would just barely *start* to tip. Once the box started to tip, we could ask ourselves whether it might then manage to right itself again. The answer is no, because once it starts to tip, both of the $r_\perp$ values start to decrease, but their ratio is still a/b.)

(b) The force of friction is $F_f = \mu F_n = \mu mg$. (In the case of static friction, this is the maximum force, which is what we're interested in because we want to see if we can unstick the box and get it to slide without tipping. In the case of kinetic friction, this is simply the value of the frictional force.) We now have a torque from friction, which is in the same clockwise direction as the torque from the rope. Cancellation of the torques gives $bF_r + b\mu mg = amg$, yielding $F_r = (a/b - \mu)mg$.

(c) The answer to part b, unlike the one from part a, could come out negative under certain circumstances. The negative solution would be unphysical, and the interpretation would be that it would be impossible to accelerate the box without making it tip over. This happens for $b > a/\mu$. That makes sense. A big value of b means a tall box, which would be less stable. In the limit $\mu \rightarrow 0$, we get $b \rightarrow \infty$, meaning that there is no maximum value of b, recovering the behavior of the frictionless case.

4-9 (a) There are three forces on the ladder, F_1, F_2, and gravity, which can be thought of as acting at its center for the purpose of calculating torques. If it's going to remain at rest, the total horizontal force, total vertical force, and total torque must all be zero. Taking the axis of rotation as the center of the ladder, the gravitational force makes no torque. If ϕ was less than θ, both torques would be of the same sign (both clockwise or both counterclockwise), so they could not cancel.

(b) The three conditions for equibrium are:

$$\text{zero total horizontal force:} \qquad F_1 \cos\phi - F_2 = 0 \qquad (1)$$

$$\text{zero total vertical force:} \qquad F_1 \sin\phi - mg = 0 \qquad (2)$$

$$\text{zero total torque:} \qquad \ell F_2 \sin\theta - \ell F_1 \sin(\phi - \theta) = 0$$

or

$$F_2 \sin\theta - F_1 \sin(\phi - \theta) = 0 \qquad (3)$$

For our numerical check, let's arbitrarily choose $mg = 1.000$ N. ℓ is irrelevant, since it canceled out in eq. (3). Eq. (2) gives $F_1 = 1.118$ N. Eq. (1) gives $F_2 = 0.500$ N. As advertised, eq. (3) checks out.

4-10 In the previous problem, we got three equations, which involved several variables we want to get rid of: F_1, F_2 and m. It should be possible to solve three equations in three unknowns. We can simplify things right off the bat by realizing that eq. (2) is useless; it introduces another variable, m, which we don't know and don't want to know. So we're better off just ignoring it and considering this as two equations, (1) and (3), in two unknowns. The general strategy in solving simultaneous equations like this is to solve one of the equations for one of the unknowns, then substitute into the other equation(s) to eliminate it. Let's solve eq. (1) for $F_2 = F_1 \cos\phi$. Eliminating F_2 from eq. (3), we have

$$F_1 \cos\phi \sin\theta - F_1 \sin(\phi - \theta) = 0 \qquad ,$$

which we can now simplify by dividing by F_1, resulting in

$$\cos\phi \sin\theta - \sin(\phi - \theta) = 0 \qquad .$$

We've eliminated all the unknowns, and we now have a relationship involving only θ and ϕ, the things we're interested in. Using the trig identity as suggested, $\sin(\phi - \theta) = \sin\phi\cos\theta - \sin\theta\cos\phi$, so our equation becomes

$$\cos\phi \sin\theta - \sin\phi \cos\theta - \sin\theta \cos\phi = 0 \qquad .$$

Simplifying, we get $\tan\phi = 2\tan\theta$, so $\phi = \tan^{-1}(2\tan\theta)$. As claimed, m and ℓ are irrelevant.

4-11 (a) When the bare minimum amount of force is applied, the wheel will be in equilibrium, and will have zero force between it and the ground. The only forces are then gravity, a normal force from the curb, and the applied force. We don't know the curb's force, and we don't want to know it, so a convenient way to solve the problem is to require zero total torque, and pick the edge of the curb as our axis, so that the curb's force makes no torque. Let h be the height of the curb, let m and r be the mass and radius of the wheel. The gravity force and the applied force both make angles with the radial line connecting the axle to the curb, and these are the angles that are involved in the torques. The angle for the gravity force is $\cos^{-1}((r-h)/r$, and the angle for the applied force is 90 degrees minus that, i.e. $\sin^{-1}((r-h)/r$. Setting the magnitudes of the two torques equal, we have

$$\begin{aligned}
(r)(mg)\sin\left(\cos^{-1}((r-h)/r)\right) &= (r)(F)\sin\left(\sin^{-1}((r-h)/r)\right) \\
(mg)\sin\left(\cos^{-1}((r-h)/r)\right) &= (F)\sin\left(\sin^{-1}((r-h)/r)\right) \\
mg\frac{\sqrt{r^2-(r-h)^2}}{r} &= (F)\frac{r-h}{r} \\
mg\sqrt{r^2-(r-h)^2} &= (r-h)F \\
F &= mg\frac{\sqrt{r^2-(r-h)^2}}{r-h}
\end{aligned}$$

(b) It's infinite if $r = h$. This makes sense, because the curb doesn't lift the wheel unless the curb can get under the wheel. If $h = 0$, the force becomes zero, and of course that makes sense as well.

4-12 Astrid is right and Michelle is wrong. The force on the ball acts along the line of the string. This line is perpendicular to the ball's motion, so the string does no work on the ball, and the ball must therefore keep the same kinetic energy. After all, if the ball's kinetic energy was going to change, conservation of energy says that some other form of energy would have to change in the opposite direction, but the pole and the string aren't storing or releasing any energy.

The flaw in Michelle's reasoning is that the string's force on the ball is not straight toward the center of the pole, since the string is wrapping around the pole. The string makes a nonzero torque on the ball, and the ball's angular momentum changes. The ball is transferring angular momentum to the planet Earth.

4-13 Conservation of energy gives

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = E \qquad .$$

Intuitively, this tells us that if r is going to get small, v has to get big. But notice that v is squared. That means that v doesn't really increase all that fast, because not much of a change in v is required in order to make v^2 get quite a lot bigger. At the same time, the bomb has to conserve angular momentum, which also requires that as r gets smaller, v has to get bigger to compensate. However, the angular momentum is $L = mvr\sin\theta$, with no squares or anything, and if v isn't getting bigger all that fast, then the only way to pull the angular momentum back up is if $\sin\theta$ increases. But $\sin\theta$ can only get as big as 1, when the bomb is flying horizontally, so that puts a limit on how small r can get:

$$\begin{aligned} L &= mvr\sin\theta \\ &< mvr \end{aligned}$$

From now on, let's just use r to mean the minimum value, rather than the clumsy sumbol r_{min}. Then solving the conservation of energy equation for v gives

$$v = \sqrt{\frac{2E}{m} + \frac{2GM}{r}} \qquad ,$$

so

$$\begin{aligned} L &= mvr \\ &= mr\sqrt{\frac{2E}{m} + \frac{2GM}{r}} \\ 0 &= \left(\frac{2E}{m}\right) + \frac{2GM}{r} - \frac{L^2}{m^2r^2} \\ 0 &= \left(\frac{E}{GMm}\right) r^2 + r - \frac{L^2}{2GMm^2} \end{aligned} \qquad ,$$

which we can solve using the quadratic formula to give the result originally claimed.

4-14 (a) Let the axis be the hinge, and let the length of the bar br ℓ. The gravitational torque can be calculated by pretending that it acts at the bar's center of mass, with $r = \ell/2$. Cancellation of the torques gives $(1/2)\ell mg = \ell T\sin\theta$, and the result is $T = mg/(2\sin\theta)$.

(b) To minimize T, we want to maximize $\sin\theta$, which means letting θ be 90°. In other words, the cable is vertical.

(c) As θ approaches zero, the tension approaches infinity. No matter how hard you pull, you can't support the weight of the bar with a horizontal cable.

4-15 (a) Let the hinge be the axis, and set the total torque on one bar equal to zero. With this choice of axis, the normal force at the hinge makes zero torque, so we have

$$\frac{1}{2}\ell mg\cos\alpha = \ell T\sin(\beta - \alpha) \qquad .$$

This is a single equation in two unknowns, α and T, so we're not ready to solve anything yet.

The brute-force technique would be to set the total horizontal and vertical force on this bar equal to zero, and this would give us two more equations, while introducing one more unknown (the horizontal normal force at the hinge). We would then have three equations in three unknowns.

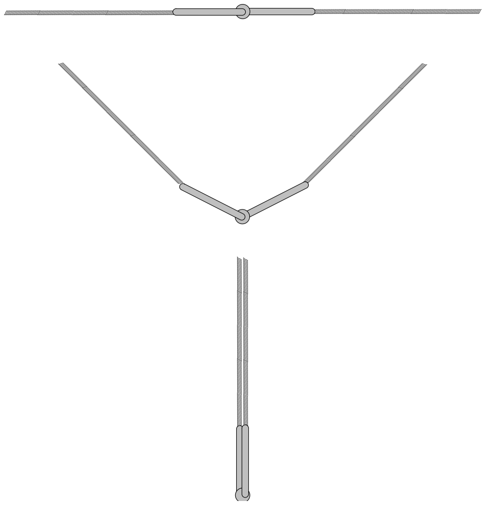
A trick that simplifies the calculation is to set the total vertical force on *both* bars equal to zero, in which case the normal force at the hinge becomes an internal force that we can ignore. We then have

$$2T\sin\beta = 2mg \qquad .$$

Solving the second equation for T and plugging in to the first equation, we get

$$\begin{aligned}\frac{1}{2}\cos\alpha &= \frac{\sin(\beta-\alpha)}{\sin\beta}\\ \frac{1}{2}\cos\alpha &= \frac{\sin\beta\cos\alpha-\cos\beta\sin\alpha}{\sin\beta}\\ \alpha &= \tan^{-1}\left(\frac{1}{2}\tan\beta\right) \qquad .\end{aligned}$$

(b) For $\beta = 0$, 45°, and 90°, we get $\alpha = 0$, 27°, and 90°, respectively. All three of these make sense physically. In particular, the intermediate case has $\alpha < \beta$, which looks intuitively right.



4-16 (a) Letting the hinge be the axis, and writing N for the magnitude of the normal force the surface of the cylinder makes on one bar, the condition for zero total torque on one of the bars is

$$\begin{aligned}(N)(\text{distance to point of contact}) &= \frac{1}{2}mg\ell\sin(90^\circ-\theta)\\ NR\tan\theta &= \frac{1}{2}mg\ell\cos\theta\end{aligned}$$

The cylinder is the only thing that supports the bars against gravity, so,

$$\begin{aligned}2N_y &= 2mg\\ N\cos\theta &= mg \qquad .\end{aligned}$$

Eliminating N, we find

$$\frac{r}{\ell} = \frac{\cos^2\theta}{2\tan\theta}$$

(b) For very large values of r/ℓ, the cylinder might as well be flat. Similarly, you don't notice the curvature of the earth in everyday life. On a flat surface, the bars will lie flat, and θ will be zero. The equation is consistent with this: dividing by the tangent of zero, we get infinity. For very small values of r/ℓ, either the denominator would have to be very large, or the numerator would have to be very small. Both these things happen at $\theta = 90°$. This makes sense, because the cylinder would be like a thin wire. The bars would hang down nearly vertically.

4-17 What makes this a little hard is that not only do we not know the weights w and W of the ship and bottle, we also can't tell the exact positions of their centers of mass. Let the bottle's center of mass lie at a distance H to the right of the fulcrum, and let the ship's center of mass be h to the right of the ship's stern. Suppose that for our first measurement we slide the ship around until its stern is a distance b_1 to the right of the fulcrum. Let the axis be at the fulcrum, so that the (unknown) force from the fulcrum doesn't make a torque. Then setting the total torque equal to zero gives

$$(b_1+h)w + HW - S_1\ell = 0 \qquad ,$$

where S_1 is the scale's reading in this position, and ℓ is the scale's distance from the fulcrum. This is one equation in four unknowns (h, H, w, and W). It wouldn't do us any good to write down the equation that says that the total force is zero, because that would bring in another unknown: the force at the fulcrum.

It should now be fairly clear that a solution exists. Every time we slide the ship around to a new b, we generate a new equation, without introducing any new unknowns. It is therefore definitely possible to get a unique solution for all four unknowns simply by sliding the ship to locations b_1, b_2, b_3, and b_4.

Actually it turns out that only two locations are necessary. At a second location,

$$(b_2 + h)w + HW - S_2\ell = 0 \qquad .$$

Subtracting the two equations gives $(b_1 - b_2)w - (S_1 - S_2)\ell = 0$, which has only the unknown w in it.

4-18 The equal-area law is just a statement of conservation of angular momentum, which will still be true, because gravity is still directly toward the sun, and still makes zero torque on the planet.

The elliptical orbit law would have to be junked, and there's no easy way to figure out what the shape of the orbits would be instead. The arguments in section 4.1.6 depended completely on the $1/r^2$ nature of the gravitational force.

The law of periods would have to change, but it wouldn't be all that difficult to rework it. If the force law was $F = GMm/r^7$, we would have $a = F/m = GM/r^7 = v^2/r$, and $v = 2\pi r/T$, leading to $T \propto r^4$.

4-19

$$K = \frac{L^2}{2I}$$

$$\Delta\Theta = \omega_o \Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$W = \tau\Delta\Theta$$

4-20 (a) The definition of k for motion in one dimension is $k = \left.\mathrm{d}^2 U/\mathrm{d}x^2\right|_0$. Translating the linear quantities x and k into the angular ones θ and κ, this becomes $\kappa = \left.\mathrm{d}^2 U/\mathrm{d}\theta^2\right|_0 = Bm$.

(b) Translating k and m to κ and I, $\omega = \sqrt{k/m}$ becomes $\omega = \sqrt{\kappa/I} = \sqrt{Bm/I}$.

4-21 (a) For the Earth's rotation, $\omega = \Delta\theta/\Delta t = (2\pi)/(24\text{ hr}) = 7.3 \times 10^{-5}\ \text{s}^{-1}$, and similarly for its yearly revolution around the sun, $\omega = 2.0 \times 10^{-7}\ \text{s}^{-1}$.

(b) The acceleration is $\omega^2 r$. The factor of ω^2 would tend to make the acceleration due to rotation bigger by a factor of $365^2 = 1.3 \times 10^5$, but the r factor would contribute a factor of (radius of orbit)/(radius of Earth) $= 2.3 \times 10^4$ in the opposite direction. The former effect is bigger, so the acceleration due to rotation is bigger. It's surprising, however, that the two numbers would even end up being near the same order of magnitude, given how different the ω and r values were.

4-22 (a) $L = (4\pi/5)MR^2/T = 1.1 \times 10^{42}\ \text{kg} \cdot \text{m}^2/s$.

(b) Angular momentum is conserved because no outside torques are acting, so the angular momentum is the same.

(c) Solving the same equation for T gives $T = 150$ s.

4-23 Let the mass of each atom be m, and the center-to-center distance between neighboring atoms b.

$$\begin{aligned}\frac{I_{cross}}{I_{line}} &= \frac{m \cdot 0^2 + 4mb^2}{m \cdot 0^2 + 2mb^2 + 2m(2b)^2} \\ &= \frac{2}{5}\end{aligned}$$

4-24 Newton's second law is

$$a = \frac{mg - T}{m} \qquad ,$$

where T is the tension in the string. The yo-yo's angular momentum is the same as the angular momentum of a single cylinder of mass m and radius R rotating about its axis:

$$L = \frac{\pi m R^2}{P} \qquad ,$$

where to avoid confusion I've used P rather than T for the period. The period is related to the yo-yo's downward velocity by $v = 2\pi r/P$, so we have

$$L = \frac{mR^2 v}{2r} \qquad .$$

The torque on the spindle is Tr, so

$$\begin{aligned}\tau &= \frac{\Delta L}{\Delta t}\\ Tr &= \frac{mR^2a}{2r}\\ T &= \frac{mR^2a}{2r^2}\end{aligned}$$

We now have two equations in the two unknowns T and a. One equation is Newton's second law, and the other is the equation just derived. Subsituting the latter into Newton's second law to eliminate T, we find

$$\begin{aligned}a &= g - \frac{R^2a}{2r^2}\\ a &= \frac{g}{1+R^2/2r^2} \qquad .\end{aligned}$$

Let's check this answer. The units work, because g is an acceleration, and the denominator is unitless. The denominator is greater than one, so $a < g$, which matches up with our real-world experience: a yo-yo doesn't drop like a rock. The acceleration gets smaller if R is bigger in relation to r, and that makes sense too, because increasing R makes it harder to spin (the same torque produces a slower change in the rate of rotation).

4-25 Let T be the time required for one rotation. Then the velocity of a ball rolling without slipping is $v = 2\pi R/T$, and we have $L/p = (4\pi mR^2/5T)/(2\pi mR/T) = (2/5)R$.

4-26 There is a kinetic frictional force on the ball, which reduces its momentum by Δp. Associated with this force there is also a torque. Just as force is the rate of change of momentum, torque is the rate of change of angular momentum, and since the ratio of torque to force is R, the ratio of ΔL to Δp is also R: $\Delta L = R\Delta p$. The ball has no angular momentum initially, so ΔL is the same as the final angular momentum L. We then have

$$\begin{aligned}\frac{p}{L} &= \frac{5}{2R} = \frac{p_\text{o}}{R\Delta p} - \frac{1}{R}\\ \frac{5}{2} &= \frac{p_\text{o}}{\Delta p} - 1\\ \Delta p &= \frac{2}{7}p_\text{o} \qquad ,\end{aligned}$$

so the ball loses 2/7 of its initial momentum, and is left with 5/7.

4-27 Taking the vector cross product of these two vectors using the component form for the cross product, I got $-2\hat{\mathbf{x}} - 5\hat{\mathbf{y}} + \hat{\mathbf{z}}$, with all the numbers in units of $\text{kg}\cdot\text{m}^2/\text{s}$. As a check on my result, I decided to use $|\mathbf{r}\times\mathbf{p}| = |\mathbf{r}||\mathbf{p}|\sin\theta_{rp}$. The left-hand side is $\sqrt{30} = 5.48$. The angle between the two vectors can be found from $\mathbf{r}\cdot\mathbf{p} = |\mathbf{r}||\mathbf{p}|\cos\theta_{rp}$, which gives $\theta = 2.40$ radians, resulting in $|\mathbf{r}||\mathbf{p}|\sin\theta = 5.49$, which checks out to within rounding errors.

4-28 The easiest way to find such a vector is to take the cross product.

$$\begin{aligned}(\mathbf{A}\times\mathbf{B})_x &= A_yB_z - B_yA_z = -3\\ (\mathbf{A}\times\mathbf{B})_y &= A_zB_x - B_zA_x = 6\\ (\mathbf{A}\times\mathbf{B})_z &= A_xB_y - B_xA_y = -3\end{aligned}$$

So our vector is $-3\hat{\mathbf{x}} + 6\hat{\mathbf{y}} - 3\hat{\mathbf{z}}$. We could also use any other vector lying along the same line, i.e., any scalar multiple of this one. I checked my result by computing its dot product with each of the two given vectors, verifying that it was zero in each case.

4-29 The definition says that the magnitude of the cross product is given by $|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}|\sin\theta_{AB}$, and its direction is perpendicular to $\mathbf{A}$ and $\mathbf{B}$, and right-handed. The magnitude part is easy to check, since the magnitude has the property $|k\mathbf{B}| = |k||\mathbf{B}|$. If k is positive, then the direction of $k\mathbf{B}$ is the same as that of $\mathbf{B}$, so the direction comes out as claimed. If k is zero, then $\mathbf{A}\times(k\mathbf{B}) = 0$, which is the same as $0\mathbf{A}\times\mathbf{B} = 0$. If k is negative, then $k(\mathbf{A}\times\mathbf{B})$ is in the opposite direction compared to $\mathbf{A}\times\mathbf{B}$, but this agrees with the direction of $\mathbf{A}\times(k\mathbf{B})$, since flipping the direction of $\mathbf{B}$ to $k\mathbf{B}$ causes the result of the right-hand rule to flip as well.

4-30 The definition of the vector cross product in terms of the components is $(\mathbf{A}\times\mathbf{B})_x = A_yB_z - B_yA_z$, and similarly for y and z. Reversing the roles of **A** and **B** interchanges the positive and negative terms on the right, so the sign flips.

4-31 Since it's not associative in general, we expect that almost any set of three vectors, chosen at random, will provide a suitable counterexample. I decided to use combinations of the unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$, since they're easy to calculate with. The first example I tried was $\hat{\mathbf{x}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}})$, which demonstrated Murphy's law by being one of those rare examples in which associativity *does* hold, because both $\hat{\mathbf{x}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}})$ and $(\hat{\mathbf{x}}\times\hat{\mathbf{y}})\times\hat{\mathbf{z}}$ are zero. My next example worked: $\hat{\mathbf{y}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}}) = \hat{\mathbf{y}}\times\hat{\mathbf{x}} = -\hat{\mathbf{z}}$, but $(\hat{\mathbf{y}}\times\hat{\mathbf{y}})\times\hat{\mathbf{z}} = 0\times\hat{\mathbf{z}}$ equals zero, which is different.

4-32 (a) The cross product inside the parentheses results in a vector, and the second cross product results in a vector. (b) The cross product gives a vector, and then dotting it with the other vector gives a scalar. (c) The dot product gives a scalar, but you can't take a cross product of a scalar with a vector. This one is nonsense.

4-33 (a) Since it's infinitesimally small, we can aproximate it as a rectangle. Its dimensions are $\mathrm{d}r$ and (using the definition of radian measure) $r\,\mathrm{d}\theta$, so the area is $\mathrm{d}A = r\,\mathrm{d}r\,\mathrm{d}\theta$.
(b) Now we get an (approximately) rectangular box with height $\mathrm{d}z$. Multiplying by the height of the box gives a volume $\mathrm{d}V = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$.
(c) The flavor of this problem is very similar to that of the calculation of the moment of inertia of a disk, $(1/2)Mr^2$, in section 4.2.5. In fact, we can save quite a bit of time by slicing the cone into disks and just integrating over all the disks. Each disk has infinitesimal height $\mathrm{d}z$ and mass $\mathrm{d}M$. It turns out to be convenient to pick a coordinate system in which $z = 0$ is the tip of the cone, and the cone extends to $z = h$ (i.e., if we want to visualize the positive z axis as being up, then the cone is upside-down). The advantage of this choice is that the surface of the cone is defined by the simple equation $r = (b/h)z$.

$$\begin{aligned}
I &= \int \frac{1}{2}\,\mathrm{d}Mr^2 \\
&= \int_0^h \frac{1}{2}(\rho A\,\mathrm{d}z)r^2 \\
&= \int_0^h \frac{1}{2}\rho\pi r^2\,\mathrm{d}zr^2 \\
&= \frac{1}{2}\rho\pi\left(\frac{b}{h}\right)^4\int_0^h z^4\,\mathrm{d}z \\
&= \frac{\pi}{10}\rho b^4 h
\end{aligned}$$

Our answer is in terms of ρ, but we're supposed to express it in terms of M. We can either look up the formula for the volume of a cone, $(1/3)\pi b^2 h$, or evaluate it using a similar integral, omitting a factor of $(1/2)\rho r^2$ in the integrand. The final result is

$$I = \frac{3}{10}Mb^2 \qquad .$$

It makes sense that the result depends only on b, not h, since changing h without changing the mass would not change how far any of the mass was from the axis. It also makes sense that the numerical factor of 3/10 in front is smaller than the factor of 1/2 for a disk; a cone of radius b has more of its mass closer to the axis than does a disk of radius b.

4-34 This is exactly the same as the calculation of the moment of inertia of a cube in section

4.2.5, except for the limits of integration.

$$\begin{aligned}
I &= \rho \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} \left(y^2+z^2\right) \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}x \\
&= \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} \left(y^2+z^2\right) \mathrm{d}y\ \mathrm{d}z \\
&= \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} y^2\,\mathrm{d}y\ \mathrm{d}z + \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} z^2\,\mathrm{d}y\ \mathrm{d}z \\
&= \rho ab \int_{-c/2}^{c/2} y^2\,\mathrm{d}y\ + \rho ac \int_{-b/2}^{b/2} z^2\,\mathrm{d}z \\
&= \frac{1}{12}\rho abc^3 + \frac{1}{12}\rho acb^3 \\
&= \frac{1}{12}\rho abc(b^2+c^2) \\
&= \frac{M}{12}(b^2+c^2)
\end{aligned}$$

The result doesn't depend on a, and that makes sense, because changing a without changing the mass wouldn't change how far any of the mass was from the axis.

4-35 (a) The result from problem 19 was $E = L^2/2I$. This suggests plotting E as a function of L^2, which should give a line with a slope of $1/2I$. Since my spreadsheet software was rounding L^2 off to zero, I actually scaled both variables by powers of 10 before plotting them, as indicated on the labels on the axes.

The plot is indeed a pretty good straight line, although some small deviations from linearity are apparent. The fact that the y-intercept is nonzero is an indication that the form of the data is not quite $E \propto L^2$, which would give $E = 0$ for $L = 0$. The actual data do have $E = 0$ at $L = 0$, since all energies were measured relative to the lowest one. But since the graph has some curvature to it, the best-fit line doesn't go through the origin.

The slope is .1834, which has to be multiplied by 10^{-14+68}, giving $I = 2.7 \times 10^{-54}$ kg·m^2.

(b) The total volume of this nucleus is about 168 cubic femtometers, corresponding to a sphere with a radius of about 3 fm, and the moment of inertia of a sphere gives $I = (2/5)mr^2 \approx 1 \times 10^{-54}$ kg·m^2. The actual moment of inertia of the nucleus is about three times this value, which isn't bad agreement at all, considering that we're trying to describe a quantum-mechanical system using classical physics.

4-36 (a)

$$\begin{aligned}
(\mathbf{b}\times\mathbf{c})_y &= b_z c_x - c_z b_x \\
(\mathbf{b}\times\mathbf{c})_z &= b_x c_y - c_x b_y \\
(\mathbf{a}\times(\mathbf{b}\times\mathbf{c}))_x &= a_x(b_z c_x - c_z b_x) - a_z(b_x c_y - c_x b_y) \\
&= b_x(a_y c_y + a_z c_z) - c_x(a_y b_y + a_z b_z)
\end{aligned}$$

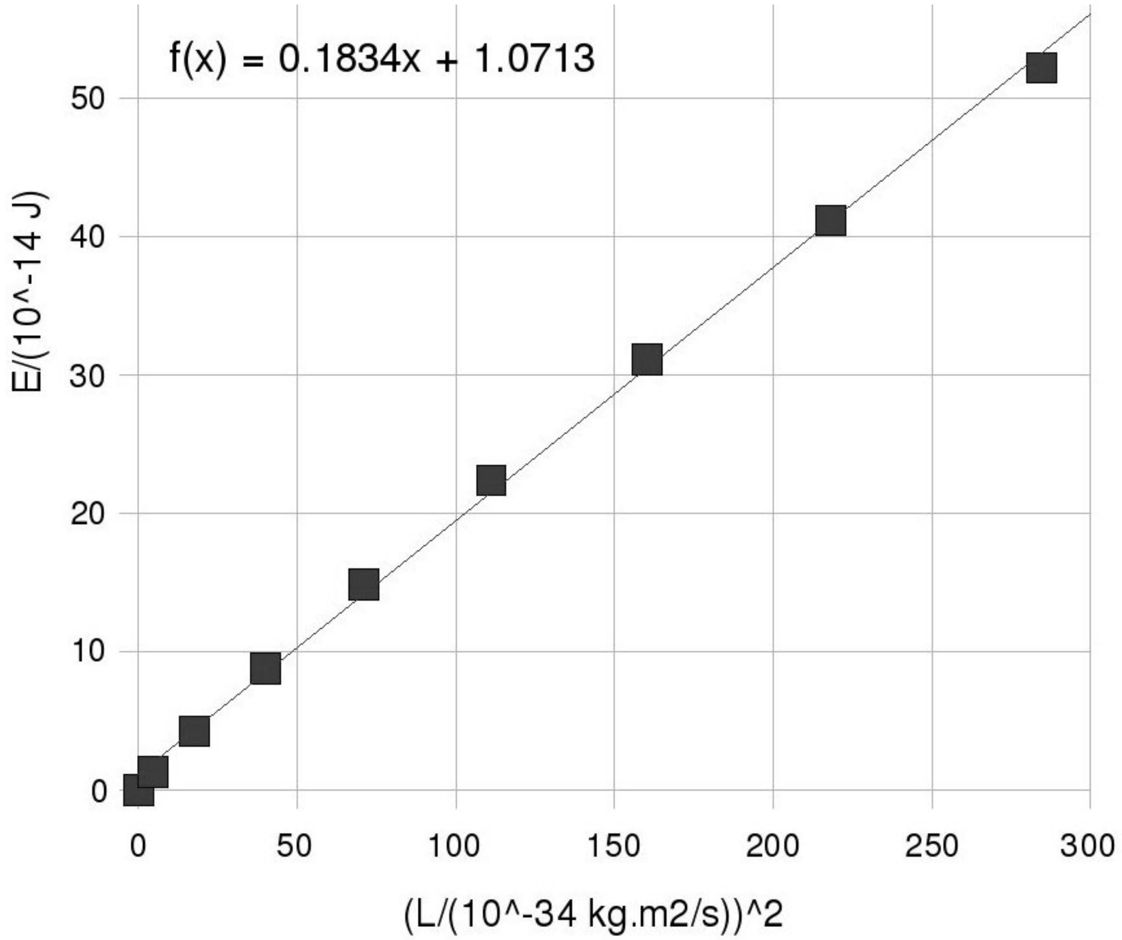
This can be put in the desired form by adding $a_x b_x c_x$ to the first term and subtracting it from the second.

(b) In the case where $\boldsymbol{\omega} = \omega\hat{\mathbf{x}}$, applying the rule found in part a to the integrand results in $\omega r^2 - x^2\omega = \omega(y^z + z^2)$.

(c) Tackled by brute force, this is messy but straightforward. To get the result much more simply, let's start with a sphere of radius 1, whose moment of inertia is $(2/5)m$. Since this equals $\int (y^2+z^2)\,\mathrm{d}m$, by symmetry we must have $\int y^2\,\mathrm{d}m = \int z^2\,\mathrm{d}m = (1/5)m$. What happens if we stretch it out along each axis, increasing its length along the x axis by a factor of a, and similarly for the other two axes? In the integral $\int y^2\,\mathrm{d}m$, the stretching has had the effect of increasing the integrand by a factor of b^2, so it contributes $(1/5)mb^2$, and similarly for the z^2 integral. The results for the diagonal elements of the moment of inertia matrix are

$$\begin{aligned}
I_{xx} &= \frac{1}{5}m(b^2+c^2) \qquad , \\
I_{yy} &= \frac{1}{5}m(a^2+c^2) \qquad \text{, and} \\
I_{zz} &= \frac{1}{5}m(a^2+b^2) \qquad .
\end{aligned}$$

4-38 Let the origin of our coordinate system be at the center of mass. By the assumption that the axes are parallel, we can ignore the existence of a dimension parallel to the axes, i.e., we can assume without loss of generality that the mass distribution is confined to a certain plane perpendicular to both axes. Offsetting the axis by a displacement $\mathbf{h}$ lying in this plane then amounts


f(x) = 0.1834x + 1.0713
E/(10^-14 J)
(L/(10^-34 kg.m2/s))^2
0
10
20
30
40
50
0
50
100
150
200
250
300

Problem 4-35.

to simply making the change $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{h}$ in the integral for the moment of inertia.

$$\begin{aligned} I &= \int (\mathbf{r}+\mathbf{h})^2 \,\mathrm{d}m \\ &= \int r^2 \,\mathrm{d}m + 2\mathbf{h}\cdot\int \mathbf{r}\,\mathrm{d}m + h^2 \int \mathrm{d}m \\ &= I_\mathrm{o} + 0 + mh^2 \qquad , \end{aligned}$$

where the cross term vanishes because it's the same integral that occurs in the definition of the center of mass, and the center of mass is at the origin.

4-39 (a) Let $\mathbf{r} = u\mathbf{A} + v\mathbf{B}$, where the interior of the triangle is defined by $u > 0$, $v > 0$, and $u + v < 1$. The parallelogram-shaped region defined by the infinitesimal intervals $\mathrm{d}u$ and $\mathrm{d}v$ then has area $\mathrm{d}A = |\mathbf{A}\times\mathbf{B}|\,\mathrm{d}u\,\mathrm{d}v$. The position of the center of mass is then given by

$$\begin{aligned} \mathbf{r}_{cm} &= \frac{\int \mathbf{r}\,\mathrm{d}A}{\int \mathrm{d}A} \\ &= \frac{\int_0^1 \int_0^{1-v} (u\mathbf{A}+v\mathbf{B})\,\mathrm{d}u\,\mathrm{d}v}{\int_0^1 \int_0^{1-v} \mathrm{d}u\,\mathrm{d}v} \qquad , \end{aligned}$$

where we can treat the differential of area $\mathrm{d}A$ as if it were the differential of mass $\mathrm{d}m$ because the mass per unit area cancels out. The numerator and denominator can both be split up into two similar integrals, and straightforward calculus gives the result as claimed.

(b) Here we do need to explicitly introduce the mass per unit area $2m/|\mathbf{A}\times\mathbf{B}|$, but we get a cancellation between this factor of $|\mathbf{A}\times\mathbf{B}|$ and the one arising from the relation between $\mathrm{d}A$ and $\mathrm{d}u\,\mathrm{d}v$. The moment of inertia is

$$\begin{aligned} I &= 2m \int_{-1/2}^{1/2}\int_{-1/2}^{1/2} r^2 \,\mathrm{d}u\,\mathrm{d}v \\ &= 2m \left[\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} A^2u^2 \,\mathrm{d}u\,\mathrm{d}v + \ldots + 0 \right] \qquad , \end{aligned}$$

where $\ldots$ represents a similar term for vector $\mathbf{B}$, and 0 is a cross-term that vanishes. The integral is separable, and the result is as claimed.

(c) Let I_o be the moment of inertia calculated in part b, and let $h = (1/6)|\mathbf{A}+\mathbf{B}|$ be the distance between the quadrilateral's center of mass and the center of mass of one of the triangles into which it can be split up, which follows from part a. Then $2(I_\mathrm{o} + mh^2) = (1/6)m(A^2 + B^2)$, and solving for I_o gives the result claimed.

4-40

$$\begin{aligned} I &= \int_0^\ell r^2 \,\mathrm{d}m \\ &= \int_0^\ell r^2 \frac{\mathrm{d}m}{\mathrm{d}r}\,\mathrm{d}r \\ &= \mu \int_0^\ell r^2 e^{r/\ell}\,\mathrm{d}r \end{aligned}$$

Although this looks like a definite integral because it has limits of integration, it's really more like an indefinite integral in the sense that the upper limit of integration is a variable. By switching to the unitless variable $u = r/\ell$, we can transform this into a definite integral that we'd be more likely to be able to find in a table of integrals, and this also has the advantage of making the calculus less messy. The result is

$$I = \mu\ell^3 \int_0^1 u^2 e^u \,\mathrm{d}u \qquad .$$

The units are all in the factor in front: (kg/m)(m^3) = kg·m^2, which is correct for a moment of inertia. The integral can be done by integration by parts, by looking it up in a table, or using computer software.

$$\begin{aligned} I &= \mu\ell^3 \, e^u(u^2 - 2u + 2)\big|_0^1 \\ &= (e-2)\mu l^3 \end{aligned}$$

4-41 I don't claim that this is a complete or optimal solution, but this is what I came up with. Rolling a coin and a ring down a book, I observed three different behaviors, which seemed to be determined by the angle θ of the book. At low angles, the coin tips over. At moderate angles, it rolls stably without slipping. At higher angles, it seems to slip, but the motion was fast so I had a hard time telling for sure. Ignoring differences in moments of inertia for rings versus disks, etc., the tipping motion is basically that of an inverted pendulum, whose characteristic time-scale is $\tau \sim \sqrt{\ell/g}$, where ℓ is a linear measure such as the radius. The motion is gyroscopically stable only when the disk's period of rotation T is short enough, and since τ is the only relevant parameter with units of time, I think the requirement must be something like $T \lesssim (\ldots)\tau$, where $(\ldots)$ indicates a unitless constant. Therefore we need the wheel to accelerate to this rate of rotation in a time that is $\lesssim \tau$. The time required to accelerate to this speed is $(\ldots)r/Tg\sin\theta$, where $(\ldots)$ is another unitless factor, which depends on the moment of inertia. Equating this to $\sqrt{r/g}$, the factors of r and g go away, and we're left with something of the form $\sin\theta = \ldots$. In other words, for a type of wheel such as a disk, there should be some universal minimum angle θ_{disk} that makes it roll without toppling. This angle is independent of the radius of the disk and the acceleration of gravity. It's not obvious to me whether or not this angle can be defined in any completely rigorous way, since it may depend on, e.g., the precision with which we can start the disk in a vertical plane.

4-42 Newton's second law applied to the hanging mass gives

$$mg - T = ma,$$

where T is the tension in the string. The torque acting on the platform is TR, so

$$\alpha = TR/I.$$

The relationship between the linear acceleration of the string and the angular acceleration of the platform is

$$a = R\alpha.$$

These are three equations in the unknowns α, T, and I. Eliminating α and T gives

$$I = mR^2\left(\frac{g}{a} - 1\right).$$

4-43 Let positive x be to the left, positive y out of the page, and opsitive z up. Let the axis be at the universal joint. The total torque is

$$\begin{aligned}&\frac{1}{2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})\times(-\hat{\mathbf{z}})\\ &+\left(\frac{1}{2}\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}}\right)\times(a\hat{\mathbf{z}})\\ &+(\hat{\mathbf{x}}+\hat{\mathbf{z}})\times(-b\hat{\mathbf{y}})\\ &+\left(\frac{1}{2}\hat{\mathbf{y}}+\hat{\mathbf{z}}\right)\times(-c\hat{\mathbf{x}}).\end{aligned}$$

Computing the cross products and setting this equal to zero gives

$$\begin{aligned}0 = &\left(-\frac{1}{2}+a+b\right)\hat{\mathbf{x}}\\ &+\left(\frac{1}{2}-\frac{1}{2}a-c\right)\hat{\mathbf{y}}\\ &+\left(-b+\frac{1}{2}c\right)\hat{\mathbf{z}}.\end{aligned}$$

This is a system of three equations in the three unknowns. The solution is $a = 1/3$, $b = 1/6$, and $c = 1/3$.

4-44 (a) Suppose that $\mathbf{u}$ is a solution. Then if $\boldsymbol{\delta}$ is any vector for which $\mathbf{a}\cdot\boldsymbol{\delta} = 0$, then $\mathbf{u}+\boldsymbol{\delta}$ is also a solution. Thus if we fix the tail of $\mathbf{u}$ at the origin, its tip traces a plane perpendicular to $\mathbf{a}$. We also need to show that a solution exists. Since $\mathbf{a}$ is nonzero, one solution is $\mathbf{a}b/|\mathbf{a}|^2$.

(b) The demonstration of nonuniqueness is similar to the one for the dot product, except that now we want $\boldsymbol{\delta}$ *parallel* to $\mathbf{a}$. The solution set forms a line parallel to $\mathbf{a}$. Existence follows because given any line segment with nonzero length, we can form a (possibly degenerate) rectangle with a given area and having that segment as one of its sides.

(c) Because the plane and line described in parts a and b are perpendicular, they have a unique point of intersection.

4-46 The moment of inertia of the rod for rotation about its center is $(1/12)m\ell^2$, where m is the mass of the rod. Using the parallel axis theorem, its moment of inertia about the fulcrum is $(1/12)m\ell^2+mb^2$. The gravitational torque, with the fulcrum as the axis, is mgb. The resulting angular acceleration is $\alpha = \tau/I$. The tangential acceleration at the bug's position is $(\ell/2 + b)\alpha$, and setting this equal to g gives $b = \ell/6$.

4-47 The moment of inertia for a uniform rod about one end is $(1/3)m\ell^2$. The torque due to gravity is $(1/2)mg\ell$. The resulting angular acceleration is $(3/2)g/\ell$. The tangential acceleration at the bug's position is $a = b\alpha$. Subtracting this from g gives

$$g_a = \left(1 - \frac{3b}{2\ell}\right) g.$$

It is perhaps a little surprising that this can be negative for $b > (2/3)\ell$, meaning that the bug will lose contact with the door unless its feet are sticky enough.

5-1 (a) According to the ideal gas law, $PV = nkT$, standard values of pressure and temperature give $V \propto n$, since k is also a constant.

(b)

$$\begin{aligned}
V &= nkT/P \\
&= (6.0 \times 10^{23})(1.38 \times 10^{-23}\ \text{J/K})(273\ \text{K}) \\
&\qquad /(1.01 \times 10^5\ \text{Pa}) \\
&= 2.2 \times 10^{-2}\ \text{J/Pa} \\
&= 2.2 \times 10^{-2}\ \text{N}\cdot\text{m}/(\text{N/m}^2) \\
&= 2.2 \times 10^{-2}\ \text{m}^3 \\
&= 2.2 \times 10^{4}\ \text{cm}^3 \\
&= 22\ \text{liters}
\end{aligned}$$

5-2 The motion of the piston through an infinitesimal distance $\mathrm{d}x$ will increase the interior volume of the cylinder by $A\,\mathrm{d}x$, where A is the cylinder's cross-sectional area. The work done is $\mathrm{d}W = F\,\mathrm{d}x = (PA)(\mathrm{d}V/A) = P\,\mathrm{d}V$.

Note that the result is not true in general for finite rather than infinitesimal changes. If we attempt to carry through the same proof for finite changes, it fails because $W \neq F\Delta x$ when F depends on x.

5-3 (a) Looking up the relevant masses, the result is 6.7×10^{-27} kg. (The electrons' masses are nearly negligible.)

(b) n=1 kg/(6.7×10^{-27} kg)= 1.5×10^{26}

(c) Approximating helium as an ideal gas, we can ignore the interactions between atoms, so there is no electrical energy to take into account, only kinetic. The thermal energy of the gas is $E = n\bar{K} = (3/2)nkT$. A change in temperature of one degree requires a change in energy of $(3/2)nk(1\ \text{K}) = 3.1 \times 10^3\ (\text{J/K})\text{K} = 3.1 \times 10^3\ \text{J}$.

5-4 The ideal gas law says $PV = nkT$. The volume is the same as before, and the temperature is the same, since the gas is back in thermal equilibrium with the room. Since n is doubled, P must be doubled as well.

5-5

$$\begin{aligned}
P &= \frac{n}{V}kT \\
&= \left(10^{-5}\ \text{cm}^{-3}\right)\left(1.38 \times 10^{-23}\ \text{J/K}\right)\left(10^3\ \text{K}\right) \\
&= \left(10\ \text{m}^{-3}\right)\left(1.38 \times 10^{-23}\ \text{J/K}\right)\left(10^3\ \text{K}\right) \\
&\sim 10^{-19}\ \text{J}\cdot\text{m}^{-3} \\
&= 10^{-19}\ \text{N}\cdot\text{m}\cdot\text{m}^{-3} \\
&= 10^{-19}\ \text{Pa}
\end{aligned}$$

5-6 (a)

$$\begin{aligned}
P_{surface} - P_{center} &= \int_{r=0}^{b} \mathrm{d}P \\
&= -\int_{r=0}^{b} \rho g\,\mathrm{d}r
\end{aligned}$$

The pressure at the surface is essentially zero, so

$$\begin{aligned}
P_{center} &= \int_{r=0}^{b} \rho g\,\mathrm{d}r \\
&= \int_{r=0}^{b} \rho\frac{Gmr}{b^3}\,\mathrm{d}r \\
&= \left.\frac{G\rho m r^2}{2b^3}\right]_{r=0}^{b} \\
&= \frac{G\rho m}{2b} \\
&= \frac{3Gm^2}{8\pi b^4}
\end{aligned}$$

(b) The units of Gm^2/b^2 are newtons, so the units of Gm^2/b^4 are N/m^2.

(c) 1.7×10^{11} Pa.

(d) Atmospheric pressure is 10^5 Pa, which is, as expected, about six orders of magnitude less than the answer to part c.

5-7 (a) The condition for a molecule to escape is that its kinetic energy has to equal the amount of gravitational energy it would gain while escaping. The kinetic energy depends on v^2, and the gravitational energy on M/r, so the ratio of the escape velocities is

$$\begin{aligned} \frac{v_m}{v_e} &= \sqrt{\frac{M_m r_e}{M_e r_m}} \\ &= 0.22 \qquad \text{[ratio of escape velocities]} \end{aligned}$$

(b) The absolute temperature is proportional to the average kinetic energy per molecule, so

$$\begin{aligned} \frac{v_m}{v_e} &= \sqrt{\frac{403\ \text{K}}{273\ \text{K}}} \\ &= 1.21 \qquad \text{[ratio of average velocities]} \end{aligned}$$

(c) For a given temperature, the average kinetic energy is a fixed value, so the mass is proportional to the inverse square of the velocity. If the temperatures on the earth and moon were the same, then the mass of the fluorocarbon molecule would have to be greater by a factor of $(0.22)^{-2}$. The higher temperature on the moon means we need the fluorocarbon molecules to be heavier by an additional factor of $(1.21)^2$. The result is that we need a mass ratio of $(0.22)^{-2}(1.21)^2 = 30$.

The atomic masses involved are N=14.0, F=19.0, C=12.0.

$$30(2 \times 14) = (12.0)n + (19.0)(2n + 2)$$

$$n \approx 16$$

So if some super-advanced alien civilization decided to cover the moon with an atmosphere of $C_{16}F_{34}$, you'd be able to walk around on the surface with only an oxygen mask!

5-8 (a)

$$\begin{aligned} \text{eff} &= \frac{Q_L}{W} \\ &= \frac{Q_L}{Q_H - Q_L} \qquad \text{[cons. of energy]} \\ &= \left(\frac{Q_H}{Q_L} - 1\right)^{-1} \\ &= \left(\frac{T_H}{T_L} - 1\right)^{-1} \qquad \text{[Carnot engine]} \\ &= \left(\frac{308\ \text{K}}{300\ \text{K}} - 1\right)^{-1} \\ &= 38 \\ \text{EER} &\approx (10/3)(\text{eff}) \\ &\approx 150 \end{aligned}$$

(b)

$$\begin{aligned} \Delta S &= \frac{Q_H}{T_H} - \frac{Q_L}{T_L} \\ &= \frac{W + Q_L}{T_H} - \frac{Q_L}{T_L} \qquad \text{[cons. of energy]} \\ &= W\left(\frac{1 + \text{eff}}{T_H} - \frac{\text{eff}}{T_L}\right) \\ &= 3 \times 10^4\ \text{J/K} \end{aligned}$$

The result is positive, so it's consistent with the second law.

5-15 (a) The equipartition theorem says that we get $(1/2)kT$ worth of energy for every term in the energy that is the square of a variable. The variables here are the n momenta of the particles, so we have $E = (1/2)nkT$. That is, the heat capacity is one third that of a gas in three dimensions.

(b) The number of dimensions matter because in three dimensions, if you want to heat the gas, you have to supply enough energy to create more motion along all three axes.

(c) The energy does not break down into terms that involve squares of variables. Therefore the equipartition theorem does not hold. The heat capacity will not be the same as in part a, and the equipartition theorem doesn't give us any easy way to calculate it.

5-16 The energy of 1.3 J equals the amount Q gained by the spoon and minus the amount $-Q$ lost by the coffee. The change in entropy is

$$\Delta S = \frac{Q}{T_{\text{spoon}}} - \frac{Q}{T_{\text{coffee}}} = 7.2 \times 10^{-4} \text{ J/K}.$$

5-17 The heat required in order to raise the temperature of a monoatomic gas by ΔT is $(3/2)nk\Delta T$. The rate of temperature increase is therefore

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= \frac{2P}{3nk}\\ &= 1.5 \times 10^{-8} \text{ K/s}.\end{aligned}$$

I was surprised by how small this was. It would take several years to heat by one degree.

5-18 (a) By equipartition, the average kinetic energy is $(3/2)kT$.

(b) Equating this to $(1/2)mv^2$ gives $\overline{v^2} = 3kT/m$. This comes out large because m is so small for an electron compared to an atom.

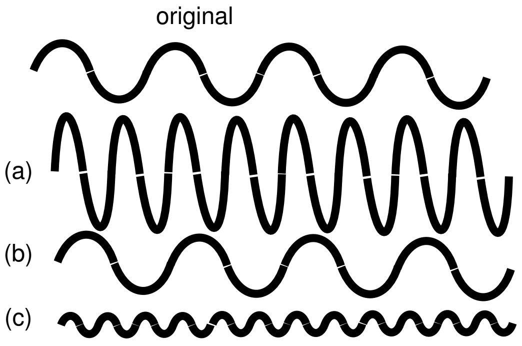
6-1 $T = 1/f = 3.82$ ms, and $\lambda = v/f = (340 \text{ m/s})/f = 1.3$ m.

6-2 Since it's a graph of the wave as a function of position, what it shows is amplitude and wavelength.

(a) The amplitude is doubled, and $\lambda = v/f$, so the wavelength is cut in half.

(b) Doubling both velocity and frequency leaves the wavelength unchanged. This is exactly the same as the orginal wave.

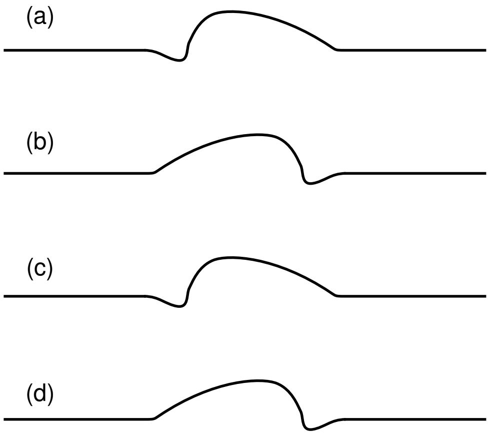
(c) The information about the velocity is irrelevant. We're given the wavelength and amplitude directly, so that's all we need to show.



6-3 (a) The scale on the time axis is not comparable to the scale of the distance axis, so there is no reason why your answer has to be the same width on the page. As the pulse passes through a certain point traveling to the right, the water will first go down a little, then up, so the graph should look similar, but reversed.

(b) As the pulse travels to the left, the water will first do the big, slow rise, then the small, short dip. The graph is not reversed compared to the original.

(c) and (d) are the same idea – the roles of question and answer are just reversed.



6-4 (a) To keep the stuff inside the cosine the same, we'd have to decrease x while t increased. The wave is moving in the negative x direction.

(b) At a particular point, the quantity $0.45\pi t$ has to change by 2π for one cycle to go by, so $T = 2\pi/0.45\pi = 4.4$, which we assume represents 4.4 seconds, since we're told that everything is set up for SI units. The frequency is $f = 1/T = 0.23$ Hz. The relationship between x and the wavelength is the same as the relationship between t and the period, so $\lambda = 2\pi/0.73\pi = 2.7$ m.

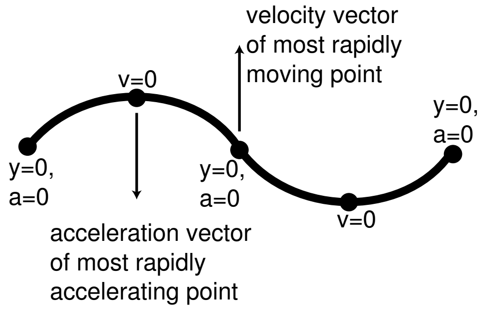
(c) We could do this by plugging in to $v = f\lambda$, but let's use a different approach. If x changes by Δx while enough time goes by for the inside of the cosine to stay the same, then $0.73\pi\Delta x + 0.45\pi\Delta t = 0$. The velocity is $\Delta x/\Delta t = -0.45/0.73 = 0.62$ m/s.

(d) The velocity of the point on the string is

$$\begin{aligned} y' &= -3.5\sin(0.73\pi x+0.45\pi t+0.37\pi)(0.45\pi) \\ &= -4.9\sin(0.73\pi x+0.45\pi t+0.37\pi) \qquad , \end{aligned}$$

which varies between −4.9 m/s and +4.9 m/s. This is completely different from the wave's velocity of 0.62 m/s, and is in the perpendicular direction.

6-5 The points with $v=0$ are the ones that have gone as far out to one side as they'll go, and are now ready to come back. The points that are whipping across the center line are the ones that are going the fastest, and they also have zero acceleration because their v vs t graph is at a peak, where the tangent line has zero slope.



6-6 (a) In the expression

$$\lambda' = \left(1-\frac{v_s}{v}\right)\lambda \qquad ,$$

we eliminate wavelength λ and replace it with v/f:

$$\frac{v}{f'} = \left(1-\frac{v_s}{v}\right)\frac{v}{f} \qquad ,$$

We simplify by dividing out the factor of v on both sides, and inverting both sides:

$$f' = \frac{f}{1-\frac{v_s}{v}}$$

(b) The ratio v_s/v is unitless, so it makes sense to subtract it from a unitless 1. Dividing a frequency by a unitless number gives a frequency.

(c) If $v_s=0$, then $f'=f$, which makes sense. For v_s greater than zero (but less than v), the denominator is less than one, so $f'>f$, indicating an upward frequency shift, which makes sense if positive v_s indicates motion toward the observer.

6-7

There are a couple of things that make it difficult to devise an accurate experiment: (1) it's hard to produce waves in a controlled way, and (2) it's hard to measure the height of the superposing waves in real time.

Possible solution: Find a machine that's capable of making spreading ripples of a consistent height, e.g., a wind-up toy car whose wheels go around. This takes care of problem 1. You could put two such cars in the water, and see if the peaks formed by constructive interference become twice as tall at a point equidistant from the two cars, compared to their height when only one car was turned on.

The toy cars also help to solve problem 2, because the waves are steady. You can put a ruler in the water, and watch the steady up-and-down vibration of the surface, measuring the height of the wave. This could be made easier by taking a video on a cell phone and then clicking through it frame by frame.

6-8 (a) Following the approach suggested in the hint in the back of the book, let the origin be at the center of the loop. The distance from the center to a point on the rope (x,y) is $\sqrt{x^2+y^2}$, which must equal the radius of the loop, so for y in terms of x we have $y=\sqrt{r^2-x^2}$. The second derivative with respect to x, evaluated at $x=0$ (where the rope is parallel to the x axis), comes out to be $-1/r$. The equation $\partial^2 y/\partial t^2 = (T/\mu)\partial^2 y/\partial x^2$ then gives $a=-T/\mu r$, where the minus sign indicates that the acceleration is in the negative y direction. The magnitude of the acceleration in circular motion is $\omega^2 r$, so $\omega^2 r = T/\mu r$, and $T=\mu\omega^2 r^2$.

(b) $v=\sqrt{T/\mu}=\omega r$. This means that the magnitude of the wave velocity is the same as the speed of a point on the rope due to its circular motion. A pulse traveling in the same direction as the rotation will have double this speed, but the cool thing is that a pulse traveling against the rotation stands still!

6-9 (a) The wave velocity at height x is $\sqrt{T/\mu}$, where the tension, T, depends on x. The tension in the string has to be enough to support the weight of the part of the string below it, so $T = \mu g x$, and $v = \sqrt{gx}$.

(b)

$$\begin{aligned} a &= \frac{\mathrm{d}v}{\mathrm{d}t} \\ &= \frac{\mathrm{d}v}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t} \\ &= \frac{\mathrm{d}v}{\mathrm{d}x} \cdot v \\ &= \frac{1}{2}(gx)^{-1/2} g \cdot v \\ &= g/2 \end{aligned}$$

(c) It has zero velocity at $x = 0$, but that doesn't mean it stops there and just disappears — that would violate conservation of energy. If you tried the experiment, you saw that the pulse turned around and headed back up the string. In other words, it has constant acceleration before, at, and after the moment when it reaches the tip.

6-10 1% of the speed of sound, or about 3 m/s

6-11 Due to the difference in speed, there will be partial reflection of the sunlight.

6-12 (a) Since the result only depends on the ratios of speeds, we can drop the powers of 10:

$$R = \frac{2.2 - 3.0}{2.2 + 3.0} = -0.15$$

(b) Energy is proportional to the square of amplitude, so the fraction of the energy reflected is $(0.15)^2 = 0.022$.

6-13 $L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{340\ \mathrm{m/s}}{2f} = 65\ \mathrm{cm}$

6-14 (a) These are all different standing wave patterns that fit within the tube. Notice how all the frequencies are multiples of 150 Hz. (Since we're getting all the multiples, not just the odd multiples, the saxophone apparently acts like a symmetric air column, and we can visualize the series of wave patterns as the one in which we fit some number of half-wavelengths in the length of the tube.)

(b) Higher frequencies correspond to shorter wavelengths. You can fit more than one half-wavelength in the tube, but you can't put less than one half-wavelength in, because the wave wouldn't be doing the right things at the ends.

6-15 (a)

C:	261.6	523.2	784.8	1046.4	1308.0
G:	392.0	784.0	1176.0	1568.0	1960.0

The 784.8 and 784.0 Hz frequencies differ by much less than 1%, and the 1046.4 and 1176.0 Hz frequencies differ by 12%. Neither combination is really in the 1-10% range that causes dissonance.

(b)

C:	261.6	523.2	784.8	1046.4	1308.0
Bb:	466.2	932.4	1398.6	1864.8	2331.0

The 1308.0 and 1398.6 Hz frequencies differ by about 7%, which is clearly going to make them dissonant.

6-16 (a) These are both reflections back into a faster medium. I don't even remember off the top of my head whether such a reflection is inverting or not, but even if it is inverting, two inversions in a row leave the wave with the same phase it had before. Since $d = 0$, the distance traveled by the two types of waves is the same, so their phase is the same when they emerge, and the interference is constructive.

(b) To get constructive interference again, we need the extra distance $2d$ traveled by the twice-reflected wave to equal one complete wavelength. In other words, $d = \lambda/2$.

(c) Each cycle means the top plate has dropped by a distance $\lambda/2$, so the diameter of the filament is $n\lambda/2$.

6-17 (a) For a medium with a fixed set of properties, the energy content of a wave is proportional to the square of its amplitude. The reflected wave is in the same medium as the original one, so the fraction of the energy that is reflected is R^2. The fraction transmitted is then $f = 1-R^2 = 1-[(\alpha-1)/(\alpha+1)]^2 = 4\alpha/(1+\alpha)^2$, which isn't the same as T^2.

(b) We now have $\alpha = v_2/v_1$ and $\beta = v_3/v_2$, and the fraction of the energy transmitted is

$$\frac{4\alpha}{(1+\alpha)^2}\cdot\frac{4\beta}{(1+\beta)^2} \qquad .$$

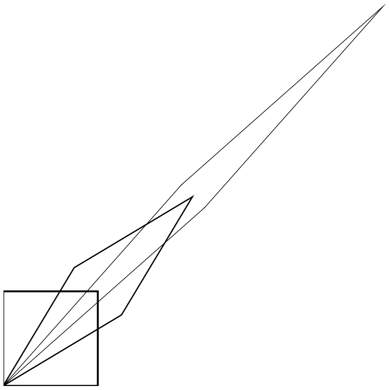
We have to maximize this quantity subject to the constraint $\alpha\beta = v_3/v_1 =$ constant. Since the variables α and β are treated completely symmetrically, we suspect that the result must be $\alpha=\beta$, in which case we have $v_2=\sqrt{v_1v_3}$.

To prove more rigorously that the solution is as claimed, it helps if we start with some simplifications. Constant factors like 4 don't affect the result, and in fact the constraint on the product $\alpha\beta$ guarantees that the entire numerator is constant. Thus maximizing the transmission amounts to simply minimizing the denominator. Since the denominator is a perfect square, all we really have to do is minimize its square root, which is $(1+\alpha)(1+\beta)$. Defining $x=\alpha\beta=v_3/v_1$ for ease of writing, and substituting x/α for β, the quantity to be minimized becomes $(1+\alpha)(1+x/\alpha) = 1+x+\alpha+x/\alpha$. The derivative with respect to α is $1-x\alpha^{-2}$, and setting this equal to zero gives $\alpha=\sqrt{x}=\beta$, as expected.

6-18 (a) We have $R=(\alpha-1)/(\alpha+1)$. Inverting α gives $R=(1/\alpha-1)/(1/\alpha+1)=(\alpha-1)/(1+\alpha)$, which means we've simply negated R. The interpretation is that the energy of the reflected pulse is unchanged, but the reflection's inverting or uninverting character is flipped.

(b) Based on part a, we expect that there will be two values of α that will do the job, and that they will be inverses of each other. Solving for α in terms of R gives $\alpha=(1+R)/(1-R)$. Taking $R=1/2$ gives $\alpha=3$, and without having to do the arithmetic for the case of the inverting reflection, we can tell that it will happen for $\alpha=1/3$.

7-1 To get good precision, you need to do this on a full-size printout, as suggested by the problem. To keep from using up a whole page on my solutions handouts, I've shown the idea below at about 25% of the actual size.



First I drew the long diagonal twice as long again, meaning about 276 mm. To keep the area the same, I had to make its "cross-bar" half as long as on the preceding parallelogram, or about 17.5 mm. The speed represented by this new parallelogram is equal to the slope of its bottom edge. To get this slope, which is the rise divided by the run, we have to draw the rise and run accurately, i.e., lined up straight with the edges of the paper. I did this by using a second piece of paper as a right angle. The result is that the slope is (rise)/(run) = 91 mm/105 mm = .86, representing 86% of the speed of light.

7-2 (a) She says her own velocity is zero.

Time dilation happens because of relative motion. We can conclude that A and B are not in motion relative to one another, but that there is relative motion between them and C.

(b) Alice says B is at rest, but C is moving. (c) Betty says A is at rest, but C is moving. (d) Cathy says both A and B are moving, and she sees them moving in the same direction at the same speed.

7-3 The equation for gamma is $\gamma=1/\sqrt{1-v^2}$. If v is negative, v^2 still comes out positive. Negative and positive values of v give the same results for gamma.

This makes sense, because positive and negative values of v indicate motion in opposite directions. Relativistic effects like time dilation and length contraction shouldn't come out different if you travel west rather than east, or north rather than south. Gamma indicates how strong these relativistic effects are, so gamma shouldn't depend on the direction of motion. This relates directly to the second postulate stated in section 7.2, that all directions in space have the same properties.

Note that it *is* OK for a directional (vector) quantity like velocity to be connected to some other direction quantity, such as acceleration, by a relation such as $a = dv/dt$. Then an observer who chooses the opposite orientation for their coordinate system will simply flip all the signs of the velocities and accelerations, and there is no way to tell in experiments which choice is "special." That's different from the situation with γ, which is not directional and can be measured without even choosing a coordinate system.

7-4 (a)

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-(v/c)^2}}\\ &= \frac{1}{\sqrt{1-\left(\frac{17000\ \text{m/s}}{3\times10^8\ \text{m/s}}\right)^2}}\\ &= 1.0000000016\end{aligned}$$

(b) Gamma gives us a comparison of the rate at which time flows on earth and on the probe. A gamma of one would indicate equal rates. Since the gamma is a tiny bit greater than one, the ratio is a tiny bit different from a one-to-one ratio. Gamma differs from one by 1.6×10^{-9}, i.e., 1.6 parts per billion. The disagreement between clocks on the probe and on earth that accumulates over each year is therefore

$$(31\times10^6\ \text{s})(1.6\times10^{-9}) = 0.050\ \text{s} \qquad .$$

This is a fairly big discrepancy. You might even be able to detect it with a consumer-grade clock. (The way NASA really notices the effect is that when the probe signals back to earth by radio, the radio waves vibrate a little more slowly than they should, i.e., the signal's frequency is shifted a tiny bit.)

7-5 Its kinetic energy is equal to its mass-energy minus its resting value,

$$\begin{aligned}K &= \mathcal{E} - mc^2\\ &= m(\gamma-1)c^2\\ &= (8.0\times10^7\ \text{kg})\left(\frac{1}{\sqrt{1-(1/2)^2}}-1\right)c^2\\ &= 1.1\times10^{24}\ \text{J}\end{aligned}$$

This is a thousand times greater than the total energy content of the world's nuclear arsenals. In other words, the Enterprise is the ultimate weapon of mass destruction. If it crashed into a planet (as it did, in one of the movies), it would destroy all life on the planet's surface.

7-6 Let $r = 6.4\times10^6$ m be the radius of the earth, $R = 1.5\times10^{11}$ m the radius of its orbit, and $T = 1$ year $= 3.2\times10^7$ s the period of its orbit. Then the velocity is $v = (2\pi R)/T$, and we have $\gamma = 1/\sqrt{1-v^2/c^2} = 1 + 4.8\times10^{-9}$. The resulting contraction of the diameter is about $(4.8\times10^{-9})(2r) = 6$ cm.

7-7 (a) $v = \text{(slope)} = \text{(rise)/(run)} = x/\gamma$, so $x = v\gamma$. (b) Because we're using units in which $c = 1$, the diagram has symmetry with respect to reflection across the 45-degree diagonal. Therefore the (t, x) coordinates of P and Q are simply related by a swap of t and x, and Q has coordinates $(v\gamma, \gamma)$. (c) Points P and Q are separated by $|\Delta t| = (1-v)\gamma$, and $|\Delta x| = (1-v)\gamma$ as well. The Pythagorean theorem results in $\text{PQ} = \sqrt{2}(1-v)\gamma$. (d) Averaging the coordinates of points P and Q, we find that C has coordinates $x = t = (1+v)\gamma/2$. The Pythagorean theorem then gives $\text{OC} = (1+v)\gamma/\sqrt{2}$. (e) The area of triangle PQO is (1/2)(base)(height) $= (1/2)\text{PQ}\cdot\text{OC} = (1/2)(1-v)(1+v)\gamma^2$, which simplifies to $(1/2)(1-v^2)\gamma^2$. Doubling this to find the area of the quadrilateral, we obtain $(1-v^2)\gamma^2$, and setting this equal to 1 and solving for γ gives the desired result.

7-8 (a) In units of 10^{-27} kg, the amount of mass lost is

$$1.67495 - 1.67265 - 0.00091 = 0.00139 \qquad ,$$

i.e., 0.00139×10^{-27} kg. This mass has been converted into energy:

$$\begin{aligned} E &= mc^2 \\ &= (0.00139 \times 10^{-27})(3 \times 10^8)^2 \\ &= 1.25 \times 10^{-13} \text{ joules} \quad . \end{aligned}$$

(In the meter-kilogram-second system of units, kilograms multiplied velocity squared give units of joules.)

(b) For this process, the total mass changes from 1.67265 to 1.67586 (again in units of 10^{-27} kg). This is an *increase* in mass, which means that we would have to have a source of energy to make the reaction happen. A free proton has no such source of energy; it's all by itself, so there's nothing for it to get energy from.

7-9 (a)

$$\begin{aligned} p &= m\gamma v \\ &= \frac{mv}{\sqrt{1-v^2}} \\ p^2\left(1-v^2\right) &= m^2v^2 \\ p^2 &= \left(m^2+p^2\right)v^2 \\ v &= \frac{p}{\sqrt{m^2+p^2}} \end{aligned}$$

(b) At low velocities, the second term inside the square root is negligible, and we have

$$\begin{aligned} v &\approx \frac{p}{\sqrt{m^2}} \\ &= \frac{p}{m} \quad , \end{aligned}$$

which is the nonrelativistic result.

(c) For very large momenta, the m^2 term is negligible compared to the p^2 term, so

$$\begin{aligned} v &= \frac{p}{\sqrt{p^2}} \\ &= \pm 1, \end{aligned}$$

which is the speed of light in natural units.

(d) Converting to SI units, we have

$$v = \frac{p}{\sqrt{m^2+p^2/c^2}}.$$

7-10 (a)

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1-v^2/c^2}} \\ &= \frac{1}{\sqrt{1-9/25}} \\ &= \frac{1}{\sqrt{16/25}} \\ &= \frac{5}{4} \end{aligned}$$

(b) The $(3/5)c$ example came out simple because 3, 4, and 5 are integers that have the relationship $3^2+4^2=5^2$. This is just like a 3-4-5 right triangle, but there's no triangle involved here! We need to find another set of numbers like this. The simplest way is to reverse the role of the 3 and 4, in which case we get

$$v = \frac{4}{5} \quad , \quad \gamma = \frac{5}{3}$$

If that seems like a cheap shot, then you need to find another set of integers that form a right triangle; I was lazy, and just did a Google search for "Pythagorean triple," which turned up $11^2 + 60^2 = 61^2$ as an example. This provides the examples

$$v = \frac{11}{61} \quad , \quad \gamma = \frac{61}{60}$$

and

$$v = \frac{60}{61} \quad , \quad \gamma = \frac{61}{11} \quad .$$

7-11 (a) If v is very close to c then the denominator in $\gamma = 1/\sqrt{1-v^2/c^2}$ gets very close to zero. Dividing by a very small number gives a very big result. Ultrarelativistic particles have very large values of γ.

(b) The energy due to its mass is mc^2, which is a big number because c is big. However, the energy due to its motion equals $m(\gamma-1)c^2$, which is even bigger, because not only does it have the c^2 factor, it also has $\gamma - 1$, which is big. The particle has much more energy due to its motion

than due to its mass.
(c) $\mathcal{E}/p = m\gamma c^2/m\gamma v = c^2/v$.
(d) For $v = c$, this becomes $\mathcal{E}/p = c^2/c = c$.
(e) The beam of light was emitted with a power of 1 watt, i.e., 1 joule per second. Since it lasted for one second, its energy is one joule.

$$\begin{aligned} p &= \mathcal{E}/c \\ &= (1\ \text{J})/(3\times 10^8\ \text{m/s}) \\ &= (1\ \text{kg}\cdot\text{m}^2/\text{s}^2)/(3\times 10^8\ \text{m/s}) \\ &= 3\times 10^{-9}\ \text{kg}\cdot\text{m/s} \end{aligned}$$

It has only a miniscule amount of momentum, which fits the correspondence principle: until Einstein came along, nobody even knew light had momentum at all.

7-12 (a) A piece of rope with length ℓ has volume $A\ell$ and mass $m = \rho A\ell$, so $\mu = m/\ell = \rho A$. Setting $v = \sqrt{T/\mu}$ equal to c and solving for T, we find $T = c^2\rho A$.
(b) Plugging in $\rho = 1\ \text{g/cm}^3 = 10^3\ \text{kg/m}^3$ gives a maximum tensile strength of $T/A = 10^{20}$ Pa, which is about a thousand times greater than that hypothesized for the strongest carbon nanotubes.
(c) Most likely the rope would break. The existence of a theoretical maximum strength for the rope also suggests that even the strongest rope theoretically possible would break.

7-15 Really this boils down to expanding γ in a Taylor series, and that task can be simplified a lot by letting $x = v^2$, and thinking of γ as a function of x. Then

$$\begin{aligned} \gamma &= (1-x)^{-1/2} \\ \gamma' &= \frac{1}{2}(1-x)^{-3/2} \\ \gamma'' &= \frac{3}{4}(1-x)^{-5/2} \end{aligned}$$

Evaluating these at $x = 0$, we get 1, 1/2, and 3/4, so the Taylor series is

$$\begin{aligned} \gamma &= 1 + \frac{1}{1!}\cdot\frac{1}{2}x + \frac{1}{2!}\cdot\frac{3}{4}x^2 + \ldots \\ &= 1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \ldots \\ K &= \frac{1}{2}mv^2 + \frac{3}{8}mv^4 + \ldots \qquad , \end{aligned}$$

which in ordinary units would be

$$K = \frac{1}{2}mv^2 + \frac{3}{8c^2}mv^4 + \ldots \qquad .$$

The odd terms vanish, because kinetic energy is a scalar that doesn't depend on the direction of motion. The first term is the nonrelativistic expression, as claimed.

7-16 We have $p = m\gamma v = m(1-v^2)^{-1/2}v$. A brute-force evaluation of the Taylor series for this expression can be done, but it's messy. One way to simplify the calculation is by evaluating the Taylor series for γ, and then simply multiplying by v. Now that we've reduced the problem to evaluating γ as a Taylor series, a second simplification is to let $x = v^2$, and think of γ as a function of x. Then

$$\begin{aligned} \gamma &= (1-x)^{-1/2} \\ \gamma' &= \frac{1}{2}(1-x)^{-3/2} \\ \gamma'' &= \frac{3}{4}(1-x)^{-5/2} \end{aligned}$$

Evaluating these at $x = 0$, we get 1, 1/2, and 3/4, so the Taylor series is

$$\begin{aligned} \gamma &= 1 + \frac{1}{1!}\cdot\frac{1}{2}x + \frac{1}{2!}\cdot\frac{3}{4}x^2 + \ldots \\ &= 1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \ldots \end{aligned}$$

Applying this to the expression for momentum, we have

$$\begin{aligned} p &= m\gamma v \\ &= mv\left(1 + \frac{1}{2}v^2 + \ldots\right) \\ &= mv + \frac{1}{2}mv^3 + \ldots \qquad , \end{aligned}$$

or, in ordinary units,

$$p = mv + \frac{1}{2c^2}mv^3 + \ldots \qquad .$$

The even-order terms vanish, because when v flips its sign, p must flip its sign as well. The first nonvanishing term is the newtonian expression mv.

7-18 Among the spacelike vectors, **a** and **e** are clearly congruent, because they're the same except for a rotation in space; this is the same as the definition of congruence in ordinary Euclidean geometry, where rotation doesn't matter. Vector **b** is also congruent to these, since it represents an interval $3^2 - 5^2 = -4^2$, just like the other two.

The lightlike vectors **c** and **d** both represent intervals of zero, so they're congruent, even though **c** is a double-scale version of **d**.

The timelike vectors **f** and **g** are not congruent to each other or to any of the others; **f** represents an interval of 2^2, while **g**'s interval is 4^2.

7-19 Since $m^2 = \mathbf{p}\cdot\mathbf{p}$, the three cases correspond to the possibility that m^2 is positive, negative, or zero. If the energy-momentum vector is timelike, then m^2 is positive, which just means that the mass is real and nonzero. If the vector is lightlike, m^2 must be zero, so the particle is massless. It's not possible for the vector to be spacelike, because then the mass would have to be an imaginary number.

7-20 *Compton scattering:* Here, as in the analysis of pair production in the example in the text, it's helpful to discuss the process in the special frame of reference (called the center of mass frame) in which the total momentum is zero, $p_\gamma = -p_e = p$. In this frame, the gamma ray and the electron collide head-on, so the most natural thing to guess for the outcome of the collision is that they both emerge along the same line, i.e., the whole process is confined to a single line. If this was a collision between two cars, we would expect a large amount of energy to be converted into other forms such as heat, sound, and permanent deformation of the cars. But none of this can happen with subatomic particles, so we expect the collision to be perfectly "bouncy" (technically the term is "elastic"). In Newtonian mechanics, in a perfectly elastic collision, the result in the center of mass frame is that both objects simply reverse their momentum vectors. This also works in Compton scattering: if we reverse each particle's momentum, then its four-momentum vector changes from $(\mathcal{E}, p)$ to $(\mathcal{E}, -p)$, and four-momentum is conserved:

$$(\mathcal{E}_\gamma, p) + (\mathcal{E}_e, -p) = (\mathcal{E}_\gamma, -p) + (\mathcal{E}_e, p)$$

We also need to check that $(\mathcal{E}_\gamma, -p)$ and $(\mathcal{E}_e, p)$ are still legal four-momentum vectors for the gamma and electron; they are, because $m^2 = \mathcal{E}^2 - p^2$ is unchanged when we flip the sign of p. The conclusion is that Compton scattering *can* occur between a gamma ray and an electron, regardless of the absence of any third particle.

Photoelectric effect: If the photoelectric effect could occur without the presence of any third particle such as an atomic nucleus, then the initial state would look just like the initial state of Compton scattering: an electron and a gamma ray. We've already analyzed a collision with this initial state by going into the center of mass frame. The final state of the photoelectric effect would be different, however, because only the electron would exist, the gamma having been absorbed. What would this final state look like in the center of mass frame? The electron would have to have zero momentum. But this would imply that it has *lost* mass-energy as a result of absorbing the gamma, which would violate conservation of mass-energy.

7-21 The first derivative of the function $D(v)$ is

$$D' = \frac{1}{2}\left(\frac{1-v}{1+v}\right)^{-1/2}\left[-\frac{1}{1+v} + (1-v)(-1)(1+v)^{-2}\right] \qquad .$$

Evaluating this at $v = 0$ gives $D'(0) = -1$. The first two terms of the Taylor series are

$$\begin{aligned} D(v) &\approx D(0) + D'(0)v \\ &= 1 - v \qquad . \end{aligned}$$

This is identical to the nonrelativistic Doppler shift for the wavelength detected from a source receding at velocity v relative to the medium (if v is expressed as a fraction of the wave's speed). (The nonrelativistic version is actually more complicated, because in principle both the source and the observer could be moving relative to the medium. For Doppler shifts of light, there is no medium, so clearly this can't be relevant. In the nonrelativistic case we get shifts of $1-v$ and $1/(1+v)$ for a moving source and a moving observer, but the latter's Taylor series is $1-v+\ldots$, which still matches the relativistic expression in its first two terms.)

7-23 As in figure f in section 27.1, the easy test is to figure out whether a piece of paper can be formed into the relevant shape without tearing or crumpling. We can form a piece of paper into a cone without tearing or crumpling it, so the answer is no, the creature could not detect any curvature. The only exception would be at the tip of the cone, because, for example, we can't extend a line beyond the tip, violating Euclid's second postulate.

7-24 (a) The number is supposed to represent the ratio of two times or the ratio of two frequencies, so it has to be unitless. The 1 is unitless, which makes sense. It remains to calculate the units of gh/c^2. These are $(\text{m/s}^2)(\text{m})/(\text{m/s})^2$, which does come out to be unitless.

(b) Yes. If the time dilation factor had been very different from 1 under ordinary conditions, then it would have been noticed hundreds of years ago, and nobody would have ever believed in the Newtonian concept of absolute time. But in reality, c is huge in metric units, while g and h in everyday life are not, so the time dilation factor will be very close to 1.

7-25 (a) The Doppler shift is a factor of $1-gh/c^2$, which corresponds to a fractional change of $gh/c^2 = 2.5\times10^{-15}$. (b) The predicted fractional effect is $(1340\ \text{m})g/c^2 = 1.5\times10^{-13}$. The observed result is $(22\ \text{ns})/(48\ \text{hr}) = 1.3\times10^{-13}$. These seem consistent to within the precision of the experiment, as inferred by the sizes of the random fluctuations in the graph.

7-26 The kinematic effect slows down time on the ISS relative to time on the ground, while the gravitational one is in the opposite direction. Let's start by calculating the gravitational time dilation. The gravitational field g does fall off with altitude, but the decrease at an altitude of 350 km is not really all that much (see problem 10-22), so let's use the surface value of about 10 m/s^2, which is good enough to get a reasonable estimate. The time dilation factor is $1-gh/c^2$, so the fractional change in the rate of time is $gh/c^2 \approx 4\times10^{-11}$. The kinematic time dilation factor is γ, and the corresponding fractional change is $\gamma-1$. Plugging in the velocity, this comes out to be about 4×10^{-10}. We find that the kinematic time dilation is an order of magnitude greater than the gravitational one, so over all, time runs more slowly on the ISS than on the ground.

7-27 Equating the escape velocity $\sqrt{2GM/r}$ to c and solving for r we find that $r = 2GM/c^2$ comes out to be roughly a centimeter.

7-29 The easiest one to use is an octant, like the one bounded on the earth's surface by the equator, the Greenwich meridian, and the 90° − *east* meridian. It has an area of $4\pi R^2/8 = (1/2)\pi R^2$ and an angular defect of $\pi/2$, so we find that $d = A/R^2$.

7-30 (a) As the relative velocity gets closer and closer to c, the length of the long diagonal of the parallelogram approaches infinity. But the area is supposed to stay the same, so this means the short diagonal's length must approach zero. The parallelogram becomes a line. But one of our basic rules here is that observers in different frames of reference must always agree on whether or not events are the same, and this rule is being violated here, because points that were distinct on the original square are on top of each other when the square is compressed to a line. The conclusion is that we can never have a frame of reference that moves at c relative to another frame of reference.

(a) The paradox implicitly assumes the existence of a frame of reference that moves along with beam number 1, so that we can then talk about the velocity of beam 2 in beam 1's frame of reference. But there aren't frames of reference that move at c.

7-31 (a) Use natural units, with $c = 1$. Let the massive particle have mass m and velocity v. For a massless particle, we have $E = |p|$. Because this is the center of mass frame, we have $p_\gamma + p_m = 0$, so $|p_\gamma| = |p_m| = m\gamma|v|$. Thus

$$E_\gamma = m\gamma|v|,$$

while

$$\mathcal{E}_m = m\gamma,$$

and therefore $E_\gamma < \mathcal{E}_m$, since $|v| < 1$ in natural units.

(b) Let $R = KE_m/E_\gamma$. Then $R = m(\gamma - 1)/m\gamma = 1 - 1/\gamma < 1$. Therefore $KE_m < E_\gamma$.

7-33 Unlike the example of the school bus paradox, the answer here is not frame-dependent. We are not asking whether all parts of the ball fit in the hole simultaneously (which would be a frame-dependent question because simultaneity is frame-dependent). We are simply asking whether it is possible that no part of the ball ever intersects any part of the wall. Intersections of world-lines are frame-independent events.

Length contraction will not help the ball to fit through the hole, because length contraction occurs only along the direction of motion, not along the transverse axes.

7-34 (a) In units with $c = 1$, we have

$$\begin{aligned} v &= L/t \\ K &= m(\gamma - 1) \\ \tau &= t/\gamma. \end{aligned}$$

The solution of these equations is

$$K = m\left[\sqrt{1 + \left(\frac{L}{\tau}\right)^2} - 1\right],$$

or, reinserting factors of c,

$$K = mc^2\left[\sqrt{1 + \left(\frac{L}{c\tau}\right)^2} - 1\right].$$

(b) For the given data, the result is something like 10^{12} megatons of TNT. Your ultrarelativistic friend's body has so much kinetic energy that if he collided with the earth, it would be the end of the world, so I think Congress should pass a law prohibiting him from doing this.

8-1 The positive and negative charges attract each other.

8-2 Let q stand for the absolute values of the charges, which are equal to each other. We can find the force either by using Newton's law of gravity or by using the kinematical equation $a = v^2/r$ for the moon's acceleration. Choosing the first approach, we have

$$k\frac{q^2}{r^2} = G\frac{m_1 m_2}{r^2} \qquad ,$$

which we solve for q:

$$\begin{aligned} q &= \sqrt{\frac{Gm_1m_2}{k}} \\ &= 5.70 \times 10^{13}\ \text{C} \end{aligned}$$

This may sound like a big number, but it's actually not that much charge at all, on the same order of magnitude as the amount of charge in a human-scale object. It's just that in real life, the charges balance out perfectly, giving zero total charge on each body. Note that the reason the charge comes out fairly small, despite the large masses, is that G is much smaller than k, i.e. gravity is in some sense a much weaker force than electricity.

8-3 Let's use positive numbers for forces to the right, and negative numbers for forces to the left. There are two forces on the right-hand electron: a repulsive force from the other electron, and an attractive force from the nucleus. These will be represented as positive and negative numbers, respectively. The charges of the electrons are $-e$, and the charge of the nucleus is $+2e$, so the electron-electron force is $+ke^2/(0.2\ \text{nm})^2$, and the nucleus-electron force is $-k(e)(2e)/(0.1\ \text{nm})^2$, for a total force of -4×10^{-8} N.

8-4 [Note: An old version of the problem listed the parent isotope incorrectly as ^{234}Pu. It should be ^{241}Pu.]

Plutonium is atomic number 94. In alpha decay, we're removing two protons and two neutrons, so the mass number will be reduced by four, and the atomic number (number of protons) by 2. The nucleus left over after alpha decay therefore has atomic number 92 (uranium), and mass number 237; it's ^{237}U.

In electron decay, we're converting one neutron to a proton. The mass number is unchanged, but the atomic number is bumped up by one, to 95, americium. The product is ^{241}Am.

8-5 There are 82 protons in a lead nucleus. Since the lead nucleus is spherical, the electric force from the 81 other protons is the same as that of an equal amount of charge (81 times the charge of a proton) concentrated at the nucleus' center. The force is

$$k(q_{proton})\times(81q_{proton})/(6.5\times10^{-15}\text{ m})^2 = 440\text{ N}$$

which is less than the attractive force of 8 kN.

8-6 (a) $+e - e = 0 + 0$

(b) The mass of a neutron is greater than the mass of a proton plus the mass of an electron, so this is a process that increases mass, and a hydrogen atom would therefore have to have a source of energy in order to do it.

8-7 (a) Let each fixed charge be Q, and let the charge of the ball be q. When the ball is at position x, its distance from one of the fixed charges is $\sqrt{b^2+x^2}$, so the magnitude of each force on it is $|\mathbf{F}| = kQq/(b^2+x^2)$. There are two such forces, acting at different angles, so we have to add them using vector addition. Their y components cancel, and their x components are the same, so for the total force on the ball, we have

$$\begin{aligned} F &= 2F_x \\ &= \frac{2kQq}{b^2+x^2}\cdot\cos\theta \\ &= \frac{2kQq}{b^2+x^2}\cdot\frac{x}{\sqrt{b^2+x^2}} \\ &= 2kQq\left(b^2+x^2\right)^{-3/2}x \end{aligned}$$

(b) By $a = F/m$, the maximum acceleration occurs at the same point as the maximum force. To find the maximum force, we have to set the derivative of the force equal to zero and solve for x. The constants out in front don't affect the location of the maximum, so we can ignore them.

$$\begin{aligned} 0 &= \frac{\mathrm{d}}{\mathrm{d}x}\left[\left(b^2+x^2\right)^{-3/2}x\right] \\ &= -\frac{3}{2}\left(b^2+x^2\right)^{-5/2}(2x)x + \left(b^2+x^2\right)^{-3/2} \end{aligned}$$

We can simplify by multiplying both sides by $\left(b^2+x^2\right)^{5/2}$.

$$\begin{aligned} 0 &= -3x^2+b^2+x^2 \\ x &= \frac{b}{\sqrt{2}} \end{aligned}$$

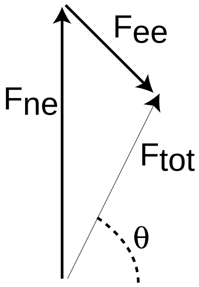
,

8-8

$$\begin{aligned} q &= \int_{x=0}^{x=\infty}\mathrm{d}q \\ &= \int_0^\infty \frac{\mathrm{d}q}{\mathrm{d}x}\,\mathrm{d}x \\ &= \int_0^\infty ae^{-bx}\,\mathrm{d}x \\ &= \frac{a}{b} \end{aligned}$$

8-9 A bowl of Cheerios with skim milk has about 150 nutritional "calories," which is actually 150 *kilocalories*, or 6×10^5 J. I'd guess it's about 0.2 kg worth of food, which would be something like 10^{25} atoms. The energy in one atom (rounded off to one sig fig) would be about 10^{-19} J.

8-10 You should always start this type of vector addition problem by drawing a rough diagram first so that you can check that your results make sense at every step.



The nucleus's force on the bottom electron, F_{ne}, is attractive, so it points up. The other electron's force on the bottom electron, F_{ee}, is repulsive, so it points down and to the right at a 45-degree angle. We'll state the final result using the angle θ defined in the figure. The magnitude of F_{ne} is given by Coulomb's law, $|F_{ne}| = k(e)(2e)/(0.1\ \text{nm})^2 = 4.6\times10^{-8}$ N. The magnitude of F_{ee} is smaller by a factor of four: a factor of two because of the smaller charge, and another factor of two because its distance from the bottom electron is greater by a factor of $\sqrt{2}$. Thus, $|F_{ee}| = 1.1\times10^{-8}$ N. To do vector addition, we need to add components. Let positive x components be to the right, positive y components up. $\mathbf{F}_{ne}$ has components $F_{ne,x} = 0$ and $F_{ne,y} = 4.6\times10^{-8}$ N. $\mathbf{F}_{ee}$ has components

$$F_{ee,x} = +(1.1\times10^{-8}\ \text{N})(\cos 45°) \qquad \text{and}$$
$$F_{ee,y} = -(1.1\times10^{-8}\ \text{N})(\sin 45°) \qquad .$$

At this point, you can check the + and − signs of the components against the figure. The total force has components

$$F_{tot,x} = F_{ne,x} + F_{ee,x} = 7.8\times10^{-9}\ \text{N} \qquad \text{and}$$
$$F_{tot,y} = F_{ne,y} + F_{ee,y} = 3.8\times10^{-8}\ \text{N} \qquad .$$

Again, you can check the + and - signs against the figure. The magnitude of the total force is $\sqrt{F^2_{tot,x} + F^2_{tot,y}} = 4\times10^{-8}$ N (one sig fig). The angle θ equals $\tan^{-1}(F_{tot,y}/F_{tot,x}) = 80°$ (one sig fig).

8-11 An electron in the tube is feeling repulsive forces from all the negatively charged particles in one end cap, and attractive forces from the positively charged ones at the opposite end. The total force on it is the vector sum of all these individual forces. According to Coulomb's law, each individual force is proportional to the charge of the electron. The situation is the same for the singly charged positive neon atom (ion). Since the magnitude of the neon atom's charge is the same as the magnitude of the electron's charge, all the forces on it are of the same magnitude, but opposite in direction. Therefore the magnitude of the total force on the neon atom is the same as the magnitude of the total force on the electron. $F = ma$, so equal forces will result in a much greater acceleration for the electron. The electron's mass is about 4×10^4 times less, so its acceleration is greater by a factor of 4×10^4.

8-12 Electricity is an inverse square force, so if r is smaller by a factor of 206.77, the force will be greater by a factor of $206.77^2 = 42754$.

8-13 (a) Coulomb's law is directly analogous to Newton's law of gravity, except for the fact that there are two types of charge, and the forces can be either repulsive or attractive. Se we can translate directly from the gravitational case to the electrical case just by changing each symbol to the analogous electrical quantity, as long as we make sure to put in the plus and minus signs by hand. Regarding the sign, two positive masses (which is the only kind that exist!) attract, but two positive charges repel. This means that, at least in the case of two positive charges, the sign has to be reversed for the electrical equation: $PE = kq_1q_2/r$. To check that this makes sense, we can imagine two positive charges being released, and accelerating away from each other because of their mutual repulsion. As r gets bigger, dividing by r gives a smaller result for the potential energy, and that makes sense, because PE has to be converted to KE. Two negative charges would also accelerate away from each other and convert PE to KE as r increased, so the same equation still works ($q_1q_2 > 0$). In fact, the equation even works when one charge is positive and one is negative; here, the product q_1q_2 is negative, the charges would accelerate toward each other, and the PE would decrease (become

more negative) as r got smaller.
(b) The equation only applies when the electron is outside the cookie, so we're estimating the energy difference between having the electron at the surface of the cookie and having the electron infinitely far away. The final potential energy is zero (dividing by $r = \infty$ gives 0). For the initial potential energy, we plug in the basic unit of charge, e, for each charge, and $r = 10^{-10}$ m, giving $PE = -2 \times 10^{-18}$ J. The change in PE is final $-$ intial $= 2 \times 10^{-18}$ J, which is roughly of the right order of magnitude compared to the result of problem #2.

8-14 There are four particles involved: two electrons and two protons. Atom A and atom B experience two repulsive forces (proton A on proton B and electron A on electron B) and two attractive forces (electron A on proton B and proton A on electron B). However, the distances involved for each of these four pairs are different. Therefore, according to Coulomb's, the four forces will all be different in magnitude, so they should not be expected to cancel. Also, the forces add like vectors, not scalars, and they are not all lined up along one line, and that's another reason why they should not cancel out exactly to zero.

8-15 The lithium is located nice and symmetrically in the lattice, so if there was no defect, the total force on it would be zero. Removing a Cl^- to make the hole removes a certain amount of force, so the total force without the Cl^- is equal to minus the force the Cl^- would have made, which we can find from Coulomb's law. The distance between the lithium and the hole is $\sqrt{(2.5 \times 0.3\ \mathrm{nm})^2 + (0.5 \times 0.3\ \mathrm{nm})^2} = 0.77$ nm, so the magnitude of the force is $ke^2/(0.77\ \mathrm{nm})^2 = 3.9 \times 10^{-10}$ N, or call it 4×10^{-10} N. The Li^+ would have been attracted by the Cl^-, so removing that attractive force leaves a total force that points directly away from the hole.

8-16 (a) Your body is mostly water, and water is mosly oxygen by weight, so for the purposes of this estimate, let's pretend that your body just consists of oxygen atoms. The mass of an oxygen atom is about 16 times the mass of a proton, which is 1.7×10^{-27} kg. If your hand has a mass of about 0.2 kg, then the number of oxygen atoms in a hand is about 0.2 kg$/(16 \times 1.7 \times 10^{-27}$ kg). The number of electrons is eight times that, $(8) \times (0.2\ \mathrm{kg})/(16 \times 1.7 \times 10^{-27}\ \mathrm{kg}) = 6 \times 10^{25}$, or call it 10^{26} electrons since its only an estimate.
(b) Coulomb's law gives $F = k(10^{26}e)(10^{26}e)/(0.1\ \mathrm{m})^2 = 2 \times 10^{26}$ N, or call it 10^{26} N.
(c) Each hand has zero net charge, and the negative charge is distributed within each hand in almost the same way as the positive charge. Another way of looking at it is that there are repulsive forces acting from one set of electrons on the other set of electrons and from one set of protons on the other set of protons, but there are attractive forces from each set of electrons acting on the protons of the opposite hand.
(d) A force of 10^{26} N is enough to pulverize a planet, so the cancellation is obviously very nearly perfect; even a very tiny percentage difference in charge would lead to a huge force. If the fractional difference was ϵ, then the total charge on each hand would be ϵq, where $q = 10^{26}e$ as estimated above. The total force would be $k\epsilon^2 q^2/r^2 = F$. You could easily notice a force of as little as 0.1 N, so $\epsilon \lesssim (r/q)\sqrt{F/k} \sim 10^{-14}$.

8-17 The maximum energy of the recoiling nucleus is attained in the case where the electron gets almost 100% of the energy, because for a fixed energy, a more massive particle carries more momentum. If the electron and neutrino were to share the energy, then their momentum vectors could also partially cancel, further reducing the recoil.

Let Q be the energy released in the decay. In the approximation that the electron is ultrarelativistic, its momentum (in the case where it carries all the energy) is $p \approx Q$, and by conservation of momentum, this is also the momentum of the recoiling nucleus. Since the nucleus is nonrelativistic, its kinetic energy (reinserting factors of c) is $K = p^2/2M \approx Q^2/2Mc^2$. Plugging in numbers, we get about 24 eV, which is clearly plenty of energy to break a chemical bond.

If we don't approximate the electron as ultrarelativistic, then the momentum of the beta, in the maximum-recoil case, isn't Q but rather $\sqrt{(m+Q)^2-m^2}$, where m is the mass of the electron. The actual maximum recoil energy for the calcium is 42 eV, so although the ultrarelativistic approximation gives the right order of magnitude, it's off by almost a factor of 2.

Remark: These energies are counterintuitively low, since the nuclear energy scale of MeV is a million times bigger than the chemical energy scale of eV. In fact, for low-energy beta decays there can be a considerable probability for the parent molecule to survive without any disruption. An experiment by Snell and Pleasanton in 1958 showed that when $^{14}CO_2$ decays, emitting an electron with an energy of 0.156 MeV, an intact $^{14}NO_2^+$ ion is produced about 80% of the time.

8-18 (a) The two particles are already at the greatest distance from one another that is allowed by conservation of energy, so neither can escape.
(b) By bringing two of the particles close together, we can reduce the gravitational potential energy to an arbitrarily large negative value. This allows the third particle to escape without violating conservation of energy. By similar reasoning, we expect that systems such as our solar system may, over long time scales, eject some of their members.
(c) The particles repel one another, so they will fly away to an infinite distance.
(d) This is the same as a.
(e) This is similar to b. We can bring one of the positive charges and one of the negative ones close together in order to release as much energy as required to eject the other positive one.

8-19 There can be no stable equilibrium in which the string is slack, since one could always move the movable charge along a path at a constant distance from the fixed charge, bringing it to a position on the circumference at which its gravitational energy would be lower.

The result can only depend on the unitless quantity $\beta = kq/mg\ell^2$. To make the writing easier, we take $\ell = 1$. The distance between the charges is $r = 2\sin\theta/2$, and for this reason it will be convenient to define the variable $p = \sin\theta/2$, which equals 0 at the bottom of the circle and 1 at the top.

For $\beta < 4$ there is an equilibrium in which the movable charge is directly above the fixed one, with the string slack. These are unstable with respect to lateral motion, and not very interesting.

For large β there is an equilibrium at $\theta = \pi$, but we need to investigate when this is stable.

Using the identity $1 - \cos\theta = 2\sin^2\theta/2$, we find for the potential energy

$$U = \frac{1}{2}\beta p^{-1} + 2p^2,$$

where the first term is electrical and the second gravitational. Although it is θ that is physically relevant, it is easier to do as much of the calculus as possible with p as the variable, since the expressions are simpler. Expressed as a function of p, the energy is always concave up, so any point with $\mathrm{d}U/\mathrm{d}p = 0$ will be a stable equilibrium. In addition, we have $\mathrm{d}U/\mathrm{d}\theta = 0$ automatically at $\theta = \pi$, since $\mathrm{d}p/\mathrm{d}\theta = 0$ there.

We have $\mathrm{d}U/\mathrm{d}p = -(1/2)\beta p^{-2} + 4p$, which equals zero for $p = (\beta/8)^{1/3}$. As remarked earlier, this is always stable. However, it only exists for $\beta \le 8$, since we must have $p \le 1$.

To evaluate the stability of the upright equilibrium, we evaluate $\mathrm{d}^2U/\mathrm{d}\theta^2$ at $\theta = \pi$, which gives $\beta/8 - 1$. This is positive if $\beta > 8$. Thus although levitation straight up, with the cord under tension, is possible for $4 < \beta < 8$, this equilibrium is not stable against lateral motion for these values of β.

In summary, we find that for $\beta \le 8$ the stable equilibrium is at

$$\sin\frac{\theta}{2} = \left(\frac{\beta}{8}\right)^{1/3},$$

while for $\beta \ge 8$ the stable and only equilibrium is the one in which the cord is taut and upright.

9-1 $\Delta t=\Delta q/I=e/I=0.16\ \mu\text{s}$

9-2 $I=\Delta q/\Delta t=(10^{10})(e)/(5\times10^{-4}\ \text{s})=3\ \mu\text{A}$

9-3 $R=\Delta V/I=(110\ \text{V})/(0.9\ \text{A})=120\ \Omega$

9-4 (a) $P=I\Delta V=(\Delta V/R)\Delta V=\Delta V^2/R$

(b) The power is proportional to ΔV^2, so cutting ΔV in half reduces the power by a factor of 4, to 50 W.

9-5 $q=\int I\,\mathrm{d}t=\int bt\,\mathrm{d}t=\frac{1}{2}bt^2+q_\text{o}$

Here q_o is a constant of integration, representing the cloud's charge at time zero.

9-6 (a) $P=I\Delta V=I(IR)=I^2R$

(b) Energy dissipated in those wires is wasted in heating the neighborhood, so we want R to be small.

9-7 $I=\mathrm{d}q/\mathrm{d}t=b\sin\omega t+\omega(a+bt)\cos\omega t$

9-8

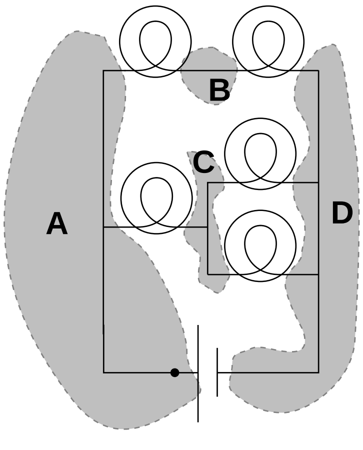
9-9

9-10 (a) If the power is the same, then $P=I\Delta V$ tells us that tripling the voltage has to go along with cutting down the current to one third of what it was.

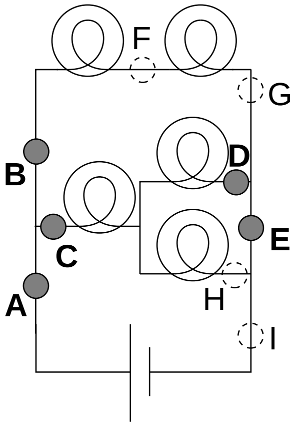
(b) By $P=I\Delta V$, the only way to get more power with the same voltage would be to increase the current. This would require fatter wires, which would be more expensive.

9-11 To get negative values of P, you'd have to have opposite signs for I and ΔV. But you can't control the signs of I and ΔV independently — switching the wires reverses both. The current flows in response to the electric force, whose direction is related to the direction of the voltage drop. Another way of looking at it is that the electrons collide with the atoms, causing them to vibrate, and the vibration is what heat is. There is no way that randomly vibrating atoms will all hit the electrons in the same direction, producing a current.

9-13 There is constant voltage throughout a conductor. The wires are good conductors, but the bulbs aren't, so all we have to do is count how many parts there are in the circuit that consist of nothing but wire. As shown in the figure, there are four such parts. For instance, if we connect one probe of the voltmeter at the dot, and one in region D, we'll get a certain voltage reading, and we'll get the *same* voltage reading even if we move the second probe to some other place in region D. We can get a total of four unique voltage readings, but region A gives zero, so there are only three unique, nonzero readings we can get.



9-14 There are five unique currents you could measure, by inserting an ammeter at A, B, C, D, and E.



The key here is that charge is conserved. That means that, for example, you'll get exactly the same current reading at B, F, and G, because any charge that flows up through B will also have to go through F and G; it can't "drive off the road," and it can't get used up in the bulbs, because that would violate conservation of charge. Likewise, I and A give the same reading.

You also can't get two different results from D and H. This isn't because of conservation of charge, it's because the two bulbs are identical, and have the same voltage drop across them.

9-15 (a) In one revolution, the amount of charge passing by a certain angular position is 1 nC. The time required for one revolution is $1/(33.3\ \text{min}^{-1})$, so $I = \Delta q/\Delta t = 3.3 \times 10^{-8}$ C/min = 600 pA.
(b) The idea is the same. The time for one orbit is $\Delta t = 2\pi r/v$, so $I = e/\Delta t = ve/2\pi r = 1$ mA.

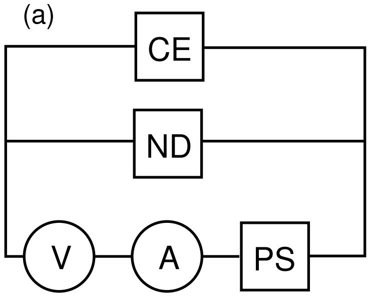
9-16

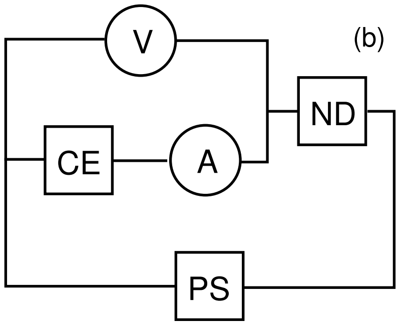
9-17 Resistance is inversely proportional to the cross-sectional area. The area depends on the square of the diameter, so the ratio of resistances is $(1.2)^2 = 1.4$.

9-18 (a) Call the voltages V_5 and V_{10}. All three currents are the same, since it's a single closed loop, and there is no place to store charge. We therefore only need one symbol for current — just call it I. Kirchoff's loop rule says $V_5 + V_{10} = 9$ volts. Ohm's law tells us $V_5 = (I)(5\ \Omega)$ and $V_{10} = (I)(10\ \Omega)$. This means that V_{10} must be twice as big as V_5, and since they have to add up to 9 volts, they must be $V_5 = 3$ V and $V_{10} = 6$ V.
(b) As explained above, all three currents are the same. One way to find the current is by solving the equation $V_5 = (I)(5\ \Omega)$ for I, giving $I = 0.6$ A. Another way would be to divide 9 V by the series resistance of 15 ohms.
(c) The 10-ohm bulb will go out, because its two sides are now connected with wire, and therefore there is no voltage difference between the two sides. The 5-ohm bulb now has the full voltage drop of the battery across it, i.e. 9 volts, and its current will be $(9\ \text{V})/(5\ \Omega) = 1.8$ A. This means it will be brighter than it was before.
(d) You'll see exactly the same thing as in the original circuit. None of the reasoning used in solving part (a) made any assumption about what order the bulbs were in. The current does not "get tired" or "get used up" by having to go through one bulb first before it gets to the other. Conservation of charge says that charge can't "get used up."

9-19 (a) They have correctly set up the CE and ND in parallel. The ammeter is measuring the total current out of the power supply, however, rather than just the current that is going to go through the ND. The voltmeter is hooked up in series, but it should be in parallel. A voltmeter has a very high resistance, so the circuit will not function when the voltmeter is hooked up this way.

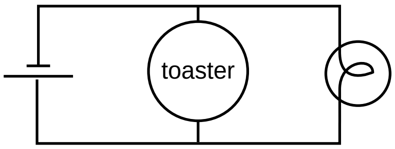
(b) The meters are hooked up correctly, but the CE and ND are in series rather than in parallel. (If they had hooked up the CE and ND in parallel, then they would have had to get the ammeter hooked up correctly to measure the current through the ND rather than the CE, but in this series circuit, both the CE and the ND have the same current through them.)



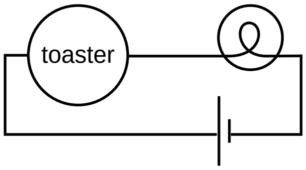


9-20 If there was no hole in the lattice, then by symmetry, the potential at any of the available gaps would be the same. Removing the Cl^- to make a hole will then change the potentials by an amount equal to minus its original contribution. Its original contribution was $kq/r = -ke/r$, so the potential at each point in space is changed by $+ke/r$. Now the lithium is going to jump from the gap it occupies in the figure, where the potential is V_1, to some neighboring gap with potential V_2, losing potential energy of $e(V_1-V_2)$. Without the hole, we would have had $V_1-V_2=0$, but removing the Cl^- to make the hole changes both V_1 and V_2. V_1 changes by an amount ke/r_1, where r_1 is the distance from the original gap to the hole, and similarly for V_2. The potential energy lost by the lithium is therefore $ke^2(1/r_1-1/r_2)$, so that is also the amount of KE it gains. To get its speed, solve $KE=(1/2)mv^2$ for $v=\sqrt{2KE/m}=e\sqrt{2k(1/r_1-1/r_2)/m}$. It is going to jump to the gap to the left, since that is the direction closest to the direction of the force on it as found in the previous homework problem. The Pythagorean theorem gives $r_1=0.77$ nm and $r_2=1.06$ nm, the mass of a lithium atom is about 7 times the mass of a proton, and plugging in to the equation for the velocity gives 3.7×10^3 m/s, or call it 4000 m/s.

9-21



(a)



(c)

(a) See figure.

(b) The E-shaped piece of wire across the top is one big piece of conducting material, so it must have the same voltage throughout it. Similarly, the whole E-shaped piece on the bottom is at the same voltage. Therefore, the voltage difference across each of the three components is the same. Since we know the supply is 110 V, the voltage drops across the toaster and lamp must also be 110 V. Ohm's law gives $I_{toaster}=(110\text{ V})/R_{toaster}\approx100$ A, and $I_{lamp}=(110\text{ V})/R_{lamp}\approx50$ A.

(c) In this setup, which is not what's really used in your house, it's not the voltage drops that are equal but the currents — whatever current goes through one component also has to go through the others. The two resistances in series add together to make a total resistance of 3 Ohms, so $I_{toaster}=I_{lamp}=V/3\ \Omega\approx40$ A. Ohm's law can now be used to find the voltage drop across each appliance, $V_{lamp}=I_{lamp}R_{lamp}=73$ V, and $V_{toaster}=I_{toaster}R_{toaster}=37$ V. (I didn't round the voltages off to one sig fig because I wanted to make it clear that they add up to 110 V.)

9-22 This setup could be made to work just fine at the extreme settings. Let R_k be the (variable) resistance of the knob, and R_h the resistance of the heater.

The "off" position would have to have $R_k=\infty$, i.e., it would be like an open circuit. The total resistance $R=R_k+R_h$ would be infinite as well, zero current would flow, and no power would be dissipated anywhere.

The full-power position would have $R_k=0$, i.e., it would be like a piece of wire. Current would flow, but there would be no voltage drop across R_k, so the only heating would be in R_h.

The problem occurs at intermediate settings, where R_k is neither zero nor infinity. Here R_k would get hot, wasting power by heating up the knob. At $R_k=R_h$, for example, the knob would be dissipating the same amount of heat as the heating element itself! This would presumably melt the knob, or burn your hand.

9-23 (a) This has to be true based on the junction rule, so it doesn't tell you anything about the resistance values.

(b) We already knew without even touching the circuit that the currents were going to be equal, and if we find that the voltages are also equal then $V=IR$ says that the resistances must be equal.

(c) The loop rule tells us that the voltage drops are equal in the parallel circuit, and if we also

find out the currents are equal, then $V = IR$ says the resistances must be equal.
(d) This is not useful information for finding out about the resistors, since it has to be true based on the loop rule.

9-24 All three in series gives one possibility. All three in parallel: one possibility. Two in series, in parallel with the third: three possibilities. Two in parallel, in series with the third: three possibilities. The total number of resistance values you can make is eight.

9-26 The total resistance seen by the voltage source is $x + y$, so the current that flows is $I = \Delta V/(x + y)$. Using the result of problem 1 from ch. 3, the power dissipated in the lamp is $P = I^2 y = \Delta V^2 y(x + y)^{-2}$. Thus we want to find the value of y that maximizes the quantity $y(x + y)^{-2}$. To find a maximum, we set the derivative equal to zero:

$$\begin{aligned} 0 &= \frac{\mathrm{d}}{\mathrm{d}y} y(x+y)^{-2} \\ &= (x+y)^{-2} - 2y(x+y)^{-3} \end{aligned}$$

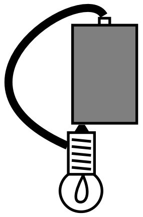
Multiplying both sides by $(x + y)^3$ simplifies this to

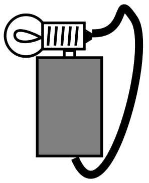
$$0 = (x + y) - 2y \qquad ,$$

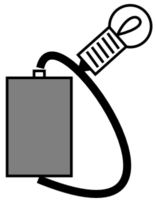
which gives $x = y$.

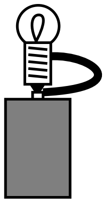
Physical explanation: Very large values of y are like an open circuit, e.g. like opening a switch. Very little current flows, so no power is dissipated in the lamp. Very small values of y result in more current, but the current is still limited by the resistance of the cord, and this current, passing through a small resistance y, dissipates very little power. This is a specific example of something known as impedance matching: the maximum power is transmitted when the resistance of the load is matched to the resistance of the transmission line.

9-28


(a) good


(b) good


(c) no good


(d) no good

The bulb has two terminals. One is the bump on the end, and the other is the screw-threaded metal jacket. The battery also has two terminals: the bump on one end and the flat surface on the opposite end. In the two good methods, a and b, current can flow out of one terminal of the battery, into one terminal of the bulb, through the filament, out the other terminal of the bulb, and back in to the opposite terminal of the battery. In method c, there is a dead end in the bulb: the current could get in through the bump-shaped terminal, but there would be no way for it to get out of the screw-threaded terminal once it had gone through the filament. The other problem with c is that there is an easy way for current to go through the low-resistance wire, so the wire will get very hot and the battery will burn out. In method d, there is a complete loop of wire consisting of the bump-shaped terminal of the bulb, the filament, the screw-threaded terminal, and the wire that comes back to the bump-shaped terminal. This loop is all made out of conductors, and the voltage is equal at all points within a piece of conducting material. Therefore the voltage difference between the two ends of the filament is zero, and by Ohm's law, $V = IR$, no current will flow through the filament. Also, method d gives no way for current to get to the other terminal of the battery.)

9-29 (a) Conservation of energy gives

$$\begin{aligned} U_A &= U_B + K_B \\ K_B &= U_A - U_B \\ \frac{1}{2}mv^2 &= e\Delta V \\ v &= \sqrt{\frac{2e\Delta V}{m}} \end{aligned}$$

(b) Plugging in numbers, we get 5.9×10^7 m/s. This is about 20% of the speed of light, so the nonrelativistic assumption was good to at least a rough approximation.

9-30 (a) The loop rule says that when you add up the voltage drops across the batteries plus the (opposite in sign) voltage drop across the bulb, you get zero. The batteries work the same as a single battery with a voltage equal to their sum. (b) It apparently has thousands of cells in series. The "resistor" would be its prey.

9-31 An open switch in (2) is the same circuit as a closed switch in (1): the light bulb is lit, and there's no problem. When you close the switch in (2), it does cause the bulb to go out (because the two sides of the bulb are connected with wire and therefore have no voltage difference), but large amounts of current will now flow through the middle branch of the circuit, which will kill the battery.

9-33 This is a problem that has a trick to it. If you don't think of the trick, it's very difficult, because the circuit can't be broken down into parallel and series parts. The trick is to realize that although the circuit has been drawn in an asymmetric way, it's actually very symmetric. The two rightmost bulbs play identical roles in the circuit, as do the two leftmost one. You can see this if you redraw the circuit so that the top left bulb and the top right bulb are on one horizontal line, and similarly for the bottom left and bottom right. The bulb in the middle has zero current through it by symmetry.

9-34 Like the preceding problem, this one can't be solved by breaking the circuit down into parallel and series parts. Unlike that one, there is no easy way to solve it by symmetry. We simply have to use the loop rule and the junction rule to write down enough equations to solve for all the unknowns. Let's label the bulbs A, B, C, ... going from top to bottom and left to right, like reading a page of text. For simplicity, we think of the bulbs at the bottom as a single bulb, E, with a resistance of two ohms. (This is the only part of the circuit that can be simplified in this way.) Let A stand for the current through bulb A, and so on. The junction rule then gives

$$\begin{aligned} D &= B + E \qquad \text{and} \\ A + B &= C \qquad . \end{aligned}$$

Since every bulb except for E has a resistance of one ohm, the voltage drop across each bulb equals the current through it. That is, we can use the symbol A interchangeable for both the current through A (in amperes) and the voltage drop across A (in volts). The only exception is the double bulb E, which has a voltage $2E$ across it. We use the loop rule three times:

$$\begin{aligned} A + C &= 1 \\ D + 2E &= 1 \\ A &= D + B \qquad . \end{aligned}$$

The result is five equations in five unknowns, which we solve by eliminating one variable after another. The final results are $A = 6/13$, $B = 1/13$, $C = 7/13$, $D = 5/13$, and $E = 4/13$. The total current is $A + D = 11/13$ amps, so the power drawn from the one-volt battery is (11/13 A)(1 V) = 11/13 W.

9-35 Using the junction rule at the junction A feeds into, we find that the current through A is 300 mA. Using the junction rule at the junction on the left, we then have 600 mA for the total current drawn from the battery, which is also the current through D. Applying the junction rule at the junction in the middle, we find a current of 100 mA for both B and C. All our unknowns are now determined, so we can use the junction on the right side to check that our solution is correct: 500 mA plus 100 mA gives 600 mA, which is indeed what we had for D.

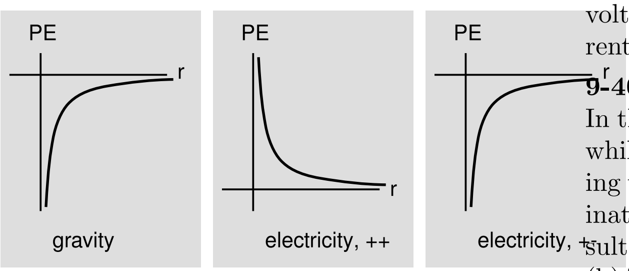
9-36

$$\begin{aligned} I &= \mathrm{d}q/\,\mathrm{d}t \\ &= 2ct \\ P &= I\Delta V \\ &= I\cdot(IR) \\ &= I^2R \\ &= 4c^2t^2 \end{aligned}$$

9-37 (a) The voltage at the surface of the large sphere is kq_a/a, and on the small one kq_b/b. Since they're connected by a wire, they must be at equal voltage. Equating the two expressions gives $q_a/q_b = a/b$.
(b) The area of a sphere is proportional to r^2, so the charge per unit area is proportional to qr^{-2}. Therefore

$$\frac{\text{density of charge on sphere } b}{\text{density of charge on sphere } a} = \frac{q_b b^{-2}}{q_a a^{-2}} = \frac{a}{b}$$

9-38 (a) Dividing by a small r gives a large negative result, while dividing by a big r gives a small negative result. The graph for gravity looks like the one on the left:



As the rock falls to smaller values of r, gaining KE, the graph shows its PE decreasing, which is what we expect from conservation of energy.

(b) Coulomb's law is directly analogous to Newton's law of gravity, except for the fact that there are two types of charge, and the forces can be either repulsive or attractive. Se we can translate directly from the gravitational case to the electrical case just by changing each symbol to the analogous electrical quantity, as long as we make sure to put in the plus and minus signs by hand. Regarding the sign, two positive masses (which is the only kind that exist!) attract, but two positive charges repel. This means that, at least in the case of two positive charges, the sign has to be reversed for the electrical equation: $PE = kq_1q_2/r$. To check that this makes sense, we can imagine two positive charges being released, and accelerating away from each other because of their mutual repulsion. As r gets bigger, dividing by r gives a smaller result for the potential energy, and that makes sense, because PE has to be converted to KE. Two negative charges would also accelerate away from each other and convert PE to KE as r increased, so the same equation still works ($q_1q_2 > 0$). In fact, the equation even works when one charge is positive and one is negative; here, the product q_1q_2 is negative, the charges would accelerate toward each other, and the PE would decrease (become more negative) as r got smaller.

9-39 The resistor on the left looks different from the others, but actually it's connected to exactly the same things they are. The whole circuit is just four resistors in parallel, but drawn in a funny way. By the loop rule, the voltage across each of the resistors is the same as the battery's voltage V, and therefore each resistor has a current $I = V/R$. The total current is $4V/R$.

9-40 (a) Let the voltage of the battery be V. In the original circuit, we have $P = IV = V^2/R$, while in the new one $P' = V^2/(R+r)$. By forming the ratio of these two quantities, we can eliminate the voltage, and $P/P' = 1 + r/R$. The result is $r = (P/P' - 1)R$.
(b) The ratio P/P' is unitless, which makes sense because we're subtracting a unitless 1 from it. The result is that ohms equal ohms, which makes sense.
(c) For $P' = P$, we have $r = 0$, which makes sense: no additional resistance was introduced, no no change occurred. For $P' = 0$, we have $r = \infty$, which also makes sense. For example if the dirt is a perfect insulator, then we have an open circuit.
(d) The current in the modified circuit is $I = V/(R + r)$, which gives $P' = I^2R = V^2R/(R +$

$r)^2$. After a little algebra, the result is

$$r = \left(\sqrt{\frac{P}{P'}} - 1\right) R,$$

which, remarkably, differs from the result of part a only by the presence of the square root.

10-1 (a) $V = Ed = 1.8 \times 10^3$ V
(b) Since $V = Ed$, a larger d would require a larger V.

10-2 (a) The approximation $(1+\epsilon)^p \approx 1 + p\epsilon$ is by now familiar. To get the equation in this form, we need to divide out a factor of d:

$$\begin{aligned} E &= \frac{kQ}{L}\left(\frac{1}{d-L/2} - \frac{1}{d+L/2}\right) \\ &= \frac{kQ}{dL}\left(\frac{1}{1-\frac{L}{2d}} - \frac{1}{1+\frac{L}{2d}}\right) \\ &\approx \frac{kQ}{dL}\left[\left(1+\frac{L}{2d}\right) - \left(1-\frac{L}{2d}\right)\right] \\ &= \frac{kQ}{dL}\left(\frac{L}{d}\right) \\ &= \frac{kQ}{d^2} \end{aligned}$$

(b)

$$\begin{aligned} E &= 2\pi\sigma k\left(1 - \frac{z}{\sqrt{b^2+z^2}}\right) \\ &= 2\pi\sigma k\left(1 - \frac{1}{\sqrt{\frac{b^2}{z^2}+1}}\right) \\ &\approx 2\pi\sigma k\left(1 - \left(1 - \frac{1}{2}\frac{b^2}{z^2}\right)\right) \\ &= \pi\sigma k\frac{b^2}{z^2} \end{aligned}$$

but πb^2 is the area of the disk, so $\pi\sigma b^2$ is the disk's total charge, and we have

$$E \approx \frac{kq}{z^2}$$

10-3

$$\begin{aligned} m &= \int \mathrm{d}m \\ &= \int \frac{\mathrm{d}m}{\mathrm{d}v}\,\mathrm{d}v \\ &= \int_0^\infty a e^{-br}\,\mathrm{d}v \\ &= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{\infty} a e^{-br} r^2 \sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi \\ &= 2\pi\int_{\theta=0}^{\pi}\int_{r=0}^{\infty} a e^{-br} r^2 \sin\theta\,\mathrm{d}r\,\mathrm{d}\theta \\ &= 4\pi\int_{r=0}^{\infty} a e^{-br} r^2\,\mathrm{d}r \end{aligned}$$

The r integral can be done by integration by parts, or with computer software. The result is

$$m = \frac{8\pi a}{b^3}$$

10-4 (a) $F = ma$ and $F = qE$, and since the field is uniform, the force and acceleration will be constant. For motion with constant acceleration, we have $v = at$, and eliminating the unknown F gives $v = qEt/m$.
(b) You can conclude that they have the same ratio of q/m, but you can't tell whether they have the same q and the same m. One could have twice the charge and twice the mass compared to the other, for instance.

10-5 The units of $\tau/D_t \sin\theta$ are

$$\frac{\mathrm{N{\cdot}m}}{\mathrm{C{\cdot}m}} = \frac{\mathrm{N}}{\mathrm{C}} \qquad ,$$

which is consistent with the original definition of the electric field.

10-6 Let the origin of the coordinate system be at the charge on top. Going thruogh the four charges in the same order as before (left, middle, top, right), we have

$$\begin{aligned} \mathbf{D} &= \sum q_i \mathbf{r}_i \\ &= (q)(-b\hat{\mathbf{x}} - b\hat{\mathbf{y}}) + (-q)(-b\hat{\mathbf{y}}) + (q)(\mathbf{0}) + (-q)(b\hat{\mathbf{x}} - b\hat{\mathbf{y}}) \\ &= -2bq\hat{\mathbf{x}} + bq\hat{\mathbf{y}} \end{aligned}$$

The result is unaffected by the change in the origin of the coordinate system. This will always happen for a charge distribution with a total charge of zero.

10-7 (a) For simplicity, let's put everything on the x axis, so the dipole moment has only an x component. If we put -1 C at $x = 0$ and $+1$ C at $x = 1$ m, then the dipole moment is $D_x = 1$ C·m. Now let's do a different coordinate system, whose origin is located 7 m to the left of the first one's. In this coordinate system, the charges are located at $x = 7$ m and 8 m. The dipole moment is the same, however, $(8 \times 1 + 7 \times -1) = 1$ C·m. For this particular pair of charges, the dipole moment doesn't seem to depend on the choice of coordinates.
(b) If we have charges $+q$ and $-q$ located at $\mathbf{r}_+$ and $\mathbf{r}_-$, then the dipole moment is $\mathbf{D} = q(\mathbf{r}_+ - \mathbf{r}_-)$. This only depends on the vector $\mathbf{r}_+ - \mathbf{r}_-$, which points from the negative charge to the positive one. Shifting the origin has no effect on $\mathbf{r}_+ - \mathbf{r}_-$, so it has no effect on the dipole moment, either.
(c) We conjecture that the choice of origin doesn't matter as long as the total charge is zero. The most elegant way to prove this is to imagine that we can build the given charge distribution out of tiny point charges, each with charge $+q$ or $-q$, where q is very small. (Mathematically, q can even be less than the fundamental charge, e.) If the total charge is zero, then we have an equal number of $+q$ and $-q$ charges, which we can arbitrarily pair up like a roomful of married couples. The total dipole moment equals the sum of the dipole moments of the couples, and since none of the couples' dipole moments depend on the choice of origin, neither does the sum.

10-8 They're the same. When we have a charge distribution with zero total charge, its dipole moment is unchanged when we move it around without rotating it, since the dipole moment is defined by the potential energy, and the potential energy is unchanged when we do this. These two distributions therefore have the same dipole moment, since we can make one into the other by sliding one of the $+-$ pairs.

10-9 A simple example is $+--+$. The total charge is zero, and the dipole moment cancels by symmetry. This is known as an electric *quadrupole.*

10-10

$$\begin{aligned}
E &= -\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{kQ}{\sqrt{b^2+z^2}}\right) \\
&= kQ\cdot\frac{1}{2}\left(b^2+z^2\right)^{-3/2}\frac{\mathrm{d}\big(z^2\big)}{\mathrm{d}z} \\
&= \frac{1}{2}kQ\left(b^2+z^2\right)^{-3/2}\cdot 2z \\
&= kQz\left(b^2+z^2\right)^{-3/2}
\end{aligned}$$

10-11 Let the square's sides be of length a. The field at the center is the vector sum of the fields that would have been produced individually by the three charges. Each of these individual fields is kq/r^2, where $r_1 = a/\sqrt{2}$ for the two charges q_1, and $r_2 = a/2$ for q_2. Vector addition can be done by adding components. Let x be horizontal and y vertical. The y components cancel by symmetry. The sum of the x components is

$$E_x = kq_1 r_1^{-2}\cos 45° + kq_1 r_1^{-2}\cos 45° - kq_2 r_2^{-2} \qquad .$$

Substituting $\cos 45° = 1/\sqrt{2}$ and setting this whole expression equal to zero, we find $q_2/q_1 = 1/\sqrt{2}$.

10-12 (a) They're both 1.000 C·m.
(b)

$$\begin{aligned}
E_A &= -\frac{k|-1.000\ \mathrm{C}|}{(10.000\ \mathrm{m})^2} + \frac{k|1.000\ \mathrm{C}|}{(9.000\ \mathrm{m})^2} \\
&= 2.11\times10^{7}\ \mathrm{N/C} \\
E_B &= -\frac{k|-2.000\ \mathrm{C}|}{(10.000\ \mathrm{m})^2} + \frac{k|2.000\ \mathrm{C}|}{(9.500\ \mathrm{m})^2} \\
&= 1.94\times10^{7}\ \mathrm{N/C}
\end{aligned}$$

They come out almost the same, but not quite. In general, the farther we are from a dipole, the more accurate it is to say that its field depends only on its dipole moment. Here, our distance from the dipoles was 10 or 20 times greater than their sizes, so it was a pretty good approximation.

10-13 $E=V/d=70\text{ mV}/6.0\text{ nm}=1.2\times10^{7}$ V/m

10-14 The voltage is

$$\begin{aligned}V&=-\int E\,\mathrm{d}x\\&=-ax-\frac{1}{4}bx^4\qquad,\end{aligned}$$

where the constant of integration is irrelevant because the electrical energy is only meaningful up to an arbitrary additive constant. Conservation of energy gives

$$\begin{aligned}K_1+U_1&=K_2+U_2\\0&=\frac{1}{2}mv^2+eV(x_2)\\v&=\pm\sqrt{-\frac{2eV(x_2)}{m}}\\&=\pm\sqrt{\frac{2e(ax_2+\frac{1}{4}bx_2^4)}{m}}\qquad.\end{aligned}$$

Note that for certain values of a, b, and x_2, we could end up with a negative quantity inside the square root. If so, then that would be Mother Mathematics' way of telling us that the proton would turn around before reaching position x_2.

If the quantity inside the square root is positive, then we have two square roots, a positive one and a negative one. This tells us that the proton could oscillate back and forth through position x_2 more than once.

10-15 (a) The voltage is the integral of the field with respect to position. We can use an indefinite integral and not worry about the constant of integration, since physically it just relates to the arbitrary choice of a reference point.

$$\begin{aligned}V&=-\int E\,\mathrm{d}r\\&\propto-\int\frac{D}{r^3}\,\mathrm{d}r\\&=\frac{D}{2r^2}\end{aligned}$$

(b) Let charge $+q$ be at $x=\ell/2$, and $-q$ at $x=-\ell/2$. The exact voltage is

$$V=\frac{kq}{|x-\ell/2|}-\frac{kq}{|x+\ell/2|}\qquad.$$

For large distances, it doesn't matter whether we do positive or negative x, because everything is symmetric, so let's just assume x is large and positive, in which case we can get rid of the absolute value signs:

$$V=\frac{kq}{x-\ell/2}-\frac{kq}{x+\ell/2}\qquad.$$

To get this in the form suggested in the hint, we have to divide out the common factor of kq/x:

$$\begin{aligned}V&=\frac{kq}{x}\left(\frac{1}{1-\ell/2x}-\frac{1}{1+\ell/2x}\right)\\&\approx\frac{kq}{x}\left[\left(1+\frac{\ell}{2x}\right)-\left(1-\frac{\ell}{2x}\right)\right]\\&=\frac{kq\ell}{x^2}\\&=\frac{kD}{x^2}\end{aligned}$$

This is consistent with part a. (Since part a was only stated as a proportionality, it doesn't matter that the two results disagree by a factor of $k/2$.)

10-17 This is the opposite of the previous problem. To find the field given the voltage, we take the derivative:

$$\begin{aligned}E&=-\frac{\mathrm{d}}{\mathrm{d}r}\frac{k}{r^3}\\&=\frac{3k}{r^4}\end{aligned}$$

10-18

$$\begin{aligned}
-e &= \int \mathrm{d}q \\
&= \int \frac{\mathrm{d}q}{\mathrm{d}v}\,\mathrm{d}v \\
&= \int_0^\infty a e^{-br} z^2 \,\mathrm{d}v \\
&= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} a e^{-br} r^4 \cos^2\theta \sin\theta \,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi \\
&= 2\pi \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} a e^{-br} r^4 \cos^2\theta \sin\theta \,\mathrm{d}r\,\mathrm{d}\theta
\end{aligned}$$

Subsituting $u = \cos\theta$,

$$\begin{aligned}
-e &= 2\pi \int_{u=1}^{-1} \int_{r=0}^{\infty} a e^{-br} r^4 u^2 \,\mathrm{d}u\,\mathrm{d}r \\
&= \frac{4\pi}{3} \int_{r=0}^{\infty} a e^{-br} r^4 \,\mathrm{d}r
\end{aligned}$$

The r integral can be done by integration by parts, or with computer software. The result is

$$\begin{aligned}
-e &= \frac{32\pi a}{b^5} \\
a &= \frac{-eb^5}{32\pi}
\end{aligned}$$

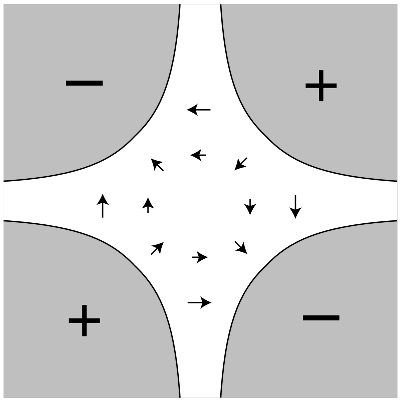
10-19 Our point lies at a distance R from the center of the dipole, and its distance from either charge is $\sqrt{R^2+\ell^2/4}$. The magnitude of the field contributed by one of the charges is therefore $E_{one} = kq/\left(R^2+\ell^2/4\right)$. The radial (in-out) components of the two fields cancel, but their z components reinforce, so $E = 2E_{one,z} = 2E_{one}\sin\phi$, where the small angle between the midplane and the individual field vector is $\phi = \tan^{-1}[(\ell/2)/R] \approx \ell/2R$. The dipole's field is therefore

$$\begin{aligned}
E_z &= 2E_{one}\sin\phi \\
&\approx 2E_{one}\phi \\
&= \frac{2E_{one}\ell}{2R} \\
&= \frac{kq\ell}{R\left(R^2+\ell^2/4\right)} \\
&\approx \frac{kD}{R^3}
\end{aligned}$$

10-20

$$\begin{aligned}
E_x &= -\frac{\mathrm{d}V}{\mathrm{d}x} \\
&= -V_{\mathrm{o}} b^{-2} y \\
E_y &= -\frac{\mathrm{d}V}{\mathrm{d}y} \\
&= -V_{\mathrm{o}} b^{-2} x
\end{aligned}$$

The fields along the axes are particularly easy to sketch, just by looking at these equations. We can then fill in some off-axis points by making use of the fact that the electric field close to the conducting surfaces must be perpendicular to the surfaces.



10-21 (a) The variable r depends on x, y, and z. To take the derivative of this voltage with respect to the coordinates, we need to make this explicit:

$$\begin{aligned} V &= br^2 \\ &= b(x^2+y^2+z^2) \\ E_x &= -\frac{\mathrm{d}V}{\mathrm{d}x} \\ &= -2bx \end{aligned}$$

Similarly for the other two components,

$$\begin{aligned} E_y &= -2by \\ E_z &= -2bz \qquad . \end{aligned}$$

(b) The field only depends on the derivatives, so we can add any constant onto the voltage and still get the same field, e.g. V could be $br^2 +$ (42 V), and the derivatives would come out the same.

10-22 (a)

$$E_R = \frac{k\lambda L}{R^2\sqrt{1+L^2/4R^2}}$$

For large L, the square root becomes $L/2R$, and the field approaches $2k\lambda/R$.

(b) By symmetry, there can only be a field in the z direction. The charge per unit length along each strip is $\lambda = \sigma\,\mathrm{d}x$, and $R = \sqrt{x^2+z^2}$, so

$$E_z = \int_{-\infty}^{\infty} \frac{2k\sigma\,\mathrm{d}x}{R}\cos\theta$$

where $\theta = \cos^{-1} z/R$ is the angle between the z direction and the field contributed by this strip, so

$$\begin{aligned} E_z &= 2k\sigma z \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{R^2} \\ &= 2k\sigma z \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+z^2} \\ &= 2k\sigma z \cdot \frac{1}{z}\tan^{-1}\frac{x}{z}\bigg]_{-\infty}^{\infty} \\ &= 2\pi k\sigma \end{aligned}$$

10-23 (a) We know that the field of a ring, on its axis, is $kQz\left(b^2+z^2\right)^{-3/2}$, so we can get the field of the cylinder by slicing it like a salami and integrating the contributions of the rings to the total field. Each ring has a width $\mathrm{d}z$, so its charge is $\mathrm{d}Q =$ (circumference)(width)$\sigma = 2\pi b\sigma\,\mathrm{d}z$.

$$\begin{aligned} E &= \int k\,\mathrm{d}Qz\left(b^2+z^2\right)^{-3/2} \\ &= \int_0^{\infty} 2k\pi b\sigma z\,\mathrm{d}z\left(b^2+z^2\right)^{-3/2} \\ &= 2k\pi b\sigma \int_0^{\infty} z\,\mathrm{d}z\left(b^2+z^2\right)^{-3/2} \end{aligned}$$

This is a good candidate for integrals.com, but if you want to do it by hand, you can substitute $u = z^2 + b^2$, which gives

$$\begin{aligned} E &= k\pi b\sigma \int_{b^2}^{\infty} u^{-3/2}\,\mathrm{d}u \\ &= -2k\pi b\sigma\, u^{-1/2}\Big]_{b^2}^{\infty} \\ &= 2k\pi\sigma \end{aligned}$$

(b) If you doubled the cylinder's radius, while keeping the charge density constant, you'd have a cylinder with four times the area, and therefore four times the charge. All the angles would be the same, but all the distances would be doubled. Doubling the distance makes the field four times smaller, and this cancels out the effect of the increase in charge.

10-24 (a) The energy per unit volume is

$$\frac{E}{V} = \frac{1}{8\pi k}|\mathbf{E}|^2 \qquad .$$

(E is energy, $\mathbf{E}$ is electric field.) At the moment when the lightning strikes, the field equals E_c. The volume of the air in which the field exists is L^2h.

$$E = \frac{1}{8\pi k}E_c^2L^2h$$

(b) A greater h gives a greater energy, so it's more dangerous if it's higher.

(c) Let's use 1000 m for both L and h. Plugging in, the result is about 10^{10} J.

10-25 (a) The surface area of a sphere is $4\pi r^2$, so the volume of the kind of spherical shell suggested in the hint is $\mathrm{d}V = 4\pi r^2\ \mathrm{d}r$. The energy is

$$\begin{aligned} U &= \int_0^\infty \frac{\mathrm{d}V}{8\pi k} E^2 \\ &= \int_0^\infty \frac{\mathrm{d}V}{8\pi k} \left(\frac{kq}{r^2}\right)^2 \\ &= \frac{kq^2}{2} \int_0^\infty r^{-2}\,\mathrm{d}r \\ &= \frac{kq^2}{2} \left(\frac{1}{0} - \frac{1}{\infty}\right) \end{aligned}$$

It's perfectly legal to take $1/\infty$, and the result is 0. However, 1/0 gives infinity. In other words, the integral diverges near $r = 0$, but not at $r = \infty$. The interpretation is that the field has infinite energy close to the charge, but finite energy in the space farther away.

(b) The energy in the exterior volume equals the result from part a, with c set to infinity, $U_{ext} = kq^2/2b$. The energy in the interior volume is

$$\begin{aligned} U_{int} &= \int_0^b \frac{\mathrm{d}V}{8\pi k} E^2 \\ &= \int_0^b \frac{\mathrm{d}V}{8\pi k} \left(\frac{kqr}{b^3}\right)^2 \\ &= \frac{kq^2}{2b^6} \int_0^b r^4\,\mathrm{d}r \\ &= \frac{kq^2}{10b} \end{aligned}$$

The total energy is $U_{int} + U_{ext} = 3kq^2/5b$.

10-26 (a) Its volume is (length) × (circumference) × (thickness) = $(L)(2\pi r)(t)$. The electric field is $E = V/t$. The energy stored is

$$\begin{aligned} \frac{1}{8\pi k}|\mathbf{E}|^2 \times (\text{volume}) &= \frac{1}{8\pi k}\left(\frac{V}{t}\right)^2 (2\pi r L t) \\ &= \frac{V^2 r L}{4kt} \end{aligned}$$

(b) Making r small means that the neuron needs less energy stored in it to have voltage V across its membrane, which may explain why they evolved to be so skinny. There is a limit on how small r can be, however, because the cell can't be so skinny that it lacks room inside for its machinery. Having a thicker membrane would also reduce the energy needed (dividing by a larger number gives a smaller result), but a cell membrane can't be arbitrarily thick, because it needs to take in nutrients and expel wastes through it.

10-27 (a) Plugging in to the equation for the electric field energy, we have $U_{one} = 4.43 \times 10^{-12}$ J.

(b) The two fields are perpendicular to each other, and add like vectors, so the result is given by the Pythagorean theorem, $E_{together} = \sqrt{2}(1\ \text{V/m}) = 1.41$ V/m.

(c) $E^2_{together}$ is exactly twice E^2_{one}, so the energy is the same when they're together as when they're apart. There is no energy consumed or released when you slide them together. (You would feel torques, but no net repulsion or attraction.)

10-28 The energy of a spherical capacitor,

$$U = \frac{kq^2}{2}\left(\frac{1}{a} - \frac{1}{b}\right) \qquad ,$$

was derived in an example, and when b approaches infinity, we have simply $U = kq^2/2a$. The definition of capacitance is $U = q^2/2C$, so we have $C = a/k$. Plugging in the radius of the earth yields 710 μF. This is a surprisingly small value, considering how big the earth is!

10-29 (a) For L approaching infinity, the first term inside thw square root becomes negligible compared to the second term, and the square root approaches $\sqrt{L^2/4R^2} = L/2R$. The desired result then follows immediately.

(b) We know the field of both infinite and finite lines of charge, so we can attack the problem either by slicing the strip lengthwise or across. Let's slice it lengthwise, since the field of an infinite line of charge is particularly simple, $E = 2k\lambda/R$. (This is the answer to another homework problem, given in the back of the book.) Let x be the direction across the strip, so that $\lambda = \sigma\,\mathrm{d}x$. We then have

$$E = \int_{-b/2}^{b/2} \frac{2k\sigma\,\mathrm{d}x}{R} \cdot \cos\theta \qquad ,$$

where $\theta = \cos^{-1} z/R$ is the angle between the z axis and the line connecting our point to a particular line.

$$\begin{aligned} E &= \int_{-b/2}^{b/2} \frac{2k\sigma\,\mathrm{d}x}{R} \cdot \frac{z}{R} \\ &= 2k\sigma z \int_{-b/2}^{b/2} \frac{\mathrm{d}x}{R^2} \\ &= 2k\sigma z \int_{-b/2}^{b/2} \frac{\mathrm{d}x}{x^2+z^2} \\ &= 4k\sigma \tan^{-1}\frac{b}{2z} \end{aligned}$$

(c) For z approaching zero, we have the arctangent of infinity, which is $\pi/2$, giving $E = 2\pi k\sigma$, and this is the correct result for the field near a charged surface. For large z, the argument of the arctangent is small, and we have $\tan^{-1}\epsilon \approx \epsilon$, yielding $E = 2kb\sigma/z = 2k\lambda/z$. From far away, the strip looks infinitely thin, like a line of charge, and this is indeed the right result for the field of an infinitely long line of charge.

10-30 We can slice the cylinder like a salami into thin slices of thickness $\mathrm{d}z$, and then integrate, because we already know that the on-axis field of a disk is $2\pi\sigma k\left(1 - z/\sqrt{b^2+z^2}\right)$. Because the slices are so thin, the surface charge density of each slice is an infinitesimal number, $\mathrm{d}\sigma$. The field is

$$\begin{aligned} E &= \int \mathrm{d}E \\ &= \int 2\pi\,\mathrm{d}\sigma k\left(1 - \frac{z}{\sqrt{b^2+z^2}}\right) \end{aligned}$$

The relationship between $\mathrm{d}\sigma$ and $\mathrm{d}z$ is $\mathrm{d}\sigma = \rho\,\mathrm{d}z$. (One way to see this is to consider a tiny cube of charge inside the cylinder, with dimensions $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$. The cube's charge is $\rho\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$, and dividing by the area $\mathrm{d}x\,\mathrm{d}y$ gives $\rho\,\mathrm{d}z$.) We then have

$$\begin{aligned} E &= \int_0^{\ell} 2\pi\rho\,\mathrm{d}zk\left(1 - \frac{z}{\sqrt{b^2+z^2}}\right) \\ &= 2\pi\rho k \int_0^{\ell} \left(1 - \frac{z}{\sqrt{b^2+z^2}}\right)\,\mathrm{d}z \end{aligned}$$

The integral can be done with computer software, or by looking it up in a table, or by substituting $u = z^2$. The result is

$$E = 2\pi k\rho\left(\ell + b - \sqrt{b^2+\ell^2}\right)$$

10-31 We use the type of coordinate system suggested in the hint, and let the disk lie directly above the origin, in the first and second quadrants. In cylindrical coordinates, a little trig shows that the equation of the disk is $r \le 2b\sin\theta$. The voltage is

$$\begin{aligned} V &= \int \frac{k\,\mathrm{d}q}{r} \\ &= k\sigma \int \frac{\mathrm{d}A}{r} \\ &= k\sigma \int \frac{r\,\mathrm{d}r\,\mathrm{d}\theta}{r} \\ &= k\sigma \int_{\theta=0}^{\pi} \int_{r=0}^{2b\sin\theta} \mathrm{d}r\ \mathrm{d}\theta \\ &= k\sigma \int_{\theta=0}^{\pi} 2b\sin\theta\ \mathrm{d}\theta \\ &= 4bk\sigma \qquad . \end{aligned}$$

10-32

$$\begin{aligned} U &= \frac{q^2}{2C} \\ &= \frac{(CV)^2}{2C} \\ &= \frac{1}{2}CV^2 \end{aligned}$$

10-33 (a) The capacitors have a total of four plates, and the two in the middle, which are connected, are isolated, so their total charge will always be zero. Therefore, the two middle plates must have the same amount of charge but with opposite signs. Thus the pattern of charges on the four plates must be $+q$, $-q$, $+q$, $-q$. If we're just connecting to them from the outside, we can think of them as one big capacitor, and we don't care about the middle plates; we just think about the charges $+q, \ldots -q$ on the ends. Each capacitor stores an energy $U_{one} = q^2/2C_{one}$, so the total energy is $U_{combined} = 2U_{one}$. Setting this equal to $q^2/2C_{combined}$, we get

$$C_{combined} = C_{one}/2 \qquad .$$

(b) Doubling the distance between the plates is like sandwiching together two capacitors in series, so it cuts the capacitance in half. The capacitance is evidently inversely proportional to the separation between the plates.

10-34 (a) As suggested in section 10.5.9, let's identify sinusoidal functions with points in the complex plane. The function $4\sin\omega t$ corresponds to the real number 4, and $3\cos\omega t$ to $3i$. Their sum is the function corresponding to $4+3i$. Now we have to go backwards and find what sinusoidal function $4+3i$ represents. The magnitude of this complex number represents the amplitude, A, of the result, so $A = \sqrt{4^2+3^2} = 5$. The argument represents the phase angle $\delta = \tan^{-1}(3/4) = 37°$.

(b)

$$\begin{aligned} 5\sin(\omega t + 37°) &= 5[\sin\omega t\cos 37° + sin 37° \cos\omega t] \\ &= 3.9\sin\omega t + 3.0\cos\omega t \end{aligned}$$

To within rounding errors, this verifies that the result was correct.

10-35 (a) By definition, one henry is 1 J/A^2. The quantity $L\,\mathrm{d}I/\,\mathrm{d}t$ therefore has units of

$$\begin{aligned} \left(\frac{\mathrm{J}}{\mathrm{A}^2}\right)\left(\frac{\mathrm{C/s}}{\mathrm{s}}\right) &= \left(\frac{\mathrm{J}}{(\mathrm{C/s})^2}\right)\left(\frac{\mathrm{C/s}}{\mathrm{s}}\right) \\ &= \frac{\mathrm{J}}{\mathrm{C}} \\ &= \mathrm{V} \end{aligned}$$

(b) A farad is defined as 1 C^2/J, so RC has units of

$$\begin{aligned} \Omega\left(\frac{\mathrm{C}^2}{\mathrm{J}}\right) &= \left(\frac{\mathrm{V}}{\mathrm{A}}\right)\left(\frac{\mathrm{C}^2}{\mathrm{J}}\right) \\ &= \left(\frac{\mathrm{J/C}}{\mathrm{C/s}}\right)\left(\frac{\mathrm{C}^2}{\mathrm{J}}\right) \\ &= \mathrm{s} \end{aligned}$$

(c) L/R has units of

$$\begin{aligned} \left(\frac{\mathrm{J}}{\mathrm{A}^2}\right)\Omega^{-1} &= \left(\frac{\mathrm{J}}{\mathrm{A}^2}\right)\left(\frac{\mathrm{A}}{\mathrm{V}}\right) \\ &= \frac{\mathrm{J}}{\mathrm{A{\cdot}V}} \\ &= \frac{\mathrm{J}}{\mathrm{W}} \\ &= \frac{\mathrm{J}}{\mathrm{J/s}} \\ &= \mathrm{s} \end{aligned}$$

10-36 The definition of inductance is $U = LI^2/2$. If we think of the pair of inductors as one big inductor, then applying this definition, I has to be the total current that goes in. This gets split in half, so the energy stored in a single inductor is $U_{one} = L_{one}(I/2)^2/2$, and the total energy is $U = 2U_{one} = L_{one}I^2/4$. Equating this to $LI^2/2$, we get $L = L_{one}/2$. You get half the inductance by putting two in parallel.

10-37 When you have an exponent with a funky base, the standard method for reexpressing it in terms of base e is to apply the identity $\ln(a^b) = b\ln a$, which gives $a^b = \exp(b\ln a)$. This gives $i^i = e^{i \ln i}$. This reduces the problem to one of finding the natural log of i. Euler's formula tells us that raising e to the power $(\pi/2)i$ gives i. (Actually there is more than one solution, but we'll come back to that.) Taking $\ln i = (\pi/2)i$, we find

$$i^i = e^{-\pi/2} \qquad .$$

Euler's formula tells us that e^x is the same as $e^{x+2\pi i}$, $e^{x+4\pi i}$, etc. That means that $\ln i$ has more than one possible value. It can be $(\pi/2 + 2\pi n)i$, where n is any integer. This means that i^i could be any real number of the form $e^{-\pi/2-2\pi n}$. It shouldn't be too surprising that an exponent such as i^i can have more than one value. For example, $4^{1/2}$ can equal either 2 or -2.

10-38 We normally think of wires as having zero resistance. If the wire has some inductance, then the added impedance is higher at high frequencies, so stray inductance will be most important at high frquencies. The impedance of a capacitor, however, varies inversely with frequency, so stray capacitance will be most important at low frequencies.

10-39 The reasoning is very similar to the reasoning by which we derived the impedance of a capacitor. Suppose an inductor has a current $I = \tilde{I} \sin \omega t$ passing through it. The voltage difference across the inductor is

$$\begin{aligned} V &= L \frac{\mathrm{d}I}{\mathrm{d}t} \\ &= L\omega\tilde{I}\cos\omega t \qquad . \end{aligned}$$

The amplitude of this voltage is $L\omega\tilde{I}$. The impedance is defined as the amplitude of the voltage divided by the amplitude of the current, so we get $L\omega$.

10-40 The result of problem 21 for the field of an infinite strip of width b was $E = 4k\sigma\tan^{-1}(b/2z)$. Slicing the box up into strips, with each strip having an infinitesimal surface charge density $\mathrm{d}\sigma = \rho\,\mathrm{d}z$, we have

$$\begin{aligned} E &= \int \mathrm{d}E \\ &= \int 4k\tan^{-1}\frac{b}{2z}\ \mathrm{d}\sigma \\ &= \int_0^b 4k\tan^{-1}\frac{b}{2z}\,\rho\,\mathrm{d}z \\ &= 4k\rho\int_0^b \tan^{-1}\frac{b}{2z}\ \mathrm{d}z \end{aligned}$$

Substituting $u = 2z/b$, we have

$$E = 2k\rho b\int_0^2 \tan^{-1}\frac{1}{u}\ \mathrm{d}u$$

The integral can be looked up in a table or done with computer software, and the result is

$$E = 2k\rho b\left(2\tan^{-1}\frac{1}{2} + \frac{1}{2}\ln 5\right)$$

10-41 Newton's laws won't give a solution in closed form, since the force and acceleration wouldn't be constant, and wouldn't be known as a function of time. Instead, we need to use conservation of energy. The loss in electrical energy $q\Delta V$ equals the gain in kinetic energy $(1/2)mv^2$. The voltage can be found by integrating over the surface of the pipe,

$$V = k\int \frac{\sigma\,\mathrm{d}A}{r} \qquad ,$$

where r is the distance between the point at which we're calculating the voltage and a point on the cylinder. The two points at which we need to calculate the voltage are the center of the cylinder and a point at infinity. In the latter case, r is infinite, so $V = 0$. In other words, ignoring signs, we can just calculate this integral for the center of the pipe, and use the result as ΔV.

$$\begin{aligned} V &= k\int_{-\ell/2}^{\ell/2} \frac{\sigma 2\pi b\,\mathrm{d}z}{r} \\ &= k\int_{-\ell/2}^{\ell/2} \frac{\sigma 2\pi b\,\mathrm{d}z}{\sqrt{b^2+z^2}} \end{aligned}$$

The integral can be done with computer software, or looked up in a table. The result is

$$V = 4\pi k\sigma b \,\text{asinh}\left(\frac{\ell}{2b}\right) \qquad ,$$

where asinh is the inverse hyperbolic sine function, which can also be written as $\text{asinh}\, x = \ln\left(x+\sqrt{x^2+1}\right)$. For the velocity when the charge is infinitely far away from the "gun," we have:

$$\begin{aligned}\frac{1}{2}mv^2 &= q|\Delta V|\\ &= q|0-V|\\ &= qV\\ v &= \sqrt{2\frac{q}{m}V}\\ &= \sqrt{8\pi k\sigma b\frac{q}{m}\,\text{asinh}\left(\frac{\ell}{2b}\right)}\end{aligned}$$

10-42 Solving $\omega \approx 1/\sqrt{LC}$ for C, we get $C = 1/L\omega^2 = 1/4\pi^2L^2f^2$. Looking at the nearest handy FM radio dial, the range of frequencies is about 88-108 MHz, so the capacitor needs to cover the range from about 2.2 to 3.3 pF.

10-43 (a)

$$\begin{aligned}Z &= (1/Z_R + 1/Z_C)^{-1}\\ &= [1/R + 1/(-i/\omega C)]^{-1}\\ &= (2.70+6.28i)^{-1}\times 10^5\ \Omega\end{aligned}$$

The magnitude of a complex number's inverse is the inverse of its magnitude, so

$$\begin{aligned}|Z| &= \frac{1}{\sqrt{2.70^2+6.28^2}}\times 10^5\ \Omega\\ &= 1.5\times 10^4\ \Omega\end{aligned}$$

(rounding the final result to two sig figs). The argument of a complex number's inverse is minus the argument of the original number, so $\arg Z = -\tan^{-1}(6.28/2.70) = -69°$.

(b) The amplitude of the current is

$$\begin{aligned}\tilde{I} &= \tilde{\Delta V}/|Z|\\ &= 68\ \text{nA} \qquad .\end{aligned}$$

Suppose the current has the phase of a sine wave. Then we associate the current with a point on the real axis, and multiplying it by the impedance will give a voltage that has an argument of −69°, i.e., closer to the negative imaginary axis. The negative imaginary axis represents something with the phase of $-\cos$, and if you sketch the graphs of sin and $-\cos$, you'll see that $-\cos$ is behind by 90 degrees in phase. Therefore the voltage is behind the current, by 69 degrees.

Another method for doing this, which avoids having to memorize what represents what in the complex plane, is to note that an impedance with an argument of −90° (on the negative imaginary axis) is capacitive, so our parallel RC circuit, with its impedance having an argument of −69°, is dominated by its capacitor. For a purely capacitive circuit, the current leads the voltage, so we know that will also be true in our circuit. The mnemonic "eVIL" may come in handy here. It tells you that the voltage (V) leads the current (I) in an inductive (L) circuit, whereas it's the other way around in a capacitive one. (Some people know this as "ELI the ICE man," with E as a notation for emf, which is a concept, closely related to voltage, that we'll encounter later.)

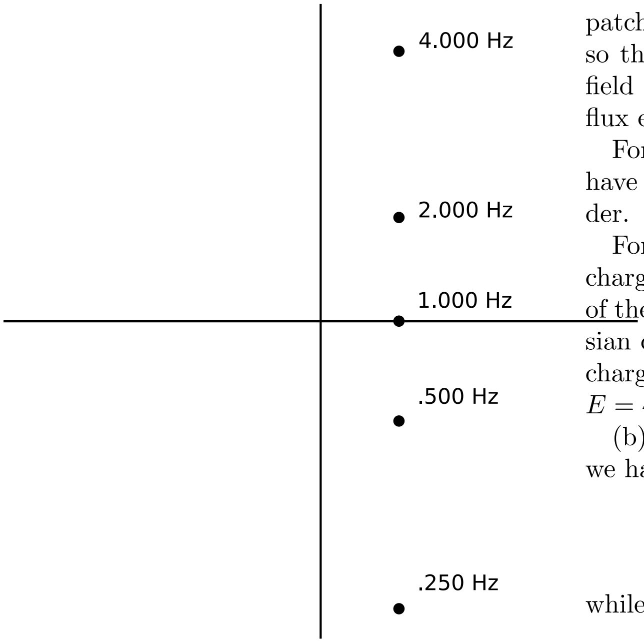
10-44 (a) Impedances add in series, so

$$\begin{aligned}Z &= R + i\omega L - \frac{i}{\omega C}\\ &= 1 + i(\omega - 1/\omega) \qquad ,\end{aligned}$$

where, for ease of writing, I've omitted the units and sig figs. The results are:

ω (Hz)	Z (Ω)
0.250	$1.000-3.750i$
.500	$1.000-1.500i$
1.000	1.000
2.000	$1.000+1.500i$
4.000	$1.000+3.750i$

Here's what it looks like in the complex plane:



(b) The resonant frequency is $\omega \approx 1/\sqrt{LC} =$ 1.000 Hz. This is the frequency at which the magnitude of the impedance is the smallest, so the largest current will flow for a given voltage.
(c) $f = \omega/2\pi = 0.1592$ Hz

10-45 In parallel, it will be the inverse impedances that add. The total inverse impedance is $1/Z = (0.4 - 0.3i)$ kΩ^{-1}. The real part of this can be contributed by the inverse resistance of a resistor with a resistance of 2.5 kΩ. For the imaginary part, we need a component whose inverse impedance $1/Z$ will be a negative imaginary value. That means that its impedance Z has to be a positive imaginary value, so it has to be an inductor, just as in the original series setup. We then have $L = Z/(i\omega) = Z/(2\pi i f) = 10$ mH.

10-46 (a) The Gaussian surface can be a finite cylinder of length ℓ and radius R, sharing the same axis with the charged cylinder. By symmetry, the field is completely radial, so there is no flux through the end-caps. Symmetry also requires that the field's magnitude depend only on R, so the field on the curved surface of our Gaussian surface is all of the same strength, and we don't have to break it down into many tiny patches. The area of the curved surface is $2\pi R\ell$, so the flux is $2\pi R\ell E$, where E is the unknown field we're trying to find. By Gauss' law, this flux equals $4\pi k q_{in}$.

For $R < b$, the enclosed charge is zero, so we have $E = 0$ throughout the interior of the cylinder.

For $R > b$, the enclosed charge equals the charge density multiplied by the area of the part of the charged cylinder that lies inside the Gaussian cylinder. This area is $2\pi b\ell$, so the enclosed charge is $2\pi b\ell\sigma$. Equating this to the flux gives $E = 4\pi k\sigma b/R$.

(b) For R approaching b from the positive side, we have

$$\lim_{R\to b^+} E = 4\pi k\sigma \qquad ,$$

while on the inside,

$$\lim_{R\to b^-} E = 0 \qquad ,$$

The discontinuity amounts to a jump of $|\mathbf{E}_{\perp,1} - \mathbf{E}_{\perp,2}| = 4\pi k$, which is the expected result for the field near a charged surface.

(c) The charge per unit length is $\lambda = (2\pi b\ell\sigma)/(\ell) = 2\pi b\sigma$, so the external field can be expressed as $E = 2k\lambda/R$, which is the same as the field of a line of charge. It makes sense that it comes out this way, because our argument only depended on the symmetry, not on the exact placement of the charge inside the Gaussian surface.

(d) For $R < a$, the field is zero, because neither conductor contributes any field. For $R > b$, the fields $E = \pm 2k\lambda/R$ cancel. Only for $a < R < b$ do we have a nonvanishing field. In this region, there is no contribution from the outer cylinder, so the field is simply that of the inner cylinder, $E = 2k\lambda/R$.

The capacitance per unit length is defined in terms of the energy contained in the field. To find this energy, we have to integrate over concentric cylindrical shells, with each shell stretching from R to $R + \mathrm{d}R$. The volume of such a shell is (circumference)(length)(thickness), or $2\pi\ell R\ \mathrm{d}R$. The total energy is

$$\begin{aligned}U &= \int_a^b \frac{E^2}{8\pi k}\ \mathrm{d}V \\ &= \int_a^b \left(\frac{k\lambda^2}{2\pi R^2}\right)\ (2\pi\ell R\ \mathrm{d}R) \\ &= k\lambda^2\ell \int_a^b \frac{\mathrm{d}R}{R} \\ &= k\lambda^2\ell\ \ln\frac{b}{a}\end{aligned}$$

The capacitance is

$$\begin{aligned}C &= \frac{q^2}{2U} \\ &= \frac{\lambda^2\ell^2}{2U} \\ &= \frac{\ell}{2k\ \ln\frac{b}{a}} \qquad .\end{aligned}$$

The capacitance per unit length is

$$\frac{C}{\ell} = \left(2k\ \ln\frac{b}{a}\right)^{-1} \qquad .$$

10-47 (a) The field is parallel to four of the box's sides, and perpendicular to the other two, so only those two fluxes are nonzero. The field passes in through one of these sides and out through the other, so one flux is positive and one is negative. Because the field is constant, these two fluxes cancel exactly, and the total flux is zero. By Gauss's law, the total charge inside the box is therefore zero.

We could imagine that there were regions of positive and negative charge density, and this particular box just happened to surround a region with equal amounts of positive and negative charge. But we haven't just proved that this particular box contains zero total charge; we've proved that there's zero total charge inside *any* box. The only way for the total charge inside *any* box to be zero is if there really is zero charge density at every point in this region of space.

(b) The field must be due to some charges that are outside this region of space.

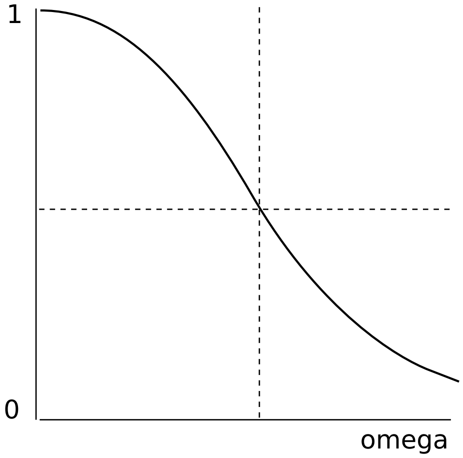
10-48 (a) A DC current can't flow through a capacitor, so the steady-state behavior of the circuit is that the capacitor is charged up, and there is no current flowing through the inductor. That means there's an electric field in the capacitor, but no magnetic field in the inductor. All of the energy is stored in the capacitor, and none in the inductor.

(b) The energy is being shuffled back and forth between the capacitor and the inductor.

(c) The fields oscillate sinusoidally, and the energy in a field is proportional to the square of the field, so the variation in each field's energy over time has the form of a square of a sinusoidal function. The square of a sine wave is a function that oscillates between 0 and 1, and its average value is 1/2, so in each case, the average energy in the field equals half the maximum energy. If we have $q = \tilde{q}\sin\omega t$ for the charge on one of the plates of the capacitor, then differentiatoin gives $I = \tilde{q}\omega\cos\omega t$. Then

$$\begin{aligned}u &= \frac{\overline{U_C}}{\overline{U_L}+\overline{U_C}} \\ &= \frac{U_{C,max}}{U_{L,max}+U_{C,max}} \\ &= \left(\frac{U_{L,max}}{U_{C,max}}+1\right)^{-1} \\ &= \left(\frac{(1/2)L\tilde{I}^2}{(1/2C)\tilde{q}^2}+1\right)^{-1} \\ &= \left(\frac{LC\tilde{q}^2\omega^2}{\tilde{q}^2}+1\right)^{-1} \\ &= \frac{1}{LC\omega^2+1}\end{aligned}$$

The figure shows the behavior of this expression as a function of frequency. The capacitor's share is 100% at zero frequency, in agreement with part a. It drops to 50% at resonance ($\omega = 1/\sqrt{LC}$), and at twice the resonant frequency, it equals 20%.



10-49 (a) The setup is similar to the preceding problem. We again choose as our Gaussian surface a finite cylinder of length ℓ and radius R. Gauss's law gives $2\pi R\ell E = 4\pi k q_{in}$, where $q_{in} = \pi R^2 \ell\rho$. Solving for the field, we have $E = 2\pi k R\rho$.

(b) At $R = 0$, we get $E = 0$. This makes sense, because the field has to vanish on the axis by symmetry. If it were to be nonzero, there would be no way to decide which direction it should point.

(c) The units of $E = 2\pi k R\rho$ are

$$\frac{\mathrm{N}}{\mathrm{C}} =? \left(\frac{\mathrm{N{\cdot}m^2}}{\mathrm{C^2}}\right)(\mathrm{m})\left(\frac{\mathrm{C}}{\mathrm{m^3}}\right),$$

which does check out.

10-50 (a) The field is parallel to all the sides of the cube except for the ones at $x = 0$ and $x = a$, so those are the only ones that might have nonzero flux through them. The field is zero at $x = 0$, so in fact the only flux is through the side at $x = a$. On this side, the field has a constant magnitude of ba, and is always perpendicular to the surface, so we don't have to do an integral to find the flux; we can just multiply the field's magnitude by the area, giving $\Phi_E = ba^3$. The charge inside the box is $q_{in} = \Phi_E/4\pi k = ba^3/4\pi k$.

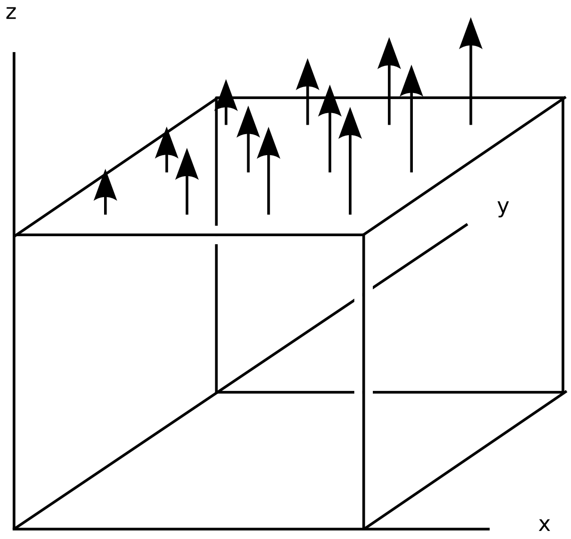
(b) The field is only perpendicular to the sides of the cube at $z = 0$ and $z = a$, but because the field doesn't depend on z, the inward flux through one of these sides is the same as the outward flux through the other. The total flux is zero, so there's zero total charge inside the box.

(c) Fluxes add, so we can calculate the fluxes of the two terms separately. The first term is really no different than the field in part a, except that it's 13 times stronger, and turned around to lie along the z axis. Therefore the flux it contributes is $13ba^3$. The second term is like the field in b, and doesn't give any flux. The total charge inside the box is $13ba^3/4\pi k$.

(d) The figure below shows the only nonvanishing flux, which is through the top side of the cube. (The fluxes on the sides are zero because the field is parallel to them, and the flux through the bottom is zero because the field is zero at $z = 0$.) The flux through the top is

$$\begin{aligned}
\Phi_E &= \int_{\text{top}} \mathbf{E}\cdot \mathrm{d}\mathbf{A} \\
&= \int_{\text{top}} bxa\hat{z}\cdot \mathrm{d}\mathbf{A} \\
&= \int_{x=0}^{a}\int_{y=0}^{a} bxa\ \mathrm{d}y\,\mathrm{d}x \\
&= ba\int_{x=0}^{a} x\int_{y=0}^{a}\ \mathrm{d}y\,\mathrm{d}x \\
&= ba\int_{x=0}^{a} xa\ \mathrm{d}x \\
&= \frac{1}{2}ba^4
\end{aligned}$$

The charge inside is $ba^4/8\pi k$.



10-51 The field points in the $\hat{\mathbf{x}}$ direction, but only depends on z. The reasoning is just like that in problem 44b. For any box aligned with the x, y, and z axes, the fluxes through the two sides perpendicular to x cancel out, because the fields don't differ for different values of x. If there's zero charge inside *any* such box, then there must not be any charge anywhere.

10-60 The energy density is proportional to the square of the field strength, so decreasing the field by a factor of 10 is equivalent to cutting the energy by a factor of 100.

10-61 (a) The answer must be of the form $\alpha q\sqrt{k/mg}$, where α is a unitless constant.

(b) We can place an image charge $-q$ above the conducting plane, at a distance 2ℓ from the real charge. The force is $(1/4)kq^2/\ell^2$. Balancing this against the gravitational force mg gives $\ell = (1/2)q\sqrt{k/mg}$.

(c) If the charge moves upward from the equilibrium position, the upward electrical force gets stronger, so there is a net upward force. Similarly, a downward displacement gives a net downward force.

10-62 (a) Because θ is unitless, it can only depend on the unitless ratio y/x.

(b) The trick here is to use three image charges: charges $-q$ at $(x, -y)$ and $(-x, y)$, and a $+q$ at $(-x, -y)$. This is exactly analogous to the formation of images in a pair of mirrors placed at right angles to one another. If we take this arrangement of charges, flip it across the y axis, and then negate all the charges, it becomes the same arrangement again. It follows that E_y vanishes, as required, on the y axis. A similar argument holds for the x axis.

By the result of part a, we can ignore factors of q and the Coulomb constant. Vector addition gives $4F_x = -1/x^2 + x/r^3$ and $4F_y = -1/y^2 + y/r^3$. The result is

$$\tan\theta = \frac{y - r^3/y^2}{x - r^3/x^2},$$

which can also be expressed in terms of y/x, but without much apparent simplification.

10-63 (a) Multiplication adds the arguments, so raising something to the third power triples its argument. The result is 3θ (or $3\theta + 2\pi n$, where n is any integer).

(b) If $a^3 = z$, then the argument of a should be 1/3 that of z. (There would be additional possibilities such that $\arg 3a = \arg z + 2\pi n$, but we're only talking about the first quadrant, so that can't happen.) The only one that looks like it has 1/3 the argument of z is x.

(c) The argument has to be 1/3 that of $a + bi$, i.e., $(1/3)\tan^{-1}(b/a)$.

(d) The easy way to do the cube root is to use the fact that if $y = x^3$, then $|y| = |x|^3$, and $\arg y = 3\arg x$. Therefore if y is the quantity inside the exponent in the denominator, we have $|x| = (\sqrt{8})^{1/3} = \sqrt{2}$, and the easiest possibility for the argument is $\arg x = (1/3)\arg y = \pi/4$. (There are other possibilities such as $11\pi/12$, because arguments that differ by a multiple of 2π represent the same thing.) Division divides the magnitudes, giving a magnitude of 1. Division subtracts the arguments, giving 0. Therefore the result of the calculation is 1.

10-64 This is impractical to carry out by the methods of freshman calculus, without using complex numbers. The trick is to use Euler's formula to rewrite $e^x\cos x$ as $(1/2)e^x(e^{ix} + e^{-ix})$. The result is -2^{50}.

11-1 The electric force vector is

$$\begin{aligned}\mathbf{F}_E &= q\mathbf{E}\\ &= (1\ \text{N})\hat{\mathbf{y}}\qquad.\end{aligned}$$

The magnitude of the magnetic force is

$$\begin{aligned}|\mathbf{F}_B| &= |q\mathbf{v}\times\mathbf{B}|\\ &= q|\mathbf{v}\times\mathbf{B}|\\ &= q|\mathbf{v}||\mathbf{B}|\sin\theta_{vB}\\ &= 2\ \text{N}\qquad,\end{aligned}$$

and its direction is determined by the right hand rule to be along the z axis, so

$$\mathbf{F}_B = (2\ \text{N})\hat{\mathbf{z}}\qquad.$$

The total force on the particle is

$$\mathbf{F} = (1\ \text{N})\hat{\mathbf{y}} + (2\ \text{N})\hat{\mathbf{z}}\qquad.$$

11-2 The original rule listed flat fingers, bent fingers, and thumb in the order

$$\mathbf{v}\quad\mathbf{B}\quad\mathbf{F}\qquad.$$

Playing with the model, you'll find that the other two ways that work are

$$\begin{aligned}&\mathbf{B}\quad\mathbf{F}\quad\mathbf{v}\qquad\text{and}\\ &\mathbf{F}\quad\mathbf{v}\quad\mathbf{B}\qquad.\end{aligned}$$

These are like starting later in the original list, and then "wrapping around" when we get to the end. The three that don't work are

$$\begin{aligned}&\mathbf{B}\quad\mathbf{v}\quad\mathbf{F}\qquad,\\ &\mathbf{v}\quad\mathbf{F}\quad\mathbf{B}\qquad,\ \text{and}\\ &\mathbf{F}\quad\mathbf{B}\quad\mathbf{v}\qquad.\end{aligned}$$

These are left-handed.

11-3 There must be either $\mathbf{E}$ or $\mathbf{B}$ or both, since some force caused the particle to accelerate. It can't be a pure magnetic field, because $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$ is zero for $\mathbf{v} = 0$, so the particle would not have accelerated from rest. It can't be a pure electric field, because a uniform electric field would just have caused the particle to accelerate in a straight line. There must be both $\mathbf{E}$ and $\mathbf{B}$ present.

11-4 (a) $\mathbf{F} = q\mathbf{v}\times\mathbf{B} = 0$, so the particle remains at rest.

(b) $\mathbf{F} = q\mathbf{v}\times\mathbf{B} = q|\mathbf{v}||\mathbf{B}|\sin 0° = 0$, so there is no force, and the particle travels in a straight line with constant speed.

(c) The force will always be perpendicular to the z axis, so since the particle initially has no velocity in the z direction, it will remain in the same plane, parallel to the x-y plane. The particle orbits in a circle in this plane, at constant speed.

11-5 (a) The length of the segment that passes a given point in time $\mathrm{d}t$ is $v\,\mathrm{d}t$, and this segment has charge $\lambda v\,\mathrm{d}t$. The current equals this charge divided by $\mathrm{d}t$, which comes out to be λv.

(b)

$$\left(\frac{\text{C}}{\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right) = \frac{\text{C}}{\text{s}} = \text{A}$$

11-6 (a)

$$B_1 = \frac{2kI_1}{c^2R}$$

(b)

$$\begin{aligned}I_2 &= \lambda_2 v_2\\ F &= q_2 v_2 B_1\\ &= \lambda_2 L v_2 B_1\\ &= I_2 L B_1\\ &= \frac{2kI_1I_2L}{c^2R}\\ &= \frac{k}{c^2}\cdot\frac{2I_1I_2L}{R}\end{aligned}$$

(c)

$$\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\cdot\frac{\text{s}^2}{\text{m}^2}\cdot\frac{\text{A}^2\cdot\text{m}}{\text{m}} = \text{N}$$

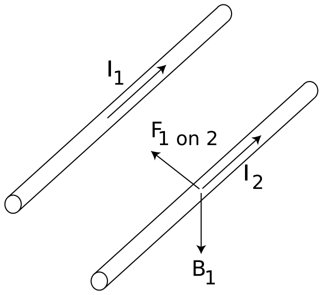
11-7 The electric force is either in the same direction as the electric field or in the opposite direction, depending on whether the particles are positively or negatively charged. Since the magnetic force is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ and the question states that the electric field is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$, the magnetic force must be either in the same direction as the electric force or in the opposite direction. Since the goal is to make the forces cancel, we'll assume the directions of the fields are chosen so the forces are in opposite directions. That means the forces cancel if they're chosen to have equal magnitudes. The electric force equals $q\mathbf{E}$. The magnitude of the magnetic force equals $qvB\sin\theta$, and since the $\mathbf{v}$ and $\mathbf{B}$ vectors are perpendicular, $\sin\theta = 1$. Therefore we need $qE = qvB$. Canceling q from both sides and solving for v, we have $v = E/B$.

11-8 The second data-point isn't very surprising, because multiplying charge by -2 and the velocity by -1 multiplies the magnetic force by $(-2)(-1) = 2$. The only thing the second data-point really tells us is that the field is purely magnetic. If the $\mathbf{F}$, $\mathbf{v}$, and $\mathbf{B}$ vectors had to be perpendicular, there would only be one possibility for this field, $\mathbf{B} = (1\ \text{kT})\hat{\mathbf{z}}$. But a magnetic field along the same direction as the motion exerts no force, so we can throw in an arbitrary x component, giving a solution such as $\mathbf{B} = (1\ \text{kT})\hat{\mathbf{z}} + (37\ \text{kT})\hat{\mathbf{x}}$.

11-9 It can't be a purely electric field, because then the velocity would have no effect on the force. It can't be purely magnetic, either, because tripling the velocity didn't simply triple the force, as it would if the only force was $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$.

11-10 (a) The magnitudes are the same, as required by Newton's third law.

(b)



(c) The force is in the opposite direction, again as required by Newton's third law; each wire is attracting the other.

(d) They would reoek each other.

11-11 (a) The field is coming out of the page, perpendicular to the plane of the coils. See the answer to the self-check.

(b) The current is counterclockwise. One way to tell this is to imagine breaking down each coil into parts. For example, we could reshape them into octagons. Each of the sixteen segments would contribute the same field at the center of the loop, and the direction of the field would be given by the right-hand rule, just as if the segments were long wires.

(c) The electrons have negative charge. To produce a counterclockwise current, they need to be moving clockwise.

(d) It's a repulsion. The electrons in the beam are being pushed inward, away from the coils. This is consistent with the result of the previous problem, where we found that there is a repulsion between currents that are in opposite directions.

11-12 (a) See the answer to the self-check.

(b)

$$\begin{aligned} a &= \frac{F}{m} \\ \frac{v^2}{r} &= \frac{qvB\sin 90°}{m} \\ \frac{v}{r} &= \frac{q}{m}B \\ r &= \frac{m}{q}\cdot\frac{v}{B} \end{aligned}$$

(c) From the definition $B = F/(qv\sin\theta)$, we see that the Tesla unit breaks down into N·s/(C·m).

$$\begin{aligned} \frac{\text{kg}}{\text{C}}\cdot\frac{\text{m/s}}{\text{T}} &= \frac{\text{kg}\cdot\text{m}}{\text{C}\cdot\text{s}}\cdot\frac{\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}} \\ &= \frac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot\frac{1}{\text{N}}\cdot\text{m} \\ &= \text{m} \end{aligned}$$

(d) The equation derived in part b has m and v on the top, so it states that increasing either of these variables will increase the radius of the circle. The opposite is true for q and B.

mass: Increasing the mass of the particles makes it harder for a given force to turn the beam away from its initial path, so the turn will be wider.

charge: Increasing the charge of the particles

makes it easier for a given field to act on them, causing them to execute a tighter curve.
velocity: This is like driving a car. Freeway off-ramps have gentle curves, because a high-velocity car is harder to turn.
field: Increasing the magnetic field results in more force on each particle, making the beam curve more tightly.
(e)

$$\begin{aligned} v &= \omega r \\ \omega &= \frac{v}{r} \\ &= \frac{qB}{m} \end{aligned}$$

11-13 (a) It can't be a purely frictional force, because then the particle wouldn't speed back up again. Magnetic fields don't do work, so it also can't be a pure magnetic field, since the particle's kinetic energy is changing. A magnetic field would make a force perpendicular to the particle's velocity vector, causing it to curve, which it isn't doing. Since there is zero magnetic force, there is probably zero magnetic field (although it is possible that the magnetic field is nonzero and parallel to the motion, causing zero force). We're left with the possibility of either a pure electrical force, or a mixture of electrical and kinetic frictional forces. If there was friction in the mix, then the friction would reverse its direction when the particle came back, while the electrical force remained constant. The forces would therefore be reinforcing for half of the motion, but at least partially canceling during the other half. The particle's acceleration, however, is the same in the two halves of the motion. We therefore conclude that an electrical force is the only one acting. The field must be down and to the left, along the direction of motion.
(b) As in part a, there is no evidence for any magnetic field. There must be friction here: if the force was purely electrical, then the particle would have constant acceleration, but it actually displays an abrupt change in acceleration from a nonzero value to zero. The simplest explanation of the observed motion is that there is no electrical field, and the particle is simply decelerated by kinetic friction. It's also possible, however, that there is an electric field along the direction of motion, either working against friction or cooperating with it in slowing the particle down. If there is an electric field, then once the particle stops, there must be a static frictional force keeping it from getting going again.
(c) The dots are equally spaced, so no work is ever being done on the particle. Therefore there can't be any electric field: the only way an electric field could avoid doing work on the particle would be if it was perpendicular to the motion, but since the particle is curving, and we're given that the electric field is uniform, it's not possible to have an electric field that is always perpendicular to the motion. Friction would also do work, so there is no friction. There must be a magnetic field into the page to produce the inward force required for circular motion.
(d) Work is being done, since the particle is slowing down and then speeding up. If we assume that the possible frictional force is exerted by some surface that is at rest with respect to the grid on the paper, then we can rule out a purely frictional force: kinetic friction would decelerate the sliding particle, but couldn't speed it up. Taking the time interval to be one and the spacing of the grid to be one, the velocity vectors are $3\hat{\mathbf{x}} + \hat{\mathbf{y}}$, $2\hat{\mathbf{x}} + 2\hat{\mathbf{y}}$, etc., with the x component always decreasing by 1 and y always increasing by 1. Therefore the acceleration is a constant one up and to the left, at a 45-degree angle with respect to the grid. This force must be coming from an electric field in this direction. If there was a kinetic friction force, it would always oppose the motion, so the force wouldn't continue to be in the same direction as the direction of the motion changed. If there was a magnetic field, the magnetic force would change its strength and direction as the particle's speed changed, so again, the force wouldn't be constant.

11-14

11-15 The units of

$$k \cdot I \cdot A \cdot c^{-2} \cdot r^{-3}$$

are

$$\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\cdot\frac{\text{C}}{\text{s}}\cdot\text{m}^2\cdot\frac{\text{s}^2}{\text{m}^2}\cdot\frac{1}{\text{m}^3} \qquad ,$$

which simplifies to N·s/C·m, and that's the same as teslas, as we can verify from the equation $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$. If we multiply something with units of teslas by the constant β, and get units of teslas, then β must be unitless.

11-16 Particle 1 isn't moving, so it experiences only an electric force. The electric field causing this force must be

$$\frac{(1\ \text{N})\hat{\mathbf{y}}}{1\ \text{C}} = (1\ \text{N/C})\hat{\mathbf{y}} \qquad .$$

Particle 2 experiences the same force as particle 1, even though it is moving along the x axis. Therefore, the magnetic field, if any, must lie along the x axis.

The magnetic and electric forces on particle 3 cancel, so the magnetic force on it must be $(-1\ \text{N})\hat{\mathbf{y}}$. A magnetic field along the x axis that produces this force must be $(-1\ \text{T})\hat{\mathbf{x}}$.

11-17 Four times the current makes four times the magnetic field, which makes $4^2 = 16$ times the energy.

11-18 The field at any point on the axis can be found by adding the contributions to the field from the two loops. By symmetry, both these fields are along the axis, so we can add them without worrying about vector addition. The whole problem just deals with comparisons, so the only part of the equation in section 24.2 we need to worry about is $(b^2+z^2)^{-3/2}$. Also, we can just let b equal 1, and interpret z as z/b.

(a) Evaluating $(1+z^2)^{-3/2}$ at $z=0$ and 0.5, and 1, we get 1.0000, 0.7155, and 0.3536. The total field at the center ($z = 0.5$ relative to both coils) is 0.7155×2, and the field in the center of one of the coils is $1.0000 + 0.3536$. The percentage drop is about 5.4% (expressed as a percentage of the stronger field).

(b) By experimenting with different values of h, we find a solution at about $h/b = 1.2$, i.e., a little larger than the Helmholtz value of $h/b = 1$.

11-19 (a) Using the equation for the magnetic field at the center of a current loop, we get 0.063 mT.

(b) Along the positive z axis.

(c) It would make a field with the same magnitude along the negative x axis.

(d) The total field is the vector sum of the two. Since they're perpendicular, the magnitude of the sum is given by the Pythagorean theorem, coming out to be 0.09 mT. It points at a 45° angle between the two, in the $x-z$ plane.

11-20 The hard part is visualizing the directions of the four fields that are adding together. Looking along the length of the apparatus, two of the fields point at a 45-degree angle down and to the left, while the other two point at 45 degrees down and to the right. By vector addition, this means that the total field is $2\sqrt{2}$ times the individual ones. Each individual field is $2kI/c^2R = 2\sqrt{2}kI/c^2b$ so the total field is $8kI/c^2b$.

11-21 At first it might seem possible, since we're trying to determine six numbers (E_x, E_y, E_z, B_x, B_y, and B_z), and two experiments would provide six pieces of raw data (F_{1x}, F_{1y}, F_{1z}, F_{2x}, F_{2y}, and F_{2z}). Ordinarily, we expect that six equations in six unknowns can be solved uniquely. However, this isn't true if the equations are redundant. For example, the system of equations

$$\begin{aligned} a &= 2b \\ 2a &= 4b \end{aligned}$$

can't be solved for a and b.

In fact, we can never solve these six equations for the six unknowns. Suppose we fix the design of your experiment by specifying $\mathbf{v}_1$ and $\mathbf{v}_2$. We may as well use unit charge in both cases, so these two velocities give a complete specification of the design of the experiment. But given this design, there will always be different fields that give the same results. To see this, let's find $\mathbf{E}$ and $\mathbf{B}$ that produce zero force in both measurements. We want fields such that

$$\begin{aligned} \mathbf{E}+\mathbf{v}_1\times\mathbf{B} &= 0 \\ \mathbf{E}+\mathbf{v}_2\times\mathbf{B} &= 0 \qquad . \end{aligned}$$

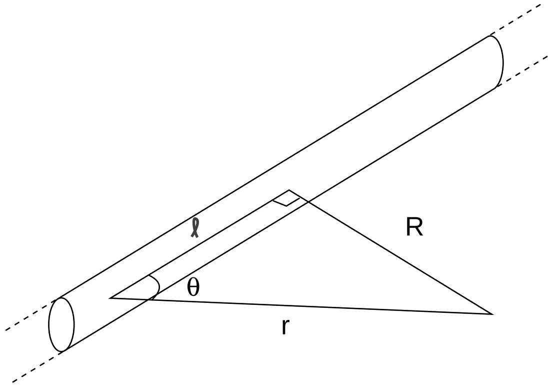
Subtracting,

$$(\mathbf{v}_1 - \mathbf{v}_2) \times \mathbf{B} = 0 \qquad .$$

It's easy to show that the procedure won't work if the two velocities are the same, so let's just assume the vector $\mathbf{v}_1 - \mathbf{v}_2$ is nonzero. We can take $\mathbf{B}$ to lie along $\mathbf{v}_1 - \mathbf{v}_2$, and let $\mathbf{E} = -\mathbf{v}_1 \times \mathbf{B}$.

11-22 The Biot-Savart law is

$$\mathrm{d}\mathbf{B} = \frac{k}{c^2} I \frac{\mathrm{d}\boldsymbol{\ell} \times \mathbf{r}}{r^3}$$



Using the variables shown in the figure, the cross product becomes $\mathrm{d}\boldsymbol{\ell} \times \mathbf{r} = r\ \mathrm{d}\ell \sin\theta\, \hat{\mathbf{z}}$, and $\sin\theta = R/r$, so we have

$$\begin{aligned} B_z &= \frac{k}{c^2} I \int_{-\infty}^{\infty} \frac{R\ \mathrm{d}\ell}{r^3} \\ &= \frac{k}{c^2} IR \int_{-\infty}^{\infty} \left(\ell^2 + R^2\right)^{-3/2} \mathrm{d}\ell \qquad . \end{aligned}$$

With the change of variable $u = \ell/R$,

$$B_z = \frac{kI}{c^2 R} \int_{-\infty}^{\infty} \left(u^2 + 1\right)^{-3/2} \mathrm{d}u \qquad .$$

I looked up the definite integral in a table. It equals 2. The result is the same as the one derived in the book by a different method.

11-23 (a) The integral for B_y was

$$B_y = \int \frac{2k\ \mathrm{d}I}{c^2 R} \cos\theta \qquad .$$

To evaluate the z component, we merely have to replace the cosine with a sine:

$$\begin{aligned} B_z &= \int \frac{2k\ \mathrm{d}I}{c^2 R} \sin\theta \\ &= \frac{2k\eta}{c^2} \int_{-a}^{b} \frac{\sin\theta}{R} \mathrm{d}y \\ &= \frac{2k\eta}{c^2} \int_{-a}^{b} \frac{y\ \mathrm{d}y}{y^2 + z^2} \end{aligned}$$

Substituting $u = y^2 + z^2$,

$$\begin{aligned} B_z &= \frac{k\eta}{c^2} \left(\int_{-a^2+z^2}^{z^2} \frac{\mathrm{d}u}{u} + \int_{z^2}^{b^2+z^2} \frac{\mathrm{d}u}{u} \right) \\ &= \frac{k\eta}{c^2} \left(\ln \frac{z^2}{a^2 + z^2} + \ln \frac{b^2 + z^2}{z^2} \right) \\ &= \frac{k\eta}{c^2} \ln \frac{b^2 + z^2}{a^2 + z^2} \end{aligned}$$

(b) The units are all contained in the factor of $k\eta/c^2$, since the logarithm is unitless. The units are therefore the same as the units of the expression for B_y derived in the book.

(c) As z approaches infinity, the argument of the logarithm approaches unity, so the logarithm approaches zero.

(d) When $a = b$, the result is zero. This makes sense, because the contributions to the field from the right and the left sides of the sheet are in opposite directions, so at a point on the centerline, they cancel exactly by symmetry.

11-24 If the little solenoid's current goes the same way as the big one's, then their fields reinforce and the energy density is higher than if they were separate. It's true that the fields are filling some of the same space, which would tend to reduce the total energy, but since the energy density depends on the square of the field, the increase in energy density is a bigger effect than the decrease in the amount of space. For instance, if the two fields are equal, and the volumes are nearly equal, then the combined field has an energy density four times greater, which more than compensates for the fact that the fields are occupying half the volume compared to the separate setup.

If the little solenoid's current goes the opposite way, their fields cancel at least partially, and the energy is less than when they were separate. The stable configuration is the one with the opposite currents.

11-25 (a) Let the z axis coincide with the axis of the solenoid, let the origin be at the center of the solenoid, and let the point at which we're evaluating the field be at $z=d$. Let the solenoid extend from $z=-b$ to $z=b$. The field is

$$\begin{aligned}B &= \int \frac{2\pi k a^2\,\mathrm{d}I}{c^2\left[a^2+(d-z)^2\right]^{3/2}}\\ &= \frac{2\pi k a^2}{c^2}\int\frac{\mathrm{d}I}{\left[a^2+(d-z)^2\right]^{3/2}}\\ &= \frac{2\pi k a^2\eta}{c^2}\int_{-b}^{b}\frac{\mathrm{d}z}{\left[a^2+(d-z)^2\right]^{3/2}}\\ &= \frac{2\pi k\eta}{c^2 a}\int_{-b}^{b}\frac{\mathrm{d}z}{\left[1+(\frac{d-z}{a})^2\right]^{3/2}}\end{aligned}$$

Substituting $u=(d-z)/a$, this becomes

$$B=-\frac{2\pi k\eta}{c^2}\int_{(d+b)/a}^{(d-b)/a}\frac{\mathrm{d}u}{\left(1+u^2\right)^{3/2}}$$

The indefinite integral is $\int(1+u^2)^{-3/2}\,\mathrm{d}u = u/\sqrt{1+u^2}$. Geometrically, the values of u at the limits of integration are -$\cot\beta$ and $\cot\gamma$, and the value of the integral at the limits of integration then equals -$\cos\beta$ and $\cos\gamma$, and the final result is as claimed.

(b) If the solenoid is very long, then both β and γ are zero, the quantity $\cos\beta+\cos\gamma$ becomes 2, and identifying NI/ℓ with η gives the result for a long solenoid.

(c) At the right-hand mouth of a very long solenoid, we have $\beta=\pi/2$ and $\gamma\approx 0$, so the field is half its maximum value.

(d) For very distant points on the right, β approaches π, and γ approaches zero, so the quantity $\cos\beta+\cos\gamma$ approaches $\cos\pi+\cos 0=0$. This makes sense, because the field should get small when we get far away.

11-26 We have to show that $\Gamma = (4\pi k/c^2)I_{through}$ is consistent with $B=2kI/c^2R$. Computing the circulation from the field, for an Ampèrian surface consisting of a loop of radius R forming a halo around the wire, we have

$$\begin{aligned}\Gamma &= B\cdot 2\pi R\\ &= \frac{2kI}{c^2R}\cdot 2\pi R\\ &= \frac{4\pi kI}{c^2}\qquad,\end{aligned}$$

which is the desired result.

11-29 (a) For points in this region, the fields contributed by the two wires are in opposite directions. Substituting $R=b-h$ and $b+h$ into $B=2kI/c^2R$, we find

$$\frac{2kI}{c^2}\left(\frac{1}{b-h}-\frac{1}{b+h}\right)$$

for the magnitude of the field.

(b) We can tile the region between the two wires with dipoles. Ignoring signs, the field is

$$\begin{aligned}B &= \int\frac{kI\,\mathrm{d}A}{c^2r^3}\\ &= \frac{kI}{c^2}\int_{x=b-h}^{b+h}\int_{y=-\infty}^{\infty}\frac{\mathrm{d}x\,\mathrm{d}y}{\left(x^2+y^2\right)^{3/2}}\\ &= \frac{kI}{c^2}\int_{x=b-h}^{b+h}\left(\frac{1}{x^3}\int_{y=-\infty}^{\infty}\frac{\mathrm{d}y}{\left(1+(y/x)^2\right)^{3/2}}\right)\mathrm{d}x\end{aligned}$$

Substituting $u=y/x$,

$$B=\frac{kI}{c^2}\int_{x=b-h}^{b+h}\left(\frac{1}{x^2}\int_{u=-\infty}^{\infty}\frac{\mathrm{d}u}{\left(1+u^2\right)^{3/2}}\right)\mathrm{d}x\qquad.$$

The indefinite integral is $u/\sqrt{1+u^2}$, so the definite integral equals 2, and

$$\begin{aligned}B &= \frac{2kI}{c^2}\int_{x=b-h}^{b+h}\frac{\mathrm{d}x}{x^2}\\ &= -\frac{2kI}{c^2}\left(\frac{1}{b+h}-\frac{1}{b-h}\right)\qquad,\end{aligned}$$

which is the same as the result in part a, except for the sign, which we didn't bother trying to set up correctly.

11-30 The result of example 9 in section 11.3.1 was $B = 4\pi kNI/c^2\ell$. The definition of inductance is

$$\frac{1}{2}LI^2 = U_m \qquad ,$$

so

$$\begin{aligned}\frac{1}{2}LI^2 &= \int \frac{\mathrm{d}U_m}{\mathrm{d}v}\,\mathrm{d}v \qquad ,\\ &= \int \frac{c^2}{8\pi k}B^2\,\mathrm{d}v \qquad ,\\ &= \frac{c^2}{8\pi k}\int B^2\,\mathrm{d}v \qquad ,\end{aligned}$$

but under the approximation of a long, skinny solenoid, the field is constant throughout the volume $v = A\ell$ inside the solenoid, and zero outside it, so

$$\begin{aligned}\frac{1}{2}LI^2 &= \frac{c^2}{8\pi k}B^2 v \qquad ,\\ &= \frac{c^2}{8\pi k}B^2\cdot A\ell \qquad ,\\ &= \frac{c^2}{8\pi k}\left(\frac{4\pi kNI}{c^2\ell}\right)^2\cdot A\ell \qquad ,\\ L &= \frac{4\pi k}{c^2}\cdot\frac{N^2A}{\ell}\end{aligned}$$

11-31 All that really matters is the cross-sectional area, a, of the smaller solenoid, and their separate fields, b and B. As the small one penetrates the big one to a depth x, we replace a volume xa from each one's interior with a new, overlapped volume xa in which the field is $B+b$.
(a) The difference in energy is

$$\frac{c^2}{8\pi k}\left[(B+b)^2 - B^2 - b^2\right]xa = \frac{c^2}{4\pi k}\cdot Bbxa \qquad .$$

(b) The energy is proportional to x, so for every millimeter deeper that we push the small one, the same amount of work is done. The force is $F = \mathrm{d}W/\,\mathrm{d}x = (c^2/4\pi k)Bba$.

11-32 The field of a wire is $B = 2kI/c^2R$, in the direction tangent to the circle centered on the wire and passing through the point of interest. As suggested in the problem, the field approaches zero as R approaches infinity, so we can neglect the contributions from the short, infinitely distant pieces of wire at the ends of the rectangle. Of the two infinite wires, each makes an equal contribution to the circulation, so we only need to integrate over one of them, and then double that result. Let the wire lie along the z axis, and let the wires be parallel to the x axis. Then

$$\begin{aligned}\Gamma_B &= 2\int_{-\infty}^{+\infty}\mathbf{B}\cdot\hat{\mathbf{x}}\,\mathrm{d}x\\ &= 2\int_{-\infty}^{+\infty}|\mathbf{B}||\hat{\mathbf{x}}|\cos\theta_{Bx}\,\mathrm{d}x\\ &= 2\int_{-\infty}^{+\infty}\frac{2kI}{c^2R}\cdot 1\cdot\frac{h}{R}\,\mathrm{d}x\\ &= \frac{4kIh}{c^2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}x}{R^2}\\ &= \frac{4kIh}{c^2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}x}{h^2+x^2}\\ &= \frac{4kIh}{c^2}\cdot\frac{1}{h}\tan^{-1}\frac{x}{h}\Big]_{-\infty}^{+\infty}\\ &= \frac{4\pi kI}{c^2}\end{aligned}$$

This is the result expected from Ampère's law.

11-33 (a) The magnetic field is proportional to N, so the magnetic field's energy depends on N^2. By the definition of inductance, this means that we have $L \propto N^2$, and that implies $Z \propto N^2$.
(b) For a given voltage, the current is inversely proportional to Z, so we have $I \propto N^{-2}$, and the field is proportional to the NI, so $B \propto N^{-1}$.
(c) The induced emf in solenoid 2 is proportional to the amplitude of the AC magnetic field supplied by solenoid 1, so $V_2/V_1 \propto N_2/N_1$.

11-34 The impedance is proportional to ω, so the current in the input side is proportional to ω^{-1}, and so is the field. The induced emf depends on the time derivative $\mathrm{d}B/\,\mathrm{d}t$. If the field varies as (constants) $\times\,\omega^{-1}\cos\omega t$, then the time derivative has an amplitude that is independent of ω, since we get a factor of ω in front from the chain rule. In other words, the gain of a transformer doesn't depend on frequency.

This result could be a little misleading in one respect. In the limit of $\omega \to 0$, we're talking about a DC circuit, and a transformer shouldn't work with DC. What's really happening is that we've made an assumption that can't possibly be correct in this limit: we've assumed that the input coil's impedance is purely inductive. That can't possibly be true at zero frequency. What should really happen is that the transformer's gain should be very nearly constant, for all frequencies high enough that the input coil's impedance is mainly inductive.

11-35 (a) Since the wire is moving through a magnetic field, the charges in the wire are also moving through a magnetic field, and they feel a force. The electrons in in the metal are free to flow, so a current is established. This current dissipates energy into heat, and by conservation of energy, that heat energy has to come from somewhere. The only place it can come from is the kinetic energy of the wire, so we conclude that the wire is decelerating. Therefore the force on the wire must be to the left. (b) The electrons have a lot of random thermal motion, but let's just talk about their motion on the average. On the average, an electron in the wire is moving to the right. Using the right-hand rule, we find that the vector cross product $\mathbf{v}\times\mathbf{B}$ points out of the page, but since q is negative, the force acting on the electron, $q\mathbf{v}\times\mathbf{B}$, points into the page. This tells us that the electrons flow counterclockwise as seen from above. Once this current is established, the velocity vector of the electrons in the rolling wire acquires a new component, into the page. This creates an additional magnetic force on the electrons, with $\mathbf{v}\times\mathbf{B}$ to the right and $q\mathbf{v}\times\mathbf{B}$ to the left, in agreement with the answer to part a. (c) Since we've already established the direction of the force in excruciating detail, I'll completely ignore signs in everything that follows. Faraday's law tells us

$$-\frac{\partial\Phi_B}{\partial t}=\Gamma_E \qquad .$$

We're told that the resistance of the circuit is dominated by the resistance of the rolling wire. In a series circuit, the current is the same everywhere, and by Ohm's law, we find that the voltage drop across the high-resistance rolling wire must be much greater than the voltage drop around the U. This means that the voltage drop V across the wire is essentially the same as Γ_E. Letting R be the resistance of the rolling wire, we have

$$\begin{aligned} I &= \frac{\Gamma_E}{R} \\ &= \frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}t}(BA) \\ &= \frac{1}{R}B\frac{\mathrm{d}A}{\mathrm{d}t} \end{aligned}$$

Let the wire's velocity be v, and let its length be ℓ. We then have $\mathrm{d}A/\,\mathrm{d}t = v\ell$, so

$$I = Bv\ell/R \qquad .$$

The problem now boils down to finding the force on a current-carrying wire in a perpendicular magnetic field. As worked out in problems 9 and 10 in this chapter (whose answers are in the back of the book), the force is $I\ell B$, so our final result for the force on the wire is

$$F = B^2\ell^2 v/R \qquad .$$

Let's interpret this result. Note that B is squared. This means that if we reversed the direction of the field, the force would still be in the same direction. That makes sense, because the answer to part a never required us to use the direction of the field. We also find that v is *not* squared. That also makes sense. If the wire was rolling in the other direction, then the logic of part a would still require that kinetic energy be converted into heat, so the wire would still have to decelerate, and the force acting on it would have to flip in order to be opposite to its motion.

11-36 Because $E = cB$ in an electromagnetic wave, and $v < c$ for a material particle, it follows that $F_B < F_E$. Therefore if we want to minimize the total force, the best we can do is to arrange for the magnetic force to be as strong as possible and opposite in direction to the electric force, so as to get as much cancellation as possible.

Let the wave have its electric field along the x axis, its magnetic field along the y axis, and therefore its direction of propagation along the z axis. If the particle is positively charged, then the electric force is in the positive x direction. We'd like to arrange for a magnetic force in the negative x direction. We can accomplish that by having the particle move in the positive z direction, i.e., along with the wave. Since this is perpendicular to the magnetic field, the magnetic force is maximized, which is what we want.

Nothing really changes if it's a negative charge; it's just that the directions of the two forces are reversed.

11-37 (a) The changing magnetic field induces an electric field, which causes a current to run around the loop. The resistance of the wire causes energy to be turned into heat. This energy can only be coming from the mechanical energy (KE and gravitational PE) of the loop, so it must be making the loop fall with an acceleration less than g. The force must be upward.
(b) An electron in the wire has negative charge, so the force it feels in a magnetic field is in the direction of $-\mathbf{v}\times\mathbf{B}$. This is into the page for an electron in the top, and out of the page for an electron in the bottom. Because the field is stronger at the top, that force dominates, and a clockwise current will be established. Once the current is established, an electron at the top has a nonvanishing v_y, and this new component gives rise to a new component of $-\mathbf{v}\times\mathbf{B}$, pointing up. The force at the bottom of the loop is downward, but it's weaker, so the total force is upward.
(c) As worked out in problems 9 and 10 in this chapter (whose answers are in the back of the book), the force exerted on a wire by a magnetic field is $I\ell B$. Let subscripts t and b stand for the top and bottom of the loop. Ignoring signs except where they're necessary, we have:

$$\begin{aligned}
F &= F_t - F_b \\
&= I\ell k(z_t - z_b) \\
&= IAk \\
&= Ak\Gamma_E/R \\
&= \frac{Ak}{R}\frac{\partial\Phi_B}{\partial t} \\
&= \frac{Ak}{R}\frac{\partial}{\partial t}\int kz\,\mathrm{d}y\,\mathrm{d}z \\
&= \frac{Ak^2(\text{width})}{R}\frac{\partial}{\partial t}\frac{z_t^2 - z_b^2}{2} \\
&= \frac{Ak^2(\text{width})}{R}\left(z_t\frac{\mathrm{d}z_t}{\mathrm{d}t} - z_b\frac{\mathrm{d}z_b}{\mathrm{d}t}\right) \\
&= \frac{Ak^2(\text{width})(\text{height})v}{R} \\
&= \frac{A^2k^2v}{R}
\end{aligned}$$

The behavior of this expression is sensible. The argument in part a didn't depend on the sign of k, so it makes sense that the final answer depends on k^2, and is therefore also independent of the sign of k. It also makes sense that the final answer has v to the first power. If we threw the loop upward, the original argument would still imply that mechanical energy was being taken out and dumped into ohmic heating, but the force required would be downward. Having R in the denominator makes sense. For instance, if we cut the wire, then it would be an open circuit, and there would be no way to establish a current; this corresponds to the case of $R=\infty$, which would give $F=0$ in our expression.

11-38 (a) The difference in the new derivation comes because the modified form of Gauss's law relates the free charge to ϵE rather than E, so instead of $E=4\pi k\sigma$, we have $\epsilon E=\sigma_{free}$, or, more for more convenient comparison, $(\epsilon/\epsilon_o)E = 4\pi k\sigma_{free}$. The result is that the capacitance is multiplied by the factor ϵ/ϵ_o relative to its vacuum value.
(b) $U_e=(1/2)CV^2 \propto \epsilon(A/h)(Eh)^2 = \epsilon v E^2$, so the figure of merit is ϵE^2.

11-39 (a) If your socks come out of the dryer

clinging together, the electric fields around them are caused by charges, $4\pi k q_{in}$. Your car's alternator is a generator that runs based on the principle that a curly electric field is caused by a changing magnetic field, $-\partial\Phi_B/\partial t$. You're reading right now using light, which is an electromagnetic wave; such a wave could not have its curly magnetic field unless such a field could be caused by a time-varying electric field, $-\partial\Phi_E/\partial t$. Electromagnets are used at the junkyard to pick up cars and scrap metal. The magnetic field is caused by a current, $4\pi k I_{through}$.
(b) Wherever k occurs, we replace it with $1/4\pi\epsilon_o$, and c^2 can be replaced with $4\pi k/\mu_o = 1/\mu_o\epsilon_o$. The results are

$$\begin{aligned}
\Phi_E &= \frac{1}{\epsilon_o} q_{in} \\
\Phi_B &= 0 \\
\Gamma_E &= -\frac{\partial\Phi_B}{\partial t} \\
\frac{1}{\mu_o\epsilon_o}\Gamma_B &= \frac{\partial\Phi_E}{\partial t} + \frac{1}{\epsilon_o} I_{through}
\end{aligned}$$

11-40 (a) We have

$$\begin{aligned}
\mathrm{d}U_e &= \frac{1}{8\pi k} E^2\,\mathrm{d}v \qquad \text{and} \\
\mathrm{d}U_m &= \frac{c^2}{8\pi k} B^2\,\mathrm{d}v \qquad .
\end{aligned}$$

In an electromagnetic wave, $B = E/c$, so

$$\begin{aligned}
\mathrm{d}U_m &= \frac{1}{8\pi k} E^2\,\mathrm{d}v \\
&= \mathrm{d}U_e \qquad .
\end{aligned}$$

(b)

$$\begin{aligned}
|\,\mathrm{d}\mathbf{p}| &= \frac{\mathrm{d}U}{c} \\
&= \frac{2\,\mathrm{d}U_e}{c}
\end{aligned}$$

The electric and magnetic fields are perpendicular to each other in an electromagnetic wave, so

$$\begin{aligned}
|\mathbf{E}\times\mathbf{B}|\,\mathrm{d}v &= EB\,\mathrm{d}v \\
&= \frac{E^2}{c}\,\mathrm{d}v \\
&= \frac{8\pi k}{c}\,\mathrm{d}U_e \qquad .
\end{aligned}$$

Finally,

$$|\,\mathrm{d}\mathbf{p}| = \frac{EB}{4\pi k}\,\mathrm{d}v \qquad .$$

(The Poynting vector is conventionally defined as this momentum density multiplied by an additional factor of c^2.)

11-41 (a) The definition of power gives $U = Pt$, and in time t the beam travels a distance $x = ct$ and fills a volume $v = Ax = Act$, so $U/v = Pt/Act = P/Ac$. (b) In time t, the beam travels a distance ct, and fills a volume Act. We need to relate the energy in this volume to the amplitudes of the fields. Using the result of the previous problem, we know that half the energy is due to the electric field, and half is due to the magnetic field, so as a shortcut we can find the energy due to the electric field, and double it:

$$U = 2U_e \tag{2.1}$$

The electric field is not constant throughout this volume; it's a sine wave. That means we need to do an integral over the volume,

$$\begin{aligned}
U &= 2U_e \\
&= 2\int \mathrm{d}U_e \\
&= 2\int \frac{1}{8\pi k} E^2\,\mathrm{d}v \\
&= \frac{1}{4\pi k}\int E^2\,\mathrm{d}v \\
&= \frac{1}{4\pi k}\int \tilde{E}^2 \sin^2(\text{stuff})\,\mathrm{d}v \\
&= \frac{\tilde{E}^2}{4\pi k}\int \sin^2(\text{stuff})\,\mathrm{d}v
\end{aligned}$$

But when you square a sine wave that goes from -1 to 1, you get a sine wave that goes from 0 to

1. The average of such a sine wave is 1/2, so we have

$$\begin{aligned} U &= \frac{\tilde{E}^2}{8\pi k} V \\ &= \frac{\tilde{E}^2 Act}{8\pi k} \end{aligned}$$

Now to relate this to the power,

$$\begin{aligned} P &= \frac{U}{t} \\ &= \frac{\tilde{E}^2 Ac}{8\pi k} \end{aligned}$$

$$\begin{aligned} \tilde{E} &= \sqrt{\frac{8\pi k P}{Ac}} \\ \tilde{B} &= \frac{\tilde{E}}{c} \\ &= \sqrt{\frac{8\pi k P}{Ac^3}} \end{aligned}$$

(c) Plugging in, we get $\tilde{E} = 1.01 \times 10^3$ V/m and $\tilde{B} = 3.36 \times 10^{-6}$ T. The electric field is about a thousand times weaker than the field in a thunderstorm. This sort of makes sense, because a beam of sunlight isn't enough to rip apart the atoms in the air as in a bolt of lightning. The magnetic field is also quite weak compared to everyday fields like the Earth's field.

11-42 The relationship between the field and the voltage is

$$E = \frac{V}{d} \qquad ,$$

so for the flux we have

$$\Phi_E = \frac{\pi R^2 V}{d} \qquad .$$

Based on the symmetry of the capacitor, we clearly need to choose an Amperian surface that is a circle centered on the axis. To extract any information about the magnetic field at radius R, we're going to have to make it a circle of radius R. Using Maxwell's equation for the circulation of the magnetic field,

$$\begin{aligned} c^2 \Gamma_B &= \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} \\ &= \frac{\pi R^2 s}{d} \end{aligned}$$

Now the magnetic field could have components in various directions, but only the component tangent to the circle will contribute to this circulation. That means we can't use this technique to get the components of the field in the radial direction and in the direction parallel to the axis. Let's compute the tangential component, B_t, first, and then worry about the other two.

$$\begin{aligned} c^2 \cdot 2\pi R B_t &= \frac{\pi R^2 s}{d} \\ B_t &= \frac{sR}{2c^2 d} \end{aligned}$$

The radial component must vanish because if it didn't, the magnetic field would be diverging from or converging on the capacitor, and Gauss's law for magnetism forbids this.

The axial component is a little trickier to rule out. If there was an axial component, then the field pattern would have to point in some direction along the axis, let's say up. Since the field cannot diverge or converge, the field pattern would then have to come back down in the region outside the capacitor. But then the magnetic field would have a circulation around a loop lying in an up-down plane. This would violate Maxwell's equation for the circulation of B, because there is no electric flux through such a surface (the electric field doesn't twirl around the axis), and there are no currents through such a surface (the currents are all axial and radial currents in the wires and the plates of the capacitor).

11-43 (a)

$$\begin{aligned}
a_x &= E\frac{q}{m} \\
&= \frac{\tilde{E}q}{m}\sin\omega t \\
v_x &= \frac{\tilde{E}q}{m\omega}(1-\cos\omega t) \\
a_z &= \frac{q}{m}v_x B \\
&= \left(\frac{q\tilde{B}}{m}\right)\left(\frac{\tilde{E}q}{m\omega}\right)(1-\cos\omega t)\sin\omega t \\
&= \left(\frac{q^2\tilde{E}\tilde{B}}{m^2\omega}\right)(-\frac{1}{2}\sin 2\omega t+\sin\omega t) \\
v_z &= \left(\frac{q^2\tilde{E}\tilde{B}}{m^2\omega}\right)\left(-\frac{1}{2\omega}\cos 2\omega t+\frac{1}{\omega}\cos\omega t\right) \\
z &= \left(\frac{q^2\tilde{E}\tilde{B}}{m^2\omega}\right)\left(-\frac{1}{4\omega^2}\sin 2\omega t+\frac{1}{\omega^2}\sin\omega t\right) \\
&= \left(\frac{q^2\tilde{E}\tilde{B}}{m^2\omega^3}\right)\left(-\frac{1}{4}\sin 2\omega t+\sin\omega t\right)
\end{aligned}$$

For me, this is a counterintuitive result: I would have expected the particle to oscillate at the frequency of the wave, rather than at a combination ω and 2ω.

(b) We need to find the maximum of the expression $-(1/4)\sin 2x+\sin x$. Setting the derivative equal to zero, we have

$$\begin{aligned}
-\frac{1}{2}\cos 2x+\cos x &= 0 \\
\cos^2 x-\cos x-\frac{1}{2} &= 0
\end{aligned}$$

$$\begin{aligned}
x &= \cos^{-1}\frac{1-\sqrt{3}}{2} \\
&= 111.5^\circ
\end{aligned}$$

At this value of x, the expression has a value of 1.101.

11-44 Suppose we have such a wave, with the fields lying along the y axis and the wave propagating along the x axis. Then we could construct an Ampèrian loop lying in the x-y plane such that Γ_E would be nonzero. According to Maxwell's equations, this would require there to be a time-varying magnetic flux through the loop. But if the magnetic field itself lies in the x-y plane, then the magnetic flux through a loop in the x-y plane is identically zero, since the field vector is parallel to the area vector.

11-45 Since the circuit is open, the charge has nowhere to go, and we won't observe any current in the ammeter. Since the capacitance q/V is decreasing, the reading on the voltmeter must go up. As with the closed circuit, positive work is done to overcome the attraction between the bound and free charges. In this example, however, the work goes into the increased energy of the higher electric field betweeen the plates.

11-48 (a) Let's use the same $x-y-z$ coordinate system employed in the book's derivation of B_y for a sheet of current. Each sheet experiences a B_y and B_z due to the other one. The z component of the magnetic field produces only sideways forces, which don't contribute to the attraction or repulsion. (The total sideways force also has to cancel by symmetry, so all it does is produce a stress that tends to stretch each sheet in the y direction.) The y component, however, does produce a repulsion according to the right-hand rule. As derived in the book, we have $B_y = 2k\eta\gamma/c^2$, and because $w \gg h$, we have $\gamma \approx \pi$, giving $B_y \approx 2\pi k\eta/c^2$ everywhere except at the edges. The edges, however, don't contribute appreciably to the total force, since $w \gg h$. Since B_y is approximately constant across each sheet, the force can be computed using the same equation as for a thin wire immersed in the same field:

$$F = I\ell B = \frac{2\pi k I^2\ell}{c^2 w} \quad .$$

(b)

$$\begin{aligned}
F &= I\ell B \\
&= I\ell\cdot\frac{2kI}{c^2h} \\
&= \frac{2kI^2\ell}{c^2h}
\end{aligned}$$

(c) In part a, the force is proportional to $1/w$, because the width of the strips is what determines the typical distance between points on one strip and points on the other. In b, the force depends on h for similar reasons. The factor of π is because in a, many points are closer than w, but in b, no points are closer than h.

11-49 The compass will respond to the combined fields of the magnet and the earth. There is no fundamental difference between the two- and three-dimensional cases, so for simplicity let's imagine the mystery magnet and the compass both being glued to a flat piece of wood, which we can orient at any angle in a horizontal plane. Before gluing the compass to the wood, we determine the direction of the earth's field. Then, with the compass glued down, we fiddle with the board's orientation until we get the compass to point in the direction of the earth's field again. In this orientation, the mystery magnet's field is along the same line as the earth's field. But is it north (reinforcing the earth's field) or south (opposing it)? If the magnet's field is stronger than the earth's, then we can easily tell by flipping the board 180 degrees; the compass will point in the direction of the magnet's field in both cases. If the magnet's field is weaker than the earth's, it's a little harder. The simplest technique is to turn the magnet away from the aligned direction by some small angle, say 5 degrees, and observe which way the needle is deflected. By one of these two methods, we can determine the direction of the magnet's field for once and for all, and draw it with a marking pen on the board.

Now we have to find the strength of the magnet's field. This is fairly easy. For instance, we can turn the magnet so that it's field is at a 90-degree angle with respect to the earth's, and measure the angle by which the compass deviates from magnetic north. We're now adding two field vectors that are at 90 degrees to one another, and this vector addition can be visualized in terms of a right triangle if we put the two vectors tip to tail. The tangent of the angle tells us the ratio of the two legs, which is the ratio of the magnet's field to the earth's.

11-51 (a),(b) See the solution in the back of the book.

(c) Currents reverse themselves (the charges are going the opposite way), so magnetic fields must reverse themselves as well.

(d) The electric field stays the same, but both the momentum and the magnetic field reverse themselves. Both sides of the proportionality are negated, but that leaves the proportionality (and the proportionality constant) unchanged.

11-53 See New Labs notebook. Needs more work.

11-55 As one capacitor discharges into the other, the changing electric field produces a magnetic field. This magnetic field takes up some of the energy. The system is an oscillating LC circuit, which will oscillate at its characteristic frequency. Eventually, radiation will dissipate the energy of the oscillations.

11-56 (a) Newton's second law gives $evB = mv^2/r$, and as analyzed in a previous problem, the condition for particles of velocity v not to be deflected in the first section is $E/B = v$. Eliminating v gives $h = 2r = 2Em/B^2e$, or

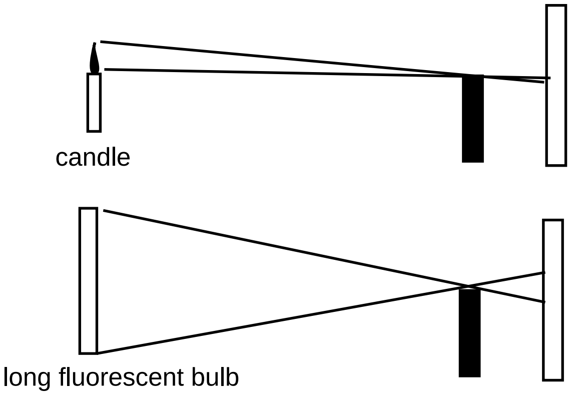
$$\frac{\Delta h}{\Delta m} = \frac{2E}{eB^2}.$$

(b) The units are

$$\begin{aligned}\frac{\mathrm{V/m}}{\mathrm{C{\cdot}T^2}} &= \frac{\mathrm{V{\cdot}C^2{\cdot}m^2}}{\mathrm{m{\cdot}C{\cdot}N^2{\cdot}s^2}}\\ &= \frac{\mathrm{N{\cdot}m^2}}{\mathrm{N^2{\cdot}s^2}}\\ &= \frac{\mathrm{m}}{\mathrm{kg}}.\end{aligned}$$

(c) It makes sense that the sensitivity increases with E, because a higher E allows the use of faster particles, which make wider circles. It also makes sense that the sensitivity decreases with B, because if B is small, there is little force causing the particles to curve, and the circles get big. Making B very small will help with sensititivity, but it comes at a price, which is that one has to build a very large vacuum chamber.

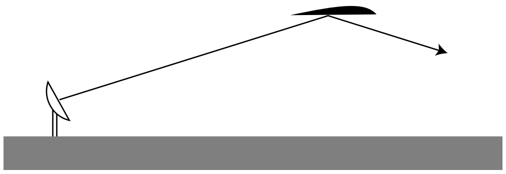
12-1 The fuzzy part of the shadow is the part that is only illuminated by some of the rays from the light source. This is the area between the extreme rays shown in the figures. The fuzzy area is larger in the second figure.



12-2 The time it takes light to travel a few meters, say 3 m, is $(3\text{ m})/(3\times10^8\text{ m/s}) = 10^{-8}$ s.

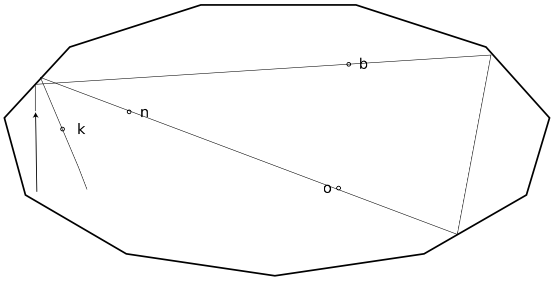
12-3 $f = c/\lambda = (3\times10^8\text{ m/s})/(10^{-9}\text{ m}) = 3\times10^{17}$ Hz, or call it 10^{17} Hz, since it's just an estimate. It would be UV or an x-ray.

12-4 Because the surfaces are flat, you get specular reflection. In specular reflection, all the reflected rays go in one direction. Unless the plane is directly overhead, that direction won't be the right direction to make the rays come back to the radar station.

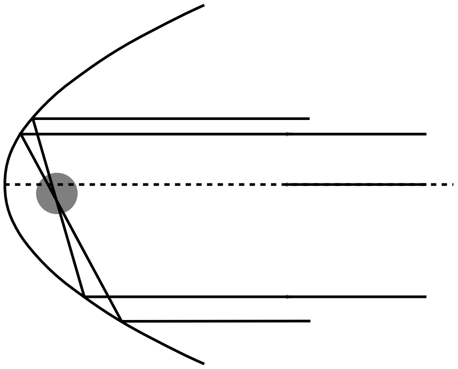


This is different from a normal plane, which has complicated, bumpy surfaces. These surfaces give diffuse reflection, which spreads the reflected rays randomly in more or less every possible direction.

12-5 It spells "bonk."



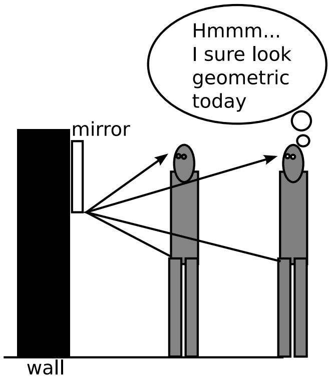
12-6 (a) The rays all cross at pretty much the same place, given the accuracy with which we can draw them.
(b) It could be used to cook food, for instance. All the sunlight is concentrated in a small area.
(c) Put the lightbulb at the point where the rays cross. The outgoing rays will then form a parallel beam going out to the right.



12-7 The image is at an equal distance behind the mirror, so the distance between the man and his image is twice his distance from the mirror, and his separation from his image is reducing at a rate of 2.0 m/s.

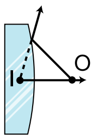
12-8 If you're going to see a particular part of your body, then rays from that part of your body have to be able to go to the mirror, be reflected, and go into your eye. If we're trying to find the lowest part you can see, then we're interested in the case where the ray hits the very bottom of the mirror. Backing up doesn't help, because the triangles are isoceles by the law of specular reflection, and the top of the base is fixed in height.

Another way of understanding the result is that if you go twice as far away, the mirror subtends half the angle, but your image also subtends half the angle, so the mirror still encompasses the same amount of your image.



12-9 (a) The image is closer to the surface of the mirror, so the magnification is less than one.

(b) It's virtual, because the rays only appear to have passed through point I. There is no way it could make a real image, because in order to make a real image it would have to bring the rays back together.

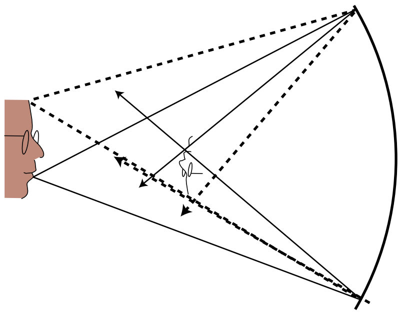


12-10 See figures a-f.
a and b: converging mirror, virtual image; increased object distance gives increased image distance
c and d: diverging mirror, virtual image: increased object distance gives increased image distance
e and f: converging mirror, real image: increased object distance gives decreased image distance

12-11 The magnification is the ratio of the image's size to the object's size. It has nothing to do with the person's location. The angular magnification, however, does depend on the person's location, because things farther away subtend smaller angles. The distance to the actual object is not changed significantly, since it's zillions of miles away in outer space, but the distance to the image does change if the observer's point of view changes. If you can get closer to the image, the angular magnification is greater.

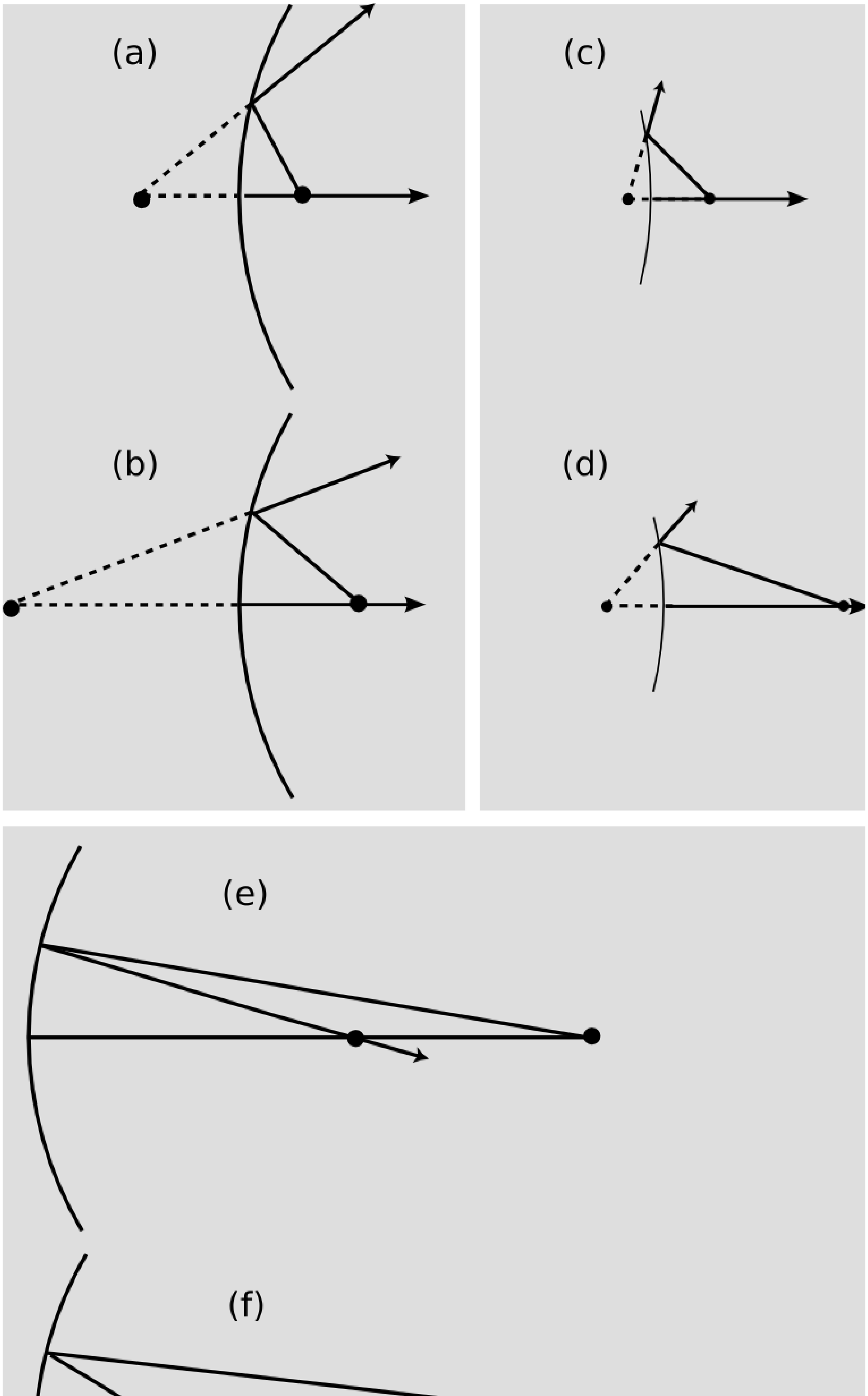
12-12 Usually you need to draw in a normal in order to get an accurate drawing of the reflected rays. Here, however, we already have one set of rays, and it's pretty easy to do the new rays so that they fan in or fan out by the correct angle compared to the old ones.

As claimed, the image of the forehead (where the dashed rays cross) is below the image of the lips.



12-13 An image is where *all* the rays that started from a certain point come together for one big family reunion. Only 3 represents an image. Points like 1, 2, and 4 look special, but only because of the particular rays we chose to trace, out of the infinite number of rays fanning out from the point on the object. If we had chosen a different set of rays, they would all have crossed at 3 as before, but not at 1, 2, and 4.

12-14 The problem can be simplified conceptually by replacing the hypothetical square watch with a square hole in a piece of paper. This version is also easy to construct and test experimentally. The bundle of rays transmitted through the hole is identical to the bundle that would be reflected by a specular-reflecting square of the same size. Let A be the aperture size, i.e., a linear dimension of the hole, and let d be the distance from the aperture to the surface on which the spot is being projected. The results depend on how A and d compare. When d is small com-


(a)
(c)
(b)
(d)
(e)
(f)

pared to A, the spot is square, but when d is large compared to A, we get a pinhole camera, which forms a circular spot shaped like the sun.

12-15 For a flat mirror, d_i and d_o are equal, so the magnification is 1, i.e., the image is the same size as the object.

12-17 We can tell that the object in the drawing is infinitely far away, because the rays are parallel. The focal length is defined in terms of the image distance in the case where the object is infinitely far away, so the focal length equals the image distance. Tracing the rays, we find that they cross at about 11 mm from the mirror, so that's its focal length.

12-18 The diameters of the mirrors are irrelevant. The focal length simply relates to the curvature of the mirror, a more strongly curved mirror having a shorter focal length. From shortest focal length to longest, the ranking of the mirrors is C, B, A.

12-20 (a) In problem #2 we found that the equation relating the object and image distances was of the form $1/f = -1/d_i + 1/d_o$. Let's make $f = 1.00$ m. To get a virtual image we need $d_o < f$, so let $d_o = 0.50$ m. Solving for d_i, we find $d_i = 1/(1/d_o - 1/f) = 1.00$ m. The magnification is $M = d_i/d_o = 2.00$. If we change d_o to 0.55 m, the magnification becomes 2.22. The magnification changes somewhat with distance, so the store's ad must be assuming you'll use the mirror at a certain distance. It can't have a magnification of 5 at all distances.

(b) Theoretically yes, but in practical terms no. If you go through a calculation similar to the one in part a, you'll find that the images of both planets are formed at almost exactly the same d_i, $d_i = f$, since $1/d_o$ is pretty close to zero for any astronomical object. The more distant planet has an image half as big ($M = d_i/d_o$, and d_o is doubled), but we're talking about *angular* magnification here, so what we care about is the angular size of the image compared to the angular size of the object. The more distant planet has half the angular size, but its image has half the angular size as well, so the angular magnification is the same. If you think about it, it wouldn't make much sense for the angular magnification to depend on the planet's distance — if it did, then determining astronomical distances would be much easier than it actually is!

12-21 (a) This occurs when the d_i is infinite. Let's say it's a converging mirror creating a virtual image, as in problems 2 and 3. Then we'd get an infinite d_i if we put $d_o = f$, i.e., the object is at the focal point of the mirror. The image is infinitely large, but it's also infinitely far away, so its angular size isn't infinite; an angular size can never be more than about 180° since you can't see in back of your head!.

(b) It's not possible to make the magnification infinite by having $d_o = 0$. The image location and object location are related by $1/f = 1/d_o - 1/d_i$, so $1/d_i = 1/d_o - 1/f$. If d_o is zero, then $1/d_o$ is infinite, $1/d_i$ is infinite, and d_i is zero as well. In other words, as d_o approaches zero, so does d_i, and d_i/d_o doesn't blow up. Physically, the mirror's curvature becomes irrelevant from the point of view of a tiny flea sitting on its surface: the mirror seems flat to the flea. So physically the magnification would be 1, not infinity, for very small values of d_o.

12-22 If the sound gets concentrated again at your ear, it must be a real image. You're at the center of the sphere formed by extending the surface. We've already found in the text that for a real image formed by a converging mirror, the equation for the image's location is $1/f = 1/d_\text{o} + 1/d_i$. If the object and image distances are equal, then $1/f = 2/d_\text{o}$, or $f = d_\text{o}/2$. The focal length is half the distance from the surface to your head.

12-23 (a) Let the virtual image formed by the top mirror lie at a height b above the top mirror. Then

$$\frac{1}{f} = \frac{1}{h} - \frac{1}{b} \qquad .$$

Solving for b, we have

$$b = \left(\frac{1}{h} - \frac{1}{f}\right)^{-1} \qquad .$$

The second image is at a distance h from the bottom mirror, so

$$\begin{aligned}\frac{1}{f} &= \frac{1}{b+h}+\frac{1}{h}\\ &= \frac{1}{h+\left(\frac{1}{h}-\frac{1}{f}\right)^{-1}}+\frac{1}{h}\end{aligned}$$

(b) Multiplying both sides by h,

$$\begin{aligned}\frac{h}{f} &= \frac{h}{h+\left(\frac{1}{h}-\frac{1}{f}\right)^{-1}}+1\\ &= \frac{1}{1+\left(1-\frac{h}{f}\right)^{-1}}+1\\ x &= \frac{1}{1+(1-x)^{-1}}+1\end{aligned}$$

This simplifies somewhat if we define a new variable $u=1-x$:

$$\begin{aligned}-u &= \frac{1}{1+1/u}\\ -u(1+1/u) &= 1\\ -u-1 &= 1\\ u &= -2\\ x &= 3\\ h &= 3f\end{aligned}$$

Here's why this is a hard problem. The solution $h=3f$ is unphysical. If h was greater than f, then the first image would have been real, not virtual, which would be inconsistent with the assumptions we made in the first place. The trick is in realizing that there was a hidden assumption involved in the step where we multiplied both sides by $1+1/u$, which was that $1+1/u$ was a real number. However, if $u=0$, then $1+1/u$ is infinite, and we can't multiply both sides by it. That's the second solution referred to in the problem:

$$\begin{aligned}u &= 0\\ x &= 1\\ h &= f\end{aligned}$$

In other words, the first image is at infinity. Images at infinity are the crossing-over point between real and virtual images. That means it didn't actually matter that we assumed the first image would be virtual. We could have gotten the same answer by assuming it was real.

12-24 (a) For a virtual image formed by a converging mirror, the extreme possibilities are $d_o \rightarrow 0$, which gives $M \rightarrow 1$ because you can't see the curvature of the mirror from up close, and $d_o = f$, which gives parallel reflected rays, $M=\infty$. The result is that $1 < M \leq \infty$.
(b) For a real image formed by a converging mirror, the extremes are obtained when either $d_o=\infty$ or $d_i=\infty$. We have $0 \leq M \leq \infty$.
For a diverging mirror, $d_o \rightarrow 0$ gives $M \rightarrow 1$ for the same reason as in part a, while $d_o \rightarrow \infty$ gives $M \rightarrow 0$. The result is $0 < M < 1$.

12-25 (a) Initially, the object is so close to the mirror that the mirror's curvature is irrelevant — any curve looks flat if you're close enough to it. Therefore for $d_o << f$, we have $d_i \approx d_o$, and the image's velocity equals the object's. As the object's distance approaches infinity, on the other hand, the image approaches a position that is one focal length above the mirror. Therefore,the image must accelerate from rest, move upward a distance equal to the focal length, and decelerate again, approaching zero velocity.
(b) The only two variables occurring in the statement of the problem are f and g, and there is only one way to combine these to make something with units of velocuty, which is to form the expression $\sqrt{fg}$. The answer must equal $\beta\sqrt{fg}$, where β is a unitless constant.
(c) As shown in example 5, the relatonship between the image and object locations is $1/f = 1/d_i - 1/d_o$. Since we already know the form of the solution's dependence on f and g, we can arbitrarily set $f=1$ and $g=1$ in order to simplify the writing. We then have $d_i = 1/(1+1/d_o)$, where $d_o = t^2/2$, and differentiation gives $v_i = t/(t^2/2+1)^2$. To find the maximum velocity, we differentiate again, and set the derivative equal to zero. In other words, we're looking for the time when the acceleration equals zero. The ac-

celeration is $a_i = 1/(t^2/2+1)^2 - 2t^2(t^2/2+1)^3$, which equals zero at $t=\sqrt{2/3}$. Plugging this back into the expression for v_i, we have a maximum velocity of $2^{-7/2}3^{3/2}$. This is the mysterious unitless constant β from part b. Recovering the general solution for $f \neq 1$ and $g \neq 1$, we have a maximum velocity of $2^{-7/2}3^{3/2}\sqrt{fg}$.

12-26 Snell's law is

$$n_a \sin\theta_a = n_d \sin\theta_d \qquad ,$$

where a refers to air and d to diamond, and $n_a \approx 1$. Total internal reflection occurs when there is no solution for θ_a, which occurs when the value required for $\sin\theta_a$ is greater than 1. The critical angle is $\theta_d = \sin^{-1}(1/n_d) = 24.4$ deg. It's measured from the normal, since that's how the angles in Snell's law are always measured. It's a minimum, because making θ_d bigger would require $\sin\theta_a$ to be more than 1, which is impossible. In a substance with a smaller index of refraction, $1/n$ would be bigger, and $\sin^{-1}(1/n)$ would also get bigger, since the inverse sine increases as you make its input bigger; the minimum angle for total internal reflection would be bigger, meaning that it would be harder to get total internal reflection.

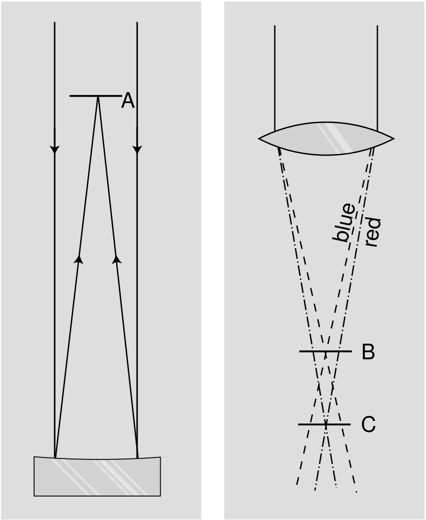
12-27 The refracted ray that was bent closer to the normal in the plastic when the plastic was in air will be bent farther from the normal in the plastic when the plastic is in water. It will become a diverging lens.

12-28 The figure shows the simplest possible reflecting and refracting telescopes, with no diagonal mirror or eyepiece.

In both cases, parallel rays enter from the star, which is infinitely far away. Each of these white rays is really a mixture of rays of all the different colors.

With the reflecting telescope, all the colors are reflected at the same angle and form a single image at one place. If we insert a piece of paper or a digital camera chip at A, only one spot is illuminated with the light from the star.

With the refracting telescope, it is never possible to get all the colors in focus at once. Each color has its own image, at a different distance from the lens. If the paper or chip is at B, where the image occurs for a specific wavelength of blue light, we get a blue dot surrounded by a fuzzy halo consisting of other colors such as red. If we move it to C, where the image occurs for a certain wavelength of red, we get a red dot surrounded by the other colors.



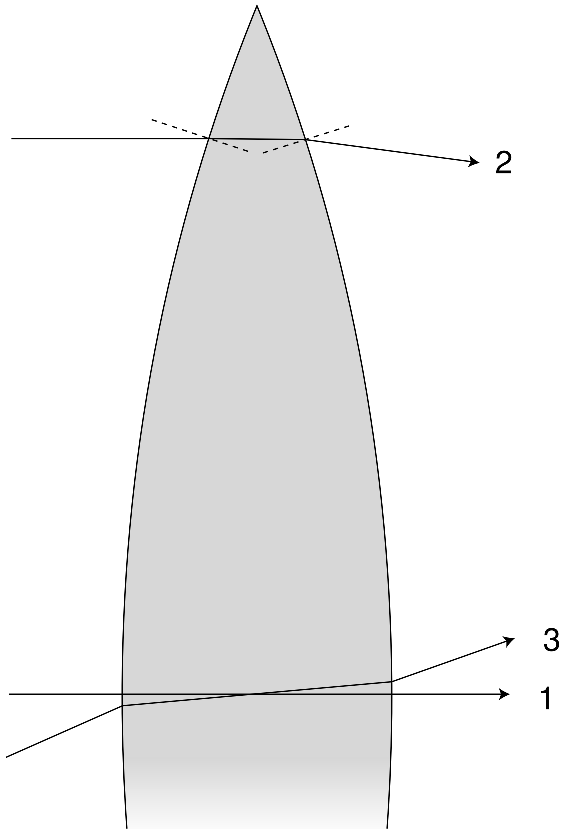
12-29 Refraction occurs only at the boundary between two substances, which in this case means the surface of the lens. Light doesn't get bent at all inside the lens, so the thickness of the lens isn't really what's important. What matters is the angles of the lens' surfaces at various points.

Ray 1 makes an angle of zero with respect to the normal as it enters the lens, so it doesn't get bent at all, and likewise at the back.

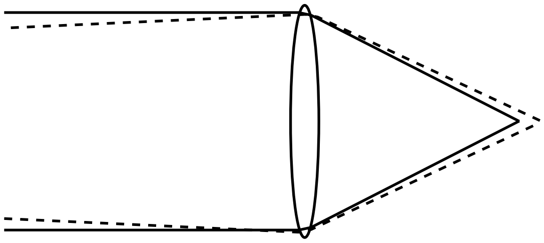
At the edge of the lens, 2, the front and back are not parallel, so a ray that traverses the lens at the edge ends up being bent quite a bit.

Although I drew both ray 1 and ray 2 coming in along the axis of the lens, it really doesn't matter. For instance, ray 3 bends on the way in, but bends an equal amount on the way out, so it still emerges from the lens moving in the same direction as the direction it originally had.

Summarizing and systematizing these observations, we can say that for a ray that enters the lens at the center, where the surfaces are parallel, the sum of the two deflection angles is zero. Since the total deflection is zero at the center, it must be larger away from the center.

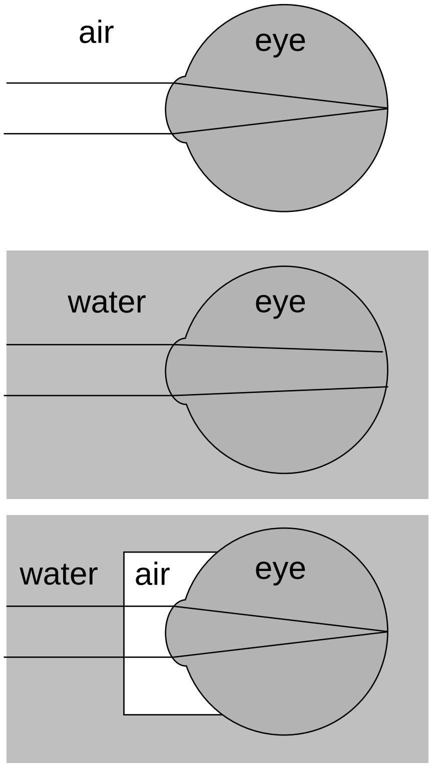


12-30 The rays from a point on a very distant object are parallel, like the solid lines in the figure. The rays from a point on a nearer object are slightly diverging, like the dashed lines. The lens cannot bend the rays from the nearer object strongly enough to make them converge as close to the lens, so the lens-to-film distance has to be greater when the object is nearer.



12-31 Normally, in air, your eyes do most of their focusing at the air-eye boundary. When you swim without goggles, there is almost no difference in speed at the water-eye interface, so light is not strongly refracted there (see figure), and the image is far behind the retina.

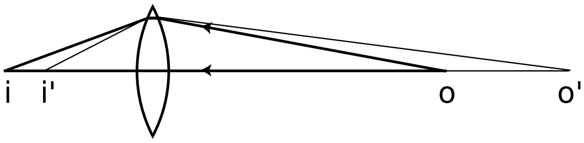
Goggles fix this problem for the following reason. The light rays cross a water-air boundary as they enter the goggles, but they're coming in along the normal, so they don't get bent. At the air-eye boundary, they get bent the same amount they normally would when you weren't swimming.



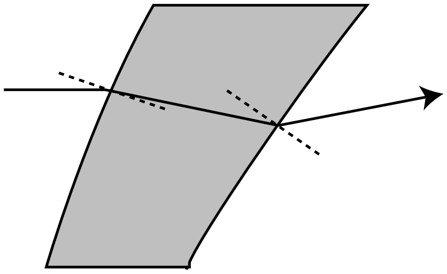
12-32 (a) See the ray diagram drawn with thick lines, showing object location o and image location i.

(b) If we move the object farther away, to o′ the cone of rays intercepted by the lens (thin lines) is less strongly diverging, and the lens is able to bring it to a closer focus, at i′. In the diagrams, we see that a smaller θ_o leads to a larger θ_i, so the signs in the equation $\pm\theta_o \pm \theta_i = \theta_f$ must be the same, and therefore both positive, since θ_f is positive by definition. The equation relating the image and object locations must be $1/f = 1/d_o + 1/d_i$.

(c) Solve the equation for $d_i = (1/f - 1/d_o)^{-1} = 32\text{cm}$.



12-33 (a) See the figure below. The first refraction clearly bends it inward. However, the back surface of the lens is more slanted, so the ray makes a bigger angle with respect to the normal at the back surface. The bending at the back surface is therefore greater than the bending at the front surface, and the ray ends up being bent *outward* more than inward.



(b) Lens 2 must act the same as lens 1. It's diverging. One way of knowing this is time-reversal symmetry: if we flip the original figure over and then reverse the direction of the ray, it's still a valid diagram.

Lens 3 is diverging like lens 1 on top, and diverging like lens 2 on the bottom. It's a diverging lens.

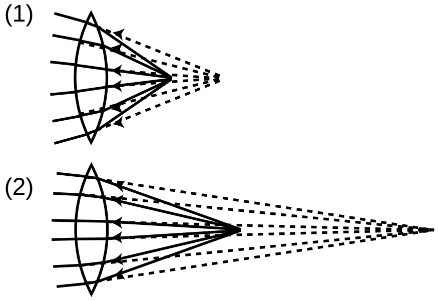
As for lens 4, any close-up diagram we draw of a particular ray passing through it will look exactly like the corresponding close-up diagram for some part of lens 1. Lens 4 behaves the same as lens 1.

12-34 I tried various circular household objects and found one whose radius of 1.5 cm seemed to make it coincide with the cornea's curvature. Because the drawing is two times bigger than life, the radius of curvature of the cornea is 1.5 cm/2=0.75 cm. The lensmaker's formula is $f = \left[(n-1)\left|1/r_1 \pm 1/r_2\right|\right]^{-1}$, but the air-cornea interface is just a single surface, so we just need to use $f = \left[(n-1)\left|1/r_1\right|\right]^{-1} = |r_1|/(n-1) = 2.5$ cm.

12-35 (a) See figure (1).

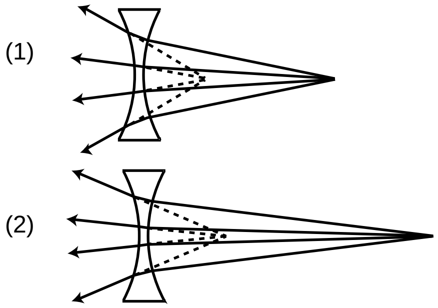
(b) If we move the object farther from the lens, as in (2), the cone of rays intercepted by the lens is less strongly diverging, so the lens is able to bring them closer together, resulting in an image that is farther from the lens. Since a smaller θ_o leads to a smaller θ_i, the signs in the equation $\pm\theta_o \pm \theta_i = \theta_f$ must be opposite. Since θ_o is greater than θ_i, we must have $\theta_o - \theta_i = \theta_f$ in order to get a positive focal angle. The equation relating the image and object locations is therefore $1/f = 1/d_o - 1/d_i$.

(c) Solving for f and plugging in, we get 18 cm.



12-36 (a) See figure (1).

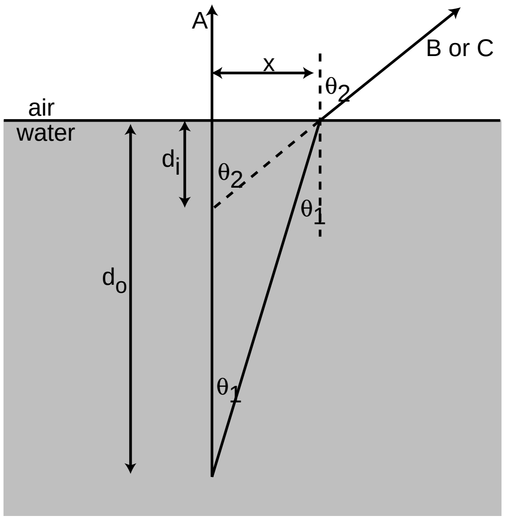
(b) Decreasing θ_o, as in figure (2), makes a cone of rays that is less strongly diverging, which the lens cannot bend outward as far, so θ_i becomes smaller. Since decreasing θ_o makes θ_i smaller as well, the signs in the equation equation $\pm\theta_o \pm \theta_i = \theta_f$ must be opposite. Since θ_i is greater than θ_o, we must have $-\theta_o + \theta_i = \theta_f$ in order to get a positive θ_f. The equation relating the image and object locations must be $1/f = -1/d_o + 1/d_i$. Solving for d_i and plugging in, we get 30.8 cm.



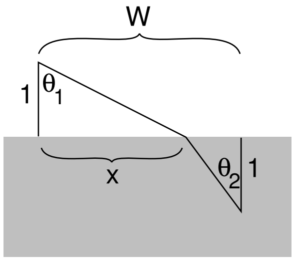
12-37 (a) Doing trig on the diagram below gives $\tan\theta_1 = x/d_o$, and $\tan\theta_2 = x/d_i$. Eliminating the unknown x and solving for d_i gives $d_i = d_o \tan\theta_1/\tan\theta_2$. Ray B has $\theta_1 = 1.0°$, and applying Snell's law with $n = 1.3$ gives $\theta_2 = 1.3°$, so rays A and B intersect at a depth $d_i = 0.769$ m. (Note that although we only know n to 2 significant figures, the point of the problem is to compare image depths for various rays, so we need to keep more sig figs.) Similarly, rays A and C intersect at $d_i = 0.733$ m.

(b) It might make it a little blurry if you were looking at it from close enough so that widely divergent rays like B and C could both get into your eye. Another effect is usually far more noticeable, however: if you try looking at an object on the bottom of a swimming pool from various angles, it appears to shift around.

(c) For small angles, $\sin\theta = \tan\theta$, so $d_i = d_o \tan\theta_1/\tan\theta_2 = d_o \sin\theta_1/\sin\theta_2 = d_o/n$, which is the same number regardless of θ.



12-38 If the principle of least time is going to work at all, it must work between any two points, so without loss of generality, we assume the points to lie at unit distance from the boundary between the two media. We also simplify things by letting $c = 1$ (which can be achieved, for example, by the proper choice of units).



The time taken by the ray along the path shown in the figure is

$$T = n_1\sqrt{1+x^2} + n_2\sqrt{1+(W-x)^2} \qquad .$$

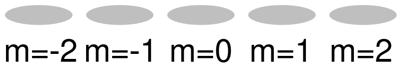
The minimum occurs when the derivative with respect to x equals zero:

$$0 = \frac{\mathrm{d}T}{\mathrm{d}x}$$

$$0 = n_1\left(1+x^2\right)^{-1/2} x - n_2\left[1+(W-x)^2\right]^{-1/2}(W-x)$$

$$0 = n_1 \sin\theta_1 - n_2 \sin\theta_2$$

12-44 The natural variable to use for labeling the fringes is m, as shown in the figure.



The angle θ is also different for the different fringes. The wavelength λ is a property of all the light being used, not a property of a particular fringe. For instance, if it's blue light, it might have a wavelength of 300 nm or so, and that would be the wavelength of all the light in every fringe. The variable d refers to a dimension of the physical object the light is diffracting around, e.g., the distance between two slits, so it's also not different for the different fringes. In particular, the light and dark pattern of fringes is *not* just a shadow of the slits — you could have hundreds of fringes made by only one or two slits.

12-45 The center-to-center spacing of patterns 2 and 3 are the same. The one with the small center-to-center spacing, pattern 1, corresponds to the grating with the large slit spacing, C. Grating A has more repetition, so its pattern is the one with narrower lines, 3. To summarize, 1=C, 2=B, 3=A.

12-46 Red light is at the long-wavelength end of the spectrum. Longer wavelengths give larger diffraction angles, so if the slits had been identical, the red pattern would have been the more widely spaced one. We conclude that different slits were used.

12-47 Let's write x for the distance from the slits to the wall, and y for the spacing between fringes. Three fringes cover about 5.1 cm in the photo, so the spacing is about $y = 1.70$ cm. An example in the text worked out the case where the angles are small, and we want to relate the wavelength to the angular spacing. The result (using radians, not degrees!) is

$$\begin{aligned}\lambda &\approx d\Delta\theta \\ &= d\tan^{-1}(y/x) \\ &\approx d\cdot(y/x) \\ &= 580 \text{ nm}\end{aligned}$$

As a check on the precision of my result, I measured across five fringes and got 8.3 cm, which gives $y = 1.66$ cm, and $\lambda = 570$ nm. Typically, measuring across more fringes would reduce the relative error in the measurement and give a better result. However, the picture only shows five maxima, so measuring across them required me to estimate the position of the minima on the ends without being able to see the next bright fringe. Roughly, then, I'd say the two results of 580 and 570 nm are equally trustworthy, and it might make sense to average them to get about 575 nm as my best value, with error bars of five or ten nm (one or two percent).

12-48 You don't want the wave properties of light to cause all kinds of funny-looking diffraction effects. You want to see the thing you're looking at in the same way you'd see a big object. Diffraction effects are most pronounced when the wavelength of the light is relatively large compared to the size of the object the light is interacting with, so red would be the worst. Blue light is near the short-wavelength end of the visible spectrum, which would be the best.

12-49 (a) You can tell it's a single slit because of the double-width central fringe.

(b) Four fringes on the top pattern are about 23.5 mm, while five fringes on the bottom one are about 14.5 mm. The spacings are 5.88 and 2.90 mm, with a ratio of 2.03. A smaller d leads to larger diffraction angles, so the width of the slit used to make the bottom pattern was almost exactly twice as wide as the one used to make the top one.

12-50 There is not actually a sharp edge to the visible rainbow; there is a range of wavelengths at each end of the spectrum over which your eye's sensitivity drops off. However, as stated in the problem, the range is roughly 400 to 700 nm, which is slightly less than a factor of two. You get a series of rainbows, each broader than the previous one, but with a constant center-to-center angular spacing. The angle of a given color within the rainbow of order m is determined by the equation $m\lambda = d\sin\theta$, or $\theta = \sin^{-1}(m\lambda/d)$. Since the red end is very nearly twice the wavelength of the violet end, the $m = 1$ red has very nearly the same value of θ as the $m = 2$ violet, so the first and second order spectra almost overlap, but not quite. The $m = 2$ and $m = 3$ spectra will definitely overlap, because the quantity $m\lambda/d$ is greater for $m = 2$ red than for $m = 3$ violet.

12-51 For the size of the diffraction blob, we have:

$$\begin{aligned}\frac{\lambda}{d} &\sim \sin\theta \\ &\approx \theta \\ \theta &\sim \frac{700\ \text{nm}}{10\ \text{m}} \\ &\approx 10^{-7}\ \text{radians}\end{aligned}$$

For the actual angular size of the star, the small-angle approximation gives

$$\begin{aligned}\theta &\sim \frac{10^{9}\ \text{m}}{10^{17}\ \text{m}} \\ &= 10^{-8}\ \text{radians}\end{aligned}$$

The diffraction blob is ten times bigger than the actual disk of the star, so we can never make an image of the star itself in this way.

12-52 (a) The patterns have two structures, a coarse one and a fine one. You can look up in the book which corresponds to w and which to d, or just use the fact that smaller features make bigger diffraction angles. The top and middle patterns have the same coarse spacing, so they have the same w. The fine structure in the top pattern has 7 fringes in 12.5 mm, for a spacing of 1.79 mm, while the middle pattern has 11 fringes in 41.5 mm, giving a spacing of 3.77 mm. The value of d for the middle pattern is therefore (0.50 mm)(1.79/3.77) = 0.23 mm.

(b) This one has about the same d as the top one (it's difficult to measure accurately because each group has only a small number of fringes), but the coarse spacing is different, indicating a different value of w. It has two coarse groupings in 23 mm, i.e., a spacing of 12.5 mm. The coarse groupings in the original pattern were about 23 mm apart, so there is a factor of two between the $w = 0.04$ mm of the top pattern and the $w = 0.08$ mm of the bottom one.

12-53 (a) The grating's spacing is the number of centimeters per line, which is one over the number of lines per centimeter, $d = (1/2000\ \text{lines/cm}) = 5 \times 10^{-4}$ cm/line. "Lines" are a fake unit that we can get rid of at will. Converting to meters, we have $d = 5 \times 10^{-6}$ m. The angles of the $m = -1$ and $m = 1$ spots are given by $\theta = \sin^{-1}(m\lambda/d) = +8.0$ degrees. The spacing of the spots on the screen is related to this by simple trig: (2.0 m)$(\tan\theta)$ = 28 cm.

(b) Note that the approximation $\sin\theta \approx \theta$ only works if we use units of radians. With it, we get $\theta = m\lambda/d = .139$ radians, which works out to be almost exactly the same as the answer we got without the approximation.

12-56 High frequencies have short wavelengths, which leads to small diffraction angles. This is a good thing, because we want the image to be sharply defined. Ultimately, you can't pick out any details that are smaller than one wavelength. (This only explains why they would use the highest possible frequency for imaging, not why they would use a lower one for heating. The most likely explanation is that it's just more expensive to get a speaker to vibrate really fast.)

12-58 For simplicity, let's assume that the cylinder has unit radius, so that its focal length is 1/2, and the relationship between the object and image distances is $1/d_i - 1/d_o = 2$. To see that the standard Peaucellier linkage doesn't do the right thing, consider the case when the object point is at infinity. Then the image distance is 1/2, whereas inversion in a circle would give a point at radius zero.

The standard definition of object and image distances is relative to the surface of the mirror, whereas the geometrically relevant quantities in this problem would clearly be measured from the center of the cylinder, $u = d_o + 1$ and $v = 1 - d_i$. The relationship between these two variables is $1/u + 1/v = 2$, which is the equation of a hyperbola with perpendicular asymptotes at $u = 1/2$ and $v = 1/2$.

This gives us strong reason to hope that the a construction based on the Peaucellier can be done, since the Peaucellier linkage calculates the inverse of a number, and the graph of the function $y = 1/x$ is also a hyperbola with perpendicular asymptotes. To make the connection, let $p = u - 1/2$ and $q = v - 1/2$, so that $p = 1/4q$. This shows that we need to make a Peaucellier linkage with $k = 1/2$. Positioning it so that when the rhombus is flattened, points P and P' lie on the surface of the cylinder. The point O should not be located at the center of the cylinder; we need to modify the linkage by adding a seventh rod, with length 1/2, to tie O to the center of the cylinder. We also need to add constraints to keep the axis of the Peaucellier linkage lined up with the seventh rod.

12-60 Snell's law gives $\sin\theta_1 = (1+\epsilon)\sin\theta_2$. To get the greatest deflection, we want the limit in which θ_1 appraoches 90 °. Expanding $\sin\theta_2$ in a Taylor series around 90 °, we have $\sin\theta_2 = 1 - \phi^2/2 + \ldots$, where $\theta_2 = \pi/2 - \phi$. The result is $\phi = \sqrt{2\epsilon}$.

12-62 Applying small-angle approximations to Schlick's approximation gives $f \approx f(0)1 - (1/32)\theta^{10}$. From the result of the previous problem, we have $f(0) = 4n(1+n)^{-2}$. (a) Let θ_2 be the angle in air. The ratio of the widths is $g = \cos\theta/\cos\theta_2$, and using the small-angle approximation $\cos x \approx 1 - x^2/2$ and the approximation $1/(1-\epsilon) \approx 1 + \epsilon$, we find $g = 1 + P\theta^2$, where $P = (n^2-1)/2$.

(b) Multiplying the expressions for α and β, and throwing away the θ^{12} term, we have $I_2 = f(0)(1 + P\theta^2 - Q\theta^{10})$, where $Q = 1/32$. Using calculus to maximize this expression, we find

$$\theta = \left[\frac{16}{5}(n^2-1)\right]^{1/8},$$

and plugging this back in to I_2 gives

$$I_2 = f(0)(1 + k(n^2-1)^{5/4}),$$

where $k = 4 \cdot 5^{-5/4} \approx 0.535$.

12-63 We have $n = \sin\phi/\sin\theta$. Doing implicit differentiation, we find $\mathrm{d}n = -\sin\phi(\cos\theta/\sin^2\theta)\,\mathrm{d}\theta$, which can be rewritten as $\mathrm{d}n = -n\cot\theta\,\mathrm{d}\theta$. This can be minimized by making θ as big as possible. To make θ as big as possible, we want ϕ to be as close as possible to 90 degrees, i.e., almost grazing the surface of the tank.

This result makes sense, because we're depending on refraction in order to get a measurement of n. At $\phi = 0$, we get $\theta = 0$, which provides no information at all about the index of refraction — the error bars become infinite. The amount of refraction increases as the angles get bigger.

13-1 After two years, the substance will be 75% decayed, not 100%.

13-2 The probabilities are independent, so $(1/6) \times (1/6) = 1/36$.

13-5 (a) Each rectangle has a width of 5 cm and a height of 0.005 cm^{-1}, so its area represents a probability of 0.025. The total number of rectangles under the curve is about 38, so the total area under the curve represents a probability of about $38 \times .025 = 0.95$, which is close enough to 1, considering that I had to estimate fractions of squares.

(b) The number of rectangles under the curve from 140 cm to 150 cm is about 6, so the the probability represented is about $6 \times .025 = .15$.

13-6 (a) The activity falls off by a factor of two as you go from $t = 5$ min to $t = 32$ min, so its half-life is about 27 min. It looks like it's ^{214}Pb.

(b) If it's a single isotope with a single well-defined half-life, then the ratio of the activity at 20 min to the activity at 10 min should be the same as the ratio of the activity at 30 min to the activity at 20 min. Only the stuff produced by the second ion speed gives ratios equal to three significant figures, which is the accuracy of the data.

13-7 Suppose we record the results of a long string of games played one after another. One method of testing for independence would be to use the data to estimate the probability of winning, $P(W)$, and also break down the list into sequential pairs (1st and 2nd game, 3rd and 4th, etc.) and use these pairs to measure the probability of winning twice in a row, $P(W \text{ and } W)$. The definition of independence then gives $P(W \text{ and } W) = P(W)^2$. We can test whether the left-hand and right-hand sides of this equation are in fact equal, to within the statistical sampling error imposed by the number of games observed.

Another approach is to look for extremely long runs of wins. If independence holds, then the probability of n wins in a row is $P(W)^n$, which falls off exponentially. Any observation of a very large n would therefore prove that the rolls were not statistically independent. For a statistical analysis of data from players who ran up world-record winning streaks, see Ethier and Hoppe, arxiv.org/pdf/0906.1545. The results are described in terms of the number of rolls, which is greater than the number of games.

13-8 (a) The variable r varies continuously from 0 to b — it's not as though we have rings of finite width. However, suppose we imagine an extremely narrow ring extending from r to $r + \mathrm{d}r$. Since the person is blindfolded, the probability of hitting a given region depends only on its area. This ring has a circumference of $2\pi r$, so its area is $2\pi r\,\mathrm{d}r$. This is proportional to r, so the probability of getting a result between r and $r + \mathrm{d}r$ is proportional to r.

(b)

$$\begin{aligned} 1 &= \int_0^b D(r)\,\mathrm{d}r \\ 1 &= \int_0^b kr\,\mathrm{d}r \\ 1 &= \frac{1}{2}kb^2 \\ k &= \frac{2}{b^2} \\ D(r) &= \frac{2r}{b^2} \end{aligned}$$

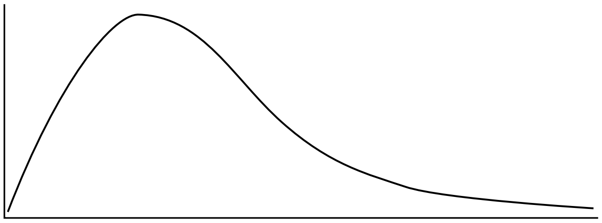
(c)

$$\begin{aligned} \bar{r} &= \int_0^b rD(r)\,\mathrm{d}r \\ &= \int_0^b \frac{2r^2}{b^2}\,\mathrm{d}r \\ &= \frac{2}{3}b \end{aligned}$$

(d) The average value of r is greater than $b/2$. This makes sense, because the area between $r = 0$ and $r = b/2$ is only one fourth of the total area.

13-9

13-10 (a) Without the factor of v in front, it would have the form of the classic bell curve. Since it's only defined for positive v, we chop off the left half of the bell. Then we multiply by v, which makes the curve drop down to zero at the origin. The result looks like this:



(b) We want to integrate D from 0 to ∞. The subsitution $u = cv^2$ gives, $\mathrm{d}u = 2cv\,\mathrm{d}v$, turning the integrand into $(b/2c)e^{-u}\,\mathrm{d}u$. The integral of $e^{-u}\,\mathrm{d}u$ from 0 to ∞ is 1, so the result is $1 = (b/2c) \cdot 1$, or $b = 2c$.

(c) I warned you not to try it by hand! (It requires a trick.) Using computer software, the result is $\int_0^\infty 2cv^2 e^{-cv^2}\,\mathrm{d}v = (1/2)\sqrt{\pi/c}$.

13-11 (a) To a good approximation, the mass of a nucleus is proportional to the number of protons plus the number of neutrons, which is the mass number. The total change in mass number from 238 to 206 is 32, so if mass is to be conserved, the number of alpha particles, with mass number four, emitted must be 32/4=8.

(b) The charge of the original nucleus was 92 (in units of the fundamental charge), and the nuclei produced have a total charge of $82 + 8 \times 2 = 98$. To conserve charge, 6 electrons must have been produced, so 6 beta decays must have occurred.

(c) The mass of a uranium atom is about 238 times the mass of a neutron or proton, so the number of surviving U atoms is

$$\frac{1230 \times 10^{-6}\ \text{kg}}{238 m_p} = 3.09 \times 10^{21} \qquad .$$

(The masses of protons and neutrons are nearly the same, so we don't have to add their masses separately.) Similarly, the number of helium atoms is

$$\frac{2.3 \times 10^{-6}\ \text{kg}}{4 m_p} = 3.44 \times 10^{20} \qquad .$$

Each of the decayed U atoms produced 8 helium atoms, so the number of decayed U atoms is $3.44 \times 10^{20}/8 = 4.30 \times 10^{19}$. The probability of decay since the rock was formed equals (number that decayed)/(number that decayed + number that survived) = 0.0137, so the probability of survival was 0.9863, and $0.9863 = 0.5^{t/t_{1/2}}$. Taking natural logarithms of both sides, $\ln 0.9863 = (t/t_{1/2}) \ln 0.5$, yielding $t = 90$ million years.

13-12 $E = hf$, and frequency corresponds to color, so if each photon was losing 20% of its energy you'd notice a pronounced change in color.

13-13 The amount of energy lost by the electrons as they leave the surface, E_s, depends on the material. With a material with a high E_s, you wouldn't be sensitive to the red end of the spectrum, which has low-energy photons.

13-14 Hertz noticed that the cutoff for producing the photoelectric effect in metals was somewhere in the visible-light spectrum. We're assuming that the energy required to ionize DNA is similar to the energy required to rip an electron out of a metal, so again, the cutoff should at a frequency that's of roughly the same order of magnitude as the frequency of visible light. The types of waves are listed in order of increasing frequency, so the only ones that can cause cancer are the ones farther down the list than the cutoff. Ultraviolet and x-rays are certainly capable of doing it. It might be possible to get cancer from visible light — our analysis is too crude to tell exactly where the cutoff is. You are not going to get cancer from power lines, FM radio, cell phones, or microwave ovens.

13-15 (a) $\lambda_1 = \lambda_4 < \lambda_2 = \lambda_3$. Since $c = f\lambda$, the frequencies go in the opposite order. The energies go in the same order as the frequencies, because $E = hf$.

(b) Conservation of energy says that the energy of both the emitted photon and the energy of the absorbed photon have to be equal to the difference in energy between the two states of the atom. Therefore only photon 2 can excite the atom in this way.

13-16 As an electron speeds up, it gains momentum, which according to $\lambda = h/p$ implies a shortening of its wavelength. Number 3 is the only one that works. (Number 2 might be a tempting alternative, but it's wrong, because the momentum isn't changing. The amplitude is increasing as we go to the right, and therefore the portion of the electron's energy that we're likely to find in the right-hand region is higher than in the left-hand region. But that's not because the electron is speeding up, it's merely because the electron has a higher probability of being on the right, so that's where its energy is likely to be.)

13-17 Visible light has a wavelength on the order of 10^{-6} m, so a typical photon of visible light is about $E = hf = hc/\lambda \approx 2 \times 10^{-19}$ J, so the number of photons emitted per second is about $(100\ \text{J/s})/(2 \times 10^{-19}\ \text{J/photon}) = 5 \times 10^{20}$ photons/s. The time each photon spends on its voyage is (3 m)/$c \approx 10^{-8}$ s. The number of photons in the beam equals the number of photons emitted during the time required for a photon to get across, $(5\times10^{20}$ photons/s$)\times(10^{-8}$ s$) = 5\times10^{12}$ photons. Given the crudeness of the initial assumptions, this is only an order-of-magnitude result, so let's call it 10^{13} photons.

13-18 (a)

$$\begin{aligned} \text{power} &= (1\ \text{mW/m}^2)(\text{cross-sectional area}) \\ &= (1\ \text{mW/m}^2)(\pi r^2) \\ &\sim 10^{-32}\ \text{W} \end{aligned}$$

(rounding to a power of 10).

(b) power $= \Delta E/\Delta t$, so

$$\begin{aligned} \Delta t &= \Delta E/\text{power} \\ &\sim 10^{13}\ \text{s} \end{aligned}$$

The classical theory says it should take thousands of years for an electron to soak up enough energy to be ejected, which contradicts experiment, so the classical theory must be wrong.

13-19 Momentum is related to kinetic energy by $E = p^2/2m$, so

$$\begin{aligned} \lambda &= h/p \\ &= h/\sqrt{2mE} \\ &= h/\sqrt{2mq\Delta V} \\ &= 1\times10^{-11}\ \text{m} \end{aligned}$$

13-20 It's conceivable that the Δx is actually *less* than the diameter of the nucleus, but it can't be *more*, since the proton stays inside the nucleus. The maximum possible value of Δx is therefore roughly the same as the diameter of the nucleus. A maximum value of Δx leads to a minimum value of Δp, since they're inversely related: $\Delta p \gtrapprox h/\Delta x$. Plugging in, we get $\Delta v = \Delta p/m \gtrapprox h/m\Delta x \gtrapprox 3\times10^7$ m/s. Note the two inequalities in a row: one because the maximum uncertainty is the diameter of the nucleus, and one because the uncertainty principle is an inequality. It's perfectly possible that they're moving much faster than this; what we've found is only a minimum.

Our result is 10% of the speed of light, so the nonrelativistic equations are good enough for this kind of estimate.

13-21 The wavelength equals $2L/n$, so we have

$$\begin{aligned} KE &= \frac{p^2}{2m} \\ &= \frac{(h/\lambda)^2}{2m} \\ &= \frac{h^2n^2}{8mL^2} \end{aligned}$$

13-22 (a) $\lambda = h/p = h/mv = 10^{-8}$ m $= 10$ nm.

(b) The area of one circuit would be 10^{-8} cm^2, which might be a square 10^{-4} cm on a side, i.e. 1000 nm.

(c) No. Diffraction is only an important effect when the size of the object the wave is diffracting around is comparable to the wavelength.

(d) If the linear size of a circuit was of the same order of magnitude as the wavelength, then we'd definitely have a lot of diffraction. A 10 nm × 10 nm circuit would have an area of 100 nm^2 $= 10^{-12}$ cm^2, so you could fit 10^{12} of them on the chip.

13-23 (a)

$$\begin{aligned} \frac{d^2\Psi}{dx^2} &= \frac{d^2}{dx^2}e^{-bx^2} \\ &= \frac{d}{dx}\left(-2bxe^{-bx^2}\right) \\ &= -2be^{-bx^2} + (-2bx)(-2bx)e^{-bx^2} \\ &= 2b\left(2bx^2-1\right)e^{-bx^2} \end{aligned}$$

$$\begin{aligned} Ee^{-bx^2} &= -\frac{\hbar^2}{2m}\left(2b\left(2bx^2-1\right)e^{-bx^2}\right) + \frac{1}{2}kx^2e^{-bx^2} \\ E &= -\frac{\hbar^2 b}{m}\left(2bx^2-1\right) + \frac{1}{2}kx^2 \\ 0 &= \left(\frac{1}{2}k - \frac{2\hbar^2b^2}{m}\right)x^2 + \left(\frac{\hbar^2 b}{m} - E\right) \end{aligned}$$

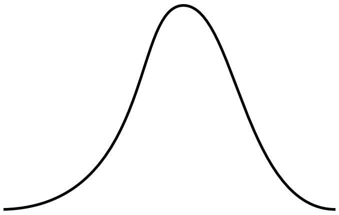
If this equation is going to be true for all values of x, then both coefficients in the polynomial have to be zero,

$$k = \frac{4\hbar^2b^2}{m}$$

and

$$E = \frac{\hbar^2 b}{m}$$

If the problem is specified by giving the potential U, then we can find the unknowns E and b that complete the specification of the wave that is the solution. The result for b is $\sqrt{km}/2\hbar$. (b) It's a bell curve, like this:



(c) If b was bigger, then we'd get the same value of the wavefunction for smaller values of x. In other words, increasing b causes the bell to get skinnier.

(d) Making k greater makes b greater, which makes it skinnier. That makes sense, because a skinnier wavefunction means that the atom is unlikely to be found very far from the equilibrium point.

13-24 (a) The sizes of the realistic wavefunctions are on the order of 0.1 nm. Plugging in $n = 1$ to the approximation $r \sim h^2 n^2 / mke^2$, we get 2 nm. The estimate was too high by about a factor of 20, which is reasonable considering the crude assumptions we made.

(b) The proton is about 2000 times more massive than the electron, so the center of mass lies about 1/2000 of the way from the center of the proton to the electron. A typical separation between the proton and electron is 0.1 nm, so the center of the proton would typically be about (0.1 nm)/2000 $\sim 10^{-13}$ m from the center of mass. The size of a proton is about 10^{-15} m, so the center of mass is normally outside the proton.

13-25 (a) The energy of a photon is proportional to its frequency, and therefore inversely proportional to its wavelength. Of the visible-light transitions, F has the greatest energy, and thus the shortest wavelength, i.e. it is near the violet end. Transition D would be closest to the red end.

(b) A, B, and C have high energies, which correspond to short wavelengths. We're told they're not visible, so they must be in the ultraviolet. (They can't be in the x-ray or gamma regions, because the energies and wavelengths are not orders of magnitude different from those of visible light.) Similar reasoning places G and H in the infrared.

(c) A long photon wavelength corresponds to a low photon frequency, a low photon energy, and a small difference between energies of atomic levels. There is no upper limit to the wavelengths, since the states get infinitely close together near the top of the hydrogen atom's energy-level diagram. On the other hand, there is a lower limit to the photon wavelength, since the bound states of the hydrogen atom span a limited range of energies, imposing a maximum on the difference between any two levels. Physically, if you put too much energy into a hydrogen atom, you strip off the electron entirely.

13-26

$$\begin{aligned} E_i &= E_f \\ E_{atom,i} &= E_{atom,f} + E_{photon} \\ E_{photon} &= E_{atom,i} - E_{atom,f} \\ &= \frac{mk^2e^4}{2\hbar^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \end{aligned}$$

But

$$\begin{aligned} E_{photon} &= hf \\ &= hc/\lambda \qquad , \end{aligned}$$

so

$$\begin{aligned} hc/\lambda &= \frac{mk^2e^4}{2\hbar^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \\ \lambda &= \frac{2\hbar^2 hc}{mk^2e^4}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)^{-1} \\ &= \frac{4\pi\hbar^3 c}{mk^2e^4}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)^{-1} \end{aligned}$$

13-27 Say the basketball has a mass of 1 kg, and in general its mass is about 0.1 m from the axis of rotation. If it's spinning at a speed of 1 m/s, then its angular momentum is on the order of 0.1 J·s. This is roughly $10^{32}\hbar$. The fact that this number is so huge is an example of the correspondence principle. Angular momentum is quantized in units of $\hbar$, so the basketball can't continuously speed up or slow down its rotation; it has to do it in quantum leaps from one value of L to another. But because L is so huge when expressed in units of $\hbar$, the process appears smooth and continuous. Therefore quantum mechanics doesn't contradict the results of the centuries of previous experiments in which rotation appeared to be smoothly variable.

13-28

13-29 The energies are inversely proportional to the wavelengths, so in terms of energies the equation would be $E_1 = E_2 + E_3$. This would happen for sets of transitions like G, D, and E in the previous problem, where the atom can get from one state to another either by a single hop or by two shorter hops.

13-30 (a) The size of the diagram is comparable to a, but b is so small that you wouldn't be able to see it on that scale.

(b) The stuff inside the exponential is $-r/a$, which is essentially zero for all values of r between 0 and b. In other words, the proton is super-tiny compared to the size of the electron cloud, and the electron's probability density therefore won't change appreciably from one part of the proton to the other. We don't need to do an integral at all. We can just multiply:

$$\begin{aligned}P &= \Psi^2 V \\ &= \left(\pi^{-1/2}a^{-3/2}e^{-r/a}\right)^2\left(\frac{4\pi}{3}b^3\right) \\ &\approx \left(\pi^{-1/2}a^{-3/2}\right)^2\left(\frac{4\pi}{3}b^3\right) \\ &= \frac{4b^3}{3a^3}\end{aligned}$$

(c) 1×10^{-15}

(d) The probability density never dies down to zero, but it gets very very close to zero. The atom is fuzzy. It doesn't have a sharply defined boundary. There is theoretically a finite probability for the electron to be a centimeter away from the proton, but it's extremely small.

13-31 In the approximate derivation in subsection 12.4.4, the only place charge came in was in the use of the expression $-ke^2/r$ for the electrical energy. If the charges were Ze and $-e$ rather than e and $-e$, we'd simply have Ze^2 instead of e^2. The result contains e^4, so we need to replace $e^4 = (e^2)^2$ with $(Ze^2)^2 = Z^2e^4$. The result would be

$$E_n = -\frac{mk^2Z^2e^4}{2\hbar^2}\cdot\frac{1}{n^2}.$$

13-32 (a) The relation

$$r \sim \frac{h^2n^2}{mke^2}$$

has the mass of the electron in the denominator. Increasing the mass by a factor of 207 scales the atom down by a factor of 207.

(b) The equation

$$E_n = -\frac{mk^2e^4}{2\hbar^2}\cdot\frac{1}{n^2}$$

would scale up all the energies by a factor of 207. If the atom's energies are scaled up, then so are the energies of its absorption and emission lines. They would be ultraviolet or x-ray lines.

13-33 As suggested in the problem, I used units in which $c = 1$, $h = 1$, and $m = 1$, where m is the mass of the electron. In these units, the initial mass-energy of the electron is 1 (mc^2 in metric units), and a photon has energy = momentum = $f = 1/\lambda$. The electron's initial mass-energy is 1, and I'll reserve the symbol E for its final mass-energy (not the photon's energies).

Conservation of energy:

$$f + 1 = f' + E$$

Conservation of momentum

$$f = -f' + \sqrt{E^2-1}$$

Solving the first equation for E and substituting into the second equation gives

$$(f+f')^2 = (f-f'+1)^2 - 1 \qquad .$$

Solving for f', we have

$$f' = \frac{f}{1+2f}$$

or

$$\lambda' = \lambda + 2$$

13-35 The original equation, for a single rectangular barrier, was

$$P = \exp\left(-\frac{2w}{\hbar}\sqrt{2m(PE-E)}\right) \qquad .$$

Suppose we had two different barriers, of heights PE_1 and PE_2 and widths w_1 and w_2. Although we might get complicated patterns of reflections back and forth, these are presumably unimportant in the approximation that the probability of penetration is low, because in that approximation, the intensity of the waves is dying out so rapidly that the ones that got through after some reflections would be negligible. The probability of penetrating the second barrier is then independent of the probability of penetrating the first one. When probablities are independent, the probability of the "and" is found by multiplying them:

$$P = \exp\left[-\frac{2\sqrt{2m}}{\hbar}\left(w_1\sqrt{PE_1-E} + w_2\sqrt{PE_2-E}\right)\right]$$

For a barrier that isn't just a series of rectangles, we can still approximate it as such a series — this is exactly the idea of the usual definition of the integral in terms of a limit. The widths w_1, ... are then simply dx, and we have

$$P = \exp\left[-\frac{2\sqrt{2m}}{\hbar}\int_a^b \sqrt{PE_1(x)-E}\,\mathrm{d}x\right] \qquad ,$$

where a and b represent the beginning and end of the region that classical physics forbids the particle to enter. This is known as the WKB approximation, after the initials of the three physicists who first published it.

13-36 The requirement of normalization is

$$\int \Psi^2\,\mathrm{d}v = 1 \qquad ,$$

where dv is an infinitesimal volume element, and the integral is over all space. The method used for carrying out this integral is similar to the one used in problem 13-8. The problem has spherical symmetry, so we can break space down into a series of concentric spherical shells, each one stretching from r to $r+\mathrm{d}r$. The volume of such a shell equals its area multiplied by its thickness, i.e., $\mathrm{d}v = 4\pi r^2\,\mathrm{d}r$.

$$\int_0^\infty \Psi^2\cdot 4\pi r^2\,\mathrm{d}r = 4a^{-3}\int_0^\infty e^{-2r/a}r^2\,\mathrm{d}r \qquad .$$

The change of variable $u = 2r/a$ transforms this into

$$\frac{1}{2}\int_0^\infty e^{-u}u^2\,\mathrm{d}u \qquad .$$

The definite integral can be done by integration by parts, but I just used computer software (the open-source program Maxima) to find out that it equals 2. The result of the entire calculation is 1, as required by normalization.

13-37 If this is to be a solution to the Schrodinger equation with a constant potential, then $\nabla^2\Psi$ has to be equal to a constant multiplied by Ψ. Calculating the partial derivatives, we have:

$$\begin{aligned}
\frac{\partial^2\Psi}{\partial x^2} &= -a^2\sin axe^{by}\\
\frac{\partial^2\Psi}{\partial y^2} &= b^2\sin axe^{by}\\
\nabla^2\Psi &= (b^2-a^2)\Psi
\end{aligned}$$

This does equal a constant multiplied by Ψ. An easy way to tell whether this is a classically allowed region or a classically forbidden one is to test whether the kinetic energy is positive. The kinetic energy is equal to $-\nabla^2\Psi$ multiplied by positive constants. For $|b| < |a|$, the kinetic energy is positive, and the solution represents a classically allowed region.

13-38 The result of problem 13-21, for a one-dimensional box of length L, was $K = \frac{h^2 n^2}{8mL^2}$. In three three dimensions, we have some number n_x of half-wavelengths that fit in the x direction, along length a, and similarly for b and c. The three contributions to the energies add, since $K \propto p^2 = p_x^2 + p_y^2 + p_z^2$ is additive. The result is

$$K = \frac{h^2}{8m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right)$$

13-39 The tritium has the same charge as the helium, so the electrical repulsion is unchanged, and therefore the height, width, and shape of the barrier are essentially unchanged. The only thing that's really changed is the mass of the particle being emitted, which has been reduced by a factor of 3/4. The tunneling probability is an exponential, and the exponent is proportional to the square root of the mass, so we're reducing it by a factor of $\sqrt{3/4}$. Since we're comparing two exponentials, it doesn't really matter what base we use. In base 10, the original tunneling probability was 1×10^{-29}, so we just need to cut the exponent from -29 to $-29\sqrt{3/4}$, which is about -25, so the result is 10^{-25}.

13-40 The energy of the radio photon is much more than the $E = mc^2$ equivalent of the putative mass, so the photon is still ultrarelativistic. Equating $mc^2\gamma$ to hf, and solving for v/c, we get $v/c = \sqrt{1-(mc^2/hf)^2}$. This probably rounds off to one on your calculator. A way around this is to use the approximation $\sqrt{1+x} \approx 1 + x/2$, which is valid for small x and is a special case of $(1+x)^p \approx 1 + px$. Using this approximation, we have $v/c = 1 - (1/2)(mc^2/hf)^2$. The relative size of the predicted discrepancy is about 10^{-34}.

13-41 All of the constants cancel out, so we only need the n-dependent part of the expressions. The result is

$$\frac{\lambda_E}{\lambda_F} = \frac{E_F}{E_E} = \frac{2^{-2} - 5^{-2}}{2^{-2} - 4^{-2}} = \frac{28}{25}$$

13-42 In both case, we're talking about 100 joules worth of light. Consulting the chart of the electromagnetiv spectrum, we find that the frequency of visible light is greater than that of radio waves by a factor of about $(10^{14}\text{ Hz})/(10^8\text{ Hz}) = 10^6$. (The radio spectrum covers a lot of ground, but near the middle of it we have FM radio at about 100 MHz $= 10^8$ Hz. The visible spectrum only spans about a factor of two in frequency, so that's less of an issue.) Since a photon's energy is given by $E = hf$, that means a visible photon has a million times more energy than a radio photon. There are a million times more photons in the radio signal than in the visible light. If you assumed a different value for the frequency of the radio waves, you might have gotten an answer that differed from this one by a factor of 10 or 100.

13-43 We have $KE = p^2/2m \propto p^2 \propto \lambda^{-2} \sim (r/n)^{-2}$. Setting this to be $\sim |r^k|$ results in $r \sim n^{2/(2+k)}$, and taking the energy is then $E_n \sim |r^k|n^j$, where $j = 2k/(2+k)$. For the three cases discussed in the problem, we have $j = -2$, $2/3$, and 1. With $j = 2/3$, we have states that get closer together as we go up in energy. With $j = 1$, the states are evenly spaced. This is consistent with the experimental results shown in the figure.

The cases with $k > 0$ are different from the ones with $k < 0$. Positive k indicates that the energy continues to increase, no matter how far apart the particles are. Physically, it then becomes impossible to permanently and completely separate them. The constant of proportionality in $E_n \propto r^j$ is positive.

The case of $k \to \infty$ corresponds to a particle in a box. The exponent j approaches 2, which is the result for a particle in a box.

13-44 (a) The uncertainty in position is on the order of magnitude of the diameter of the nucleus, so the uncertainty principle gives $\Delta p \sim h/\Delta x \sim 10^{-18}$ kg$\cdot$m/s. (b) $K = (1/2)mv^2 = p^2/(2m) \sim 10^{-7}$ J. (c) $m = E/c^2 \sim 10^{-24}$ kg, which is a thousand times greater than the actual mass of the neutron.

13-45 The probability distribution is the *square* of the wavefunction. Normalization requires that integrating the probability distribution give 1:

$$\begin{aligned}1 &= \int_0^L \Psi^2 \,\mathrm{d}x \\ &= A^2 \int_0^L \sin^2(2\pi x/L)\,\mathrm{d}x \\ &= A^2 \cdot (L/2) \qquad ,\end{aligned}$$

where the final line can be performed without any techniques of integration simply because we know that the average value of $\sin^2$ over a full cycle is 1/2. The result is $A = \sqrt{2/L}$.

13-46 (a) Partial differentiation with respect to x means treating y as a constant in addition to u and v. The result is $u\ln(vy)$. (b) Treating x as a constant, the derivative is uvx/y.

13-47 (a) The surface area of the spherical surface is $A = 4\pi r^2$. During a time interval t, the energy from the transmitter travels a distance $w = ct$, and by the time this energy reaches the distance r, it has been spread out over a spherical shell of volume Aw. The energy emitted during this time is Pt, so the energy density is $E/V = PT/(Aw) = P/4\pi c r^2$.

(b) The number of photons n in a volume $V = \lambda^3$ is

$$\begin{aligned}n &= \frac{E}{hf} \\ &= \frac{(E/V)\lambda^3}{hc/\lambda} \\ &= \frac{P\lambda^4}{4\pi hc^2 r^2}\end{aligned}$$

(c) Let's take $P = 10$ kW and $r = 10$ km. The result then comes out to be about 10^{21}. This huge number is consistent with the correspondence principle.

13-50

$$\begin{aligned}\nabla^2\Psi &= \frac{\partial^2}{\partial x^2}\Psi + \frac{\partial^2}{\partial y^2}\Psi \\ &= 4\cdot e^{2x+y} + 1\cdot e^{2x+y} \\ &= 5e^{2x+y}\end{aligned}$$

14-1 For a rotor, the excitation energies are proportional to $\ell(\ell+1)$, so that $E_4/E_2 = 4\cdot 5/(2\cdot 3) = 10/3$. The energy of the $4 \rightarrow 2$ transition equals $E_4 - E_2$, or $E_2(10/3 - 1) = 7.5$ MeV.

14-2 $\{\hat{\mathbf{x}}\}$ is not a basis, because there are vectors such as $\hat{\mathbf{y}}$ that we can't form as a linear combination (i.e., scalar multiple) of $\hat{\mathbf{x}}$. $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$ is the standard basis for this vector space. $\{\widehat{-\mathbf{x}}, \hat{\mathbf{x}}+\hat{\mathbf{y}}\}$ also works as a basis, because the two vectors are linearly independent, and it's easy to check that any vector in the plane can be formed as a linear superposition of them. $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{x}}+\hat{\mathbf{y}}\}$ is not a basis, because these three vectors are not linearly independent.

14-3 (a) The sketch for ℓ will be a 45-degree line through the origin, while r will be only the part of that line in the first quadrant. Of the two, only ℓ is a vector space. The set r isn't a vector space, because it doesn't have additive inverses.

(b) We have $(1/2)(\pi + \pi) = 0$, but $(1/2)\pi + (1/2)\pi = \pi$.

14-4 To do anything useful with these expressions describing units, we need to be able to talk about things like dividing meters by seconds to get meters per second. Thus "addition" needs to be multiplication, which corresponds to adding the exponents. Scalar "multiplication" actually has to be exponentiation, e.g., "multiplying" units of meters by the scalar 2 should give square meters.

14-5 (a) Because Ψ_0 is an even function and Ψ_1 is odd, the integral $\int_{-\infty}^{\infty} \Psi_0\Psi_1\,\mathrm{d}x$ is zero.

(b) Either energy or parity would distinguish them.

14-6 (a) $\Delta E \sim h/t\ 10^{-24}$ J $\sim 10^{-11}$ MeV. This uncertainty in the energy is negligible both compared to the excitation energies involved and compared to the energy resolution of any experiment.

(b) In this example, the uncertainty in energy comes out to be on the order of MeV, so that the uncertainty in the excitation energy is on the same order of magnitude as the excitation energy itself.

14-7 The given information tells us that $H_2(x) = ax^2 + 1$, where ignoring normalization has given us the opportunity to simplify things by arbitrarily letting the constant term equal 1. The orthogonality condition $\langle H_j(x)e^{-x^2/2}|H_k(x)e^{-x^2/2}\rangle = 0$ gives the integral $\int_{-\infty}^{+\infty} H_j(x)H_k(x)e^{-x^2}\,\mathrm{d}x = 0$. This is automatically satisfied for $j = 2$, $k = 1$ because the integrand is odd, but for $j = 2$, $k = 0$ it gives us a condition that allows us to determine the constant a:

$$a\int_{-\infty}^{+\infty} x^2 e^{-x^2}\,\mathrm{d}x = \int_{-\infty}^{+\infty} e^{-x^2}\,\mathrm{d}x.$$

It is possible to look up the definite integrals in a table of integrals, or do them using computer software, but as suggested in the hint, let's instead use integration by parts to relate them to each other. Let $u = x^{m-1}$ and $\mathrm{d}v = xe^{-x^2}\,\mathrm{d}x$. Then $\int_{-\infty}^{+\infty} x^m e^{-x^2}\,\mathrm{d}x = \int_{-\infty}^{+\infty} u\,\mathrm{d}v = -\int_{-\infty}^{+\infty} v\,\mathrm{d}v$, where we ignore the uv term because it equals zero at $x = \pm\infty$. The result is that

$$\int_{-\infty}^{+\infty} x^m e^{-x^2}\,\mathrm{d}x = \frac{1}{2}(m-1)\int_{-\infty}^{+\infty} x^{m-2}e^{-x^2}\,\mathrm{d}x,$$

which gives $a = -2$. We therefore have $H_2 = -2x^2 + 1$ and $\Psi_2 = (-2x^2+1)e^{-x^2/2}$.

14-8 The definition of hermitian operator would require

$$\langle \hat{H}u|v\rangle = \langle u|\hat{H}v\rangle.$$

On the left-hand side, we have $\langle \hat{H}u|v\rangle = \langle iu|v\rangle$ by the definition of $\hat{H}$. By using one of the properties of the inner product, this becomes $i^*\langle u|v\rangle = -i\langle u|v\rangle$. But if we carry out a similar computation on the right-hand side of the definition, we don't get the minus sign, so the equation is false (for $\langle u|v\rangle \neq 0$).

14-9 (a) The moment of inertia I will be something like $3m\ell^2$, where m is the mass of a proton, and $\ell = 0.1$ nm. A state with $L = \hbar$ will have energy $L^2/2I \sim \hbar^2/6m\ell^2 \sim 10^{-22}$ J. At room temperature, typical thermal energies are $kT \approx 4\times10^{-21}$ J, which is much larger.

(b) The typical kinetic energy along the axis connecting the molecule to the detector is $(1/2)kT$. The mass of the whole molecule is about $17m$, where m is the mass of the proton. Setting the thermal energy equal to $(1/2)mv^2$, we find $v/c \sim (1/c)\sqrt{kT/17m} \sim 10^{-6}$, so Doppler shifts are expected to be a parts-per-million effect. Given the number of lines and the range of frequencies, which spans about a factor of 2, the spacing of the lines will on average be something like 1%, so Doppler shifts should not broaden the lines enough to make one overlap with another.

14-10 (a) We require $\hat{H}|\text{ex.s.}\rangle = |f|\text{ex.s.}\rangle = -f|\text{ex.s.}\rangle$. As in the determination of the ground state's wavefunction, we can take $|\text{ex.s.}\rangle = |1\rangle + \beta|2\rangle$, for an unknown β. We then have

$$\begin{aligned}\hat{H}|\text{ex.s.}\rangle &= \hat{H}(|1\rangle + \beta|2\rangle)\\ &= f|2\rangle + \beta f|1\rangle,\end{aligned}$$

If we want this to equal $-f|\text{ex.s.}\rangle$, we need $\beta = -1$, giving

$$|\text{ex.s.}\rangle = |1\rangle - |2\rangle.$$

(b) $\langle\text{ex.s.}|\text{g.s.}\rangle = (\langle1| - \langle2|)(|1\rangle + |2\rangle) = \langle1|1\rangle + \langle1|2\rangle - \langle2|1\rangle - \langle2|2\rangle$. We already know that states 1 and 2 are orthogonal to one another, so this simplifies to $\langle1|1\rangle - \langle2|2\rangle$. Since the states 1 and 2 are nomalized, this is $1 - 1 = 0$.

(c) If we set $|\text{g.s.}\rangle = A(|1\rangle + |2\rangle)$, then by a computation similar to the one in part b, we get $\langle\text{g.s.}|\text{g.s.}\rangle = AA^*(\langle1|1\rangle + \langle2|2\rangle) = 2|A|^2$. If we require this to equal 1, we get $|A| = 1/\sqrt{2}$. We are free to take any complex number for A that we like, as long as it has this magnitude. The same thing happens for the excited state.

14-11 (a) The norm of Ψ (inner product of Ψ with itself) is

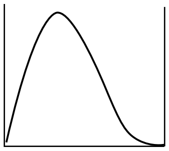
$$\langle\Psi|\Psi\rangle = (2A\langle1| + A\langle2|)(2A|1\rangle + A|2\rangle).$$

Using the linearity of the inner product, this becomes

$$\begin{aligned}&A^2(2\langle1| + \langle2|)(2|1\rangle + |2\rangle)\\ &= A^2(2^2\langle1|1\rangle + \ldots\langle1|2\rangle\\ &\quad + \ldots\langle2|1\rangle + 1^2\langle2|2\rangle).\end{aligned}$$

The second and third terms vanish because states 1 and 2 are distinguishable (or, alternatively, because one function is odd and the other is even). The result is $|A|^2(2^1 + 1^2) = 5|A|^2$, so we need $|A| = 1/\sqrt{5}$.

(b) On the left side of the box, the two functions are both positive, so the wavefunction is big and positive. On the right side of the box, there is a lot of cancellation. The resulting shape looks like this:



(c) Let x run from 0 to π. Then the two functions are basically $\sin x$ and $\sin 2x$, but we need to normalize them properly. The normalization factors come out to be $\sqrt{2/\pi}$. For the probability of being on the left side of the box, we have

$$\begin{aligned}P &= \int_0^\pi (1/5)(2/\pi)(2\sin x + \sin 2x)^2\,\mathrm{d}x \\ &= (2/5\pi)\int_0^{\pi/2} (4\sin^2 x + 4\sin x \sin 2x + \sin^2 2x)\,\mathrm{d}x\end{aligned}$$

The average values of the sine waves in the first and third terms are both 1/2. Replacing them with their average values gives

$$P = (2/5\pi)\int_0^{\pi/2} (2 + 4\sin x\sin 2x + 1/2)\,\mathrm{d}x$$

The term in $\sin x \sin 2x$ is the kind of thing that you would just do using software these days, or you can use the double-angle identity for the sine and then a change of variables to $u = \sin x$. The result is

$$\frac{1}{2} + \frac{16}{15\pi}.$$

Based on part b, it makes sense that this is greater than 1/2. (d) Probability is proportional to the square of the amplitude, so the probability of getting the ground state is four times greater than the probability of getting state 2. Therefore the probability of getting the ground state is 4/5.

(e) The shape of the wavefunction would flip, and the probability of being in the left half would now be $1 - P = 1/2 - (16/15)\pi$. Everything else would stay the same.

14-12 (a) As a warm-up let's do the one-dimensional version of the problem, for the length being L. Then $\lambda = 2L/n$, and the energy is $p^2/2m = \epsilon n^2$. The three-dimensional version is separable. The x and y dimensions contribute terms like the one we already calculated, while halving the wavelength for the z dimension gives four times the energy. Therefore the total energy, in units of ϵ, is $n_x^2 + n_y^2 + 4n_z^2$. The smallest energies we can make are the following.

1	1	1	6
2	1	1	9
1	2	1	9
3	1	1	14
1	3	1	14

(If we try bumping n_z up to 2, we get a minimum energy of 17, which is higher.) Counting spin, the lowest energy state has a degeneracy of 2, the second energy state is 4-fold degenerate, and the third is also 4-fold degenerate.

(b) Subject to the exclusion principle, the lowest total energy is obtained by putting 2 electrons in the lowest energy state and 3 in the second, for a total energy of 39 (still in units of ϵ).

(c) The "cheapest" way of forming an excited state is to pop one electron out of the lowest-energy state and into the second energy state, and this gives a total energy of 42. We can't boost the other one from the first to the second, because that would be all five electrons in the second state, which would violate the exclusion principle. The second-cheapest method is to boost one from the second energy level to the third, which produces an energy of 44. To excite one of the five electrons from the ground state to one of these two states, we would need a photon with an energy of either $42 - 39 = 3$ or $44 - 39 = 5$.

14-13 (a) First let's show that the Schrödinger equation is separable. If we write $\Psi = X(x)Y(y)Z(z)$, then $\nabla^2\Psi = X''YZ + XY''Z + XYZ''$, as in the example in the text. The Schrödinger equation has the form $(\ldots)\nabla^2\Psi + U\Psi = E\Psi$, and if we divide by Ψ on both sides this becomes $(\ldots X''/X + (1/2)kx^2) + (\ldots Y''/Y + (1/2)ky^2) + (\ldots Z''/Z + (1/2)kz^2) = E$. If X is a solution of the one-dimensional version of this Schrödinger equation, then the first term is constant, and so on.

(b) and (c): From the problem in which we solved the one-dimensional version of the problem, we know that the ground-state wavefunction is e^{-bx^2}, where $b = \sqrt{km}/2\hbar$, and from the discussion of the harmonic oscillator in the text we know that the energy of the ground state is $(1/2)\hbar\omega$, where $\omega = \sqrt{k/m}$ is the expression for the classical frequency. Generalizing this to the three-dimensional version, the ground state is

$$e^{-bx^2}e^{-by^2}e^{-bz^2} = \exp\left[-b(x^2+y^2+z^2)\right] = e^{-br^2},$$

and its energy is

$$\frac{3}{2}\hbar\omega.$$

14-14 (a) Using the same technique as in problem 11a, we need $|k|^2(2^2+2^2+1^2) = 1$, so $|k| = 1/\sqrt{9} = 1/3$.
(b) $(2^2+2^2)|k|^2 = 8/9$.
(c) Once we have determined C's spin to be down, we know that the wavefunction is $|\downarrow\downarrow\downarrow\rangle$. Therefore the probability of subsequently measuring A to have spin up is zero.

14-15 As in the discussion question, we suppose that initially there are two wave pulses, which we sketch by giving their complex amplitudes at an evenly spaced series of points. The right-going pulse is

$$1 \quad i \quad -1 \quad -i,$$

and the left-going one looks like

$$1 \quad -i \quad -1 \quad i.$$

Before collision, adding up the values of $|\Psi|^4$ gives $1^4+1^4+1^4+1^4+1^4+1^4+1^4+1^4 = 8$. When the two pulses lie on top of each other, the superposed wave is

$$2 \quad 0 \quad -2 \quad 0.$$

The sum of the $|\Psi|^4$ values is $2^4+2^4 = 32$, which is not the same as 8, so probability has not been conserved.

14-16 Let's use the fact that the ccw wavefunction should be normalized. Then we have $1 = \langle\text{ccw}|\text{ccw}\rangle = |A|^2\int_0^{2\pi} e^{-i\theta}e^{i\theta}\,\mathrm{d}\theta$. The integrand simplifies to 1, so $1 = |A|^2\int_0^{2\pi}\mathrm{d}\theta$, i.e., $|A|^2 = 1/2\pi$.

14-17 We have

$$\langle\text{c}|\text{s}\rangle = \int_0^{2\pi}\cos\theta\sin\theta\,\mathrm{d}\theta.$$

The easy way to show that this integral is zero is to note that it cancels pointwise. For every angle θ in the first quadrant, there is a corresponding angle $2\pi-\theta$ in the fourth quadrant where the integrand has the opposite sign, and similarly for the second and third quadrants. Another way to do it is to use the trig identity $\sin\theta\cos\theta = (1/2)\sin 2\theta$, whose integral vanishes because the function oscillates an even number of times over the range of integration.

14-18 From Euler's formula, we have

$$\Psi_1 = \Phi_1 + \Phi_2 \qquad \text{and}$$
$$\Psi_2 = \Phi_1 - \Phi_2.$$

Solving for the Φ's,

$$\Phi_1 = \frac{1}{2}(\Psi_1+\Psi_2) \qquad \text{and}$$
$$\Phi_2 = \frac{1}{2}(\Psi_1-\Psi_2).$$

Neither the Ψ basis nor the Φ basis is special or preferred. Either can be expressed in terms of the other.

14-19 The text has a section on cut-and-paste solutions to the Schrödinger equation, and it presents two examples, one of partial reflection from a finite step and one of a particle in a box with infinite sides. This problem can basically be solved with a mash-up of those two examples. As in the example of the particle in a box, the wavefunction is zero in the region where the potential is infinite, and we don't expect the wavefunction to be differentiable at the boundary, but we do expect it to be continuous. Our solution, to the left of the step, is of the form

$$\Psi = \Psi_I + \Psi_R,$$

where as in the example of the finite step, $\Psi_I = e^{i(kx-\omega t)}$ and $\Psi_R = Re^{i(-kx-\omega t)}$, so that

$$\Psi = e^{i(kx-\omega t)} + Re^{i(-kx-\omega t)}.$$

Since $\Psi = 0$ for $x \geq 0$, continuity requires that our expression for Ψ at $x \leq 0$ also give zero when we plug in $x = 0$. Plugging in $x = 0$ to our expression results in

$$\Psi(0) = e^{-i\omega t} + Re^{-i\omega t} = (1+R)e^{-i\omega t},$$

and the only way to make this equal to zero for all t is to have $R = -1$. Our solution is therefore

$$\Psi = e^{i(kx-\omega t)} - e^{i(-kx-\omega t)}.$$

14-21 (a) Plugging in to the Schrödinger equation for Ψ_1 gives

$$-\Psi_1 = -\frac{1}{2m}\Psi_1 + U\Psi_1,$$

which could be a solution in a classically forbidden region. Doing the same with Ψ_2, we find

$$i\Psi_2 = \frac{1}{2m}\Psi_2 + U\Psi_2,$$

which is impossible if the mass and potential are both real.
(b) Ψ_2 shows exponential growth over time, which violates unitarity.
14-22 (a) Ψ_2 can't be a solution, because it doesn't satisfy the boundary conditions: it isn't zero at $y = 0$ and $y = b$.
(b) Ψ_1 has the form of a traveling wave, which is moving to the left. By decreasing x and increasing t by the correct amount, we can get the same quantity inside the exponential, which shows that we're surfing on one point on the wave.
(c) Ψ_3 could represent motion in a classically forbidden region, since the wavefunction falls off exponentially to the left, as expected for tunneling.
(d) Ψ_4 is the only non-separable solution. The others are clearly separable because they are expressed in the form $f(x)g(y)$.

3 Mechanics

0-2

$$\begin{aligned}&3.0\times10^8\ \text{m/s}\\ &\quad\times\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)\left(\frac{1\ \text{inch}}{2.54\ \text{cm}}\right)\left(\frac{1\ \text{foot}}{12\ \text{inches}}\right)\\ &\quad\times\left(\frac{1\ \text{yard}}{3\ \text{feet}}\right)\left(\frac{1\ \text{furlong}}{220\ \text{yards}}\right)\\ &\times\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{hr}}\right)\left(\frac{24\ \text{hr}}{1\ \text{day}}\right)\left(\frac{14\ \text{days}}{1\ \text{forthnight}}\right)\\ &\qquad= 1.8\times10^{12}\ \text{furlongs/fortnight}\end{aligned}$$

0-3 (a) 10^4 μg; (b) 10^{10} μg; (c) 10^{10} μg; (d) 10^{11} μg; (e) 1 μg

0-4 (a)

$$\begin{aligned}\frac{0.5\ \text{mg}}{0.2\ \text{ng/hamburger}} &= \frac{0.5\times10^{-3}\ \text{g}}{0.2\times10^{-9}\ \text{g}}\ \text{hamburgers}\\ &= 2\ \text{million hamburgers!}\end{aligned}$$

(Note how the fake units of "hamburgers" help to reassure us that dividing the first number by the second one gives the right result.)

(b) (0.5 mg)/(300 ng/serving)=2000 servings

0-5 (a) 8 cm; (b) 1.33 m; (c) doesn't make sense; (d) 60 miles/hour

0-6 23.4 m + 2 × 0.294 m = 23.988 m, which should be rounded to 24.0 m (3 significant figures)

0-8 Both r and h have units of meters. The product rh has units of m^2, and taking its square root gives units of meters again. That makes sense, because the result is supposed to be a distance.

0-9 The range of uncertainty amounts to more than a factor of 2. That means that each of the two figures has been quoted with some completely meaningless sig figs. A better way to write the numbers would be 6 days and 14 days.

0-11 The weight is given in U.S. units with not much more than one sig fig of precision, but the metric conversion is given with three sig figs. It's silly to try to give tenths of a gram here; there must be an order of magnitude more variation than that from one bag to the next, since one pretzel is probably several grams.

0-13 The estimate of 4,480 calories is stated with three significant figures, which is absurd. Individuals often differ by a factor of two in their food consumption, and in any case this is a rough estimate for a species that has been extinct for tens of thousands of years.

1-5

$$(1\ \text{ft}^3)\times(12\ \text{in}/1\ \text{ft})^3 = 1728\ \text{in}^3$$

1-6 Volume scales like the third power of length, so the dog has 8 times more brain cells.

1-7 Solving the proportionality $A \propto L^2$ for L, we have $L \propto \sqrt{A}$. Converting this into the language of ratios gives $L_{LA}/L_{SF} = \sqrt{A_{LA}/A_{SF}} = 1.2$.

1-9 Area scales like the square of the linear dimensions, so 0.35 of the linear dimensions corresponds to $0.35^2 = 0.11$, or 11% of the area.

1-10 (a) Converting the relation $n \sim A^{1/3}$ into the language of ratios, we have $n_1/n_2 = (A_1/A_2)^{1/3}$. Plugging in the numbers, the result is $n_1 \approx 14$, which not in bad agreement with the real-world figure of 22. (b) Suppose that the exponent was equal to 1. Then if we took two adjacent tracts of land with equal area and counted up all the taxa, we would find twice as many taxa in the combined area than in either of its halves. In other words, no taxon would be found in both areas. This is of course unrealistic. Almost certainly there will be taxa that sprawl across the border.

1-11 Humans and ants are both mostly water, so the ratio of their body masses is probably about the same as the ratio of their volumes. We therefore have $m \propto V \propto L^3$, or $L \propto m^{1/3}$. Since the ratio of the body masses is $m_1/m_2 \sim 7\times10^6$, the ratio of the thicknesses is $L_1/L_2 \sim (m_1/m_2)^{1/3} \sim 200$.

1-12 In the language of proportionalities, we have $L \propto t$, and $V \propto L^3$, so $t \propto V^{1/3}$. Translating this into a statement about ratios, we have $t_2/t_1 = (V_2/V_1)^{1/3} = 100^{1/3} = 4.64$. The signals have already been propagating outward for $t_1 =$ 90 years, so to fill 100 times the volume, they would have to propagate for (90 years)(4.64), or 418 years. Adding this to 1920, we get 2338 AD.

1-13 Area scales like the square of the linear dimensions, so Earth's total surface area is four times that of Mars. Since only 30 percent of Earth's area is land, the ratio is 1.2 to 1 (Earth to Mars).

1-14 (a) You can get two A5 sheets by cutting an A4 in half, so the area of an A5 is half that of an A4. The two types are the same shape, so the area and linear dimensions are related by $A \propto L^2$, or $L \propto \sqrt{A}$. Cutting the area in half means reducing all the linear dimensions by a factor of $1/\sqrt{2}$.

(b) Let the dimensions of the B0 box, in units of meters, be a, b, and c ($a > b > c$). If you can pack two B1 boxes inside a B0 box, two of the dimensions of a B1 must be the same as the two smallest dimensions of the B0, b and c, while the third is $a/2$. Since $L \propto V^{1/3}$, the dimensions of the B1 must be smaller than those of the B0 by a factor of $1/2^{1/3}$. If the series is to continue with B2, B3, ..., the three sides of a particular box must be related by this same factor:

$$\begin{aligned} b &= \frac{a}{2^{1/3}} \\ c &= \frac{b}{2^{1/3}} = \frac{a}{2^{2/3}} \end{aligned}$$

Since the volume is one cubic meter, we have

$$\begin{aligned} 1\text{ m}^3 &= abc \\ &= (a)\left(\frac{a}{2^{1/3}}\right)\left(\frac{a}{2^{2/3}}\right) \\ &= \frac{a^3}{2} \\ a &= 2^{1/3}\text{ m} \\ b &= 1\text{ m} \\ c &= 1/2^{1/3}\text{ m} \end{aligned}$$

1-15 Apply two of the general rules for making order-of-magnitude estimates: don't try to estimate volume directly, and approximate complicated shapes as simple ones. Suppose a person is sort of a rectangle, 2 m tall, 0.5 m wide, and 0.2 m from front to back. The volume would then be $0.2\text{ m}^3 = 2\times10^5\text{ cm}^3$, which we can round off to 10^5 cm^3.

1-16 I don't even know the exact width of a football field, but let's guess the dimensions are 100 yd × 30 yd, which is close enough to 100 m × 30 m, for a total area of 3000 m^2. If we assume each blade of grass takes up on the order of 1 mm^2, then the number of blades equals $3000\text{ m}^2/1\text{ mm}^2 = 3\times10^9\text{ mm}^2/1\text{ mm}^2 = 3\times10^9$. Call it 10^9 blades.

1-17 Let's say the chip is 1 cm × 1 cm. It contains about 10^8 bits (64×10^6 bytes$\times$8 bits/byte), so the area of each circuit is about 10^{-8} cm^2. Assuming the circuits are little squares, their dimensions would be about 10^{-4} cm on a side, since $10^{-4}\text{ cm}\times10^{-4}\text{ cm} = 10^{-8}\text{ cm}^2$.

1-18 The world's population is about 5 billion, or call it 10^{10}. Suppose each person gets an apartment that's 5 m x 5 m x 2 m, for a volume of 50 m^3, or round off to 100 m^3. The total volume of the building needs to be about 10^{12} m^3. Its area at the base is 10^8 m^2, so its height needs to be $10^{12}\text{ m}^3/10^8\text{ m}^2 = 10^4\text{ m} = 10$ km.

1-19 (a) Living things typically have about the same density as water, $1\text{g/cm}^3 = 1000\text{ kg/m}^3$. The linear dimensions are about 1 μm, so the volume is about 10^{-18} m^3, giving a mass of 10^{-15} kg.

(b) Estimating the mass of the feces to be 1 kg (some of us are more full of it than others), we get about 10^{15} bacteria in the body. This is two orders of magnitude greater than the number of human cells.

(c) Since the human cells are much less numerous, but take up much more space, they must be many orders of magnitude bigger than bacterial cells.

1-20 Suppose one bongo burger is 1/4 lb, which is about 0.1 kg. If the usable meat on a cow amounts to 100 kg, then one cow makes 1000 burgers. Clearly the cow has to eat much more over the course of its life than its own adult weight. (After all, you could not raise a human to adulthood on 100-200 lb of food.) Let's say 10000 kg of feed are needed to raise a several-hundred-kg cow. Then 10000 kg of feed makes 1000 burgers, or 10 kg of feed makes 1 burger. That means $\sim 10^{11}$ kg of feed were needed to make 10^{10} burgers.

1-21 We never try to estimate mass or volume directly; we estimate linear dimensions first. Let's say the hair has a diameter of 0.1 mm, and a length of 1 cm. There's no real point in trying to compute its volume as a cylinder, since this is just an estimate. Let's just consider it to be a long, skinny box, with dimensions 0.1 mm × 0.1 mm × 1 cm. We'll assume it has the same density as water, 1 g/cm^3, so let's get all the dimensions in terms of centimeters before computing the volume and mass: $0.01\text{ cm} \times 0.01\text{ cm} \times 1\text{ cm} = 10^{-4}\text{ cm}^3$. Using the density, we get a mass of $10^{-4}\text{ g} = 10^{-7}$ kg.

1-23 First let's estimate the total volume of the Earth's atmosphere. We know that by going to the top of a mountain that's a few miles high, we can get above most of the atmosphere, so let's say the atmosphere is roughly 10 km, or 10^4 m thick. The diameter of the earth is about 12000 km, or about 10^7 m, so its surface area is on the order of 10^{14} m^2, and multiplying by the thickness of the atmosphere gives about 10^{18} m^3. A human breath is probably on the order of a liter, or 10^{-3} m^3, so Caesar's last breath has been diluted by a factor of something like $10^{18}/10^{-3} \sim 10^{21}$.

Now the question is whether your breath has a lot more than, or a lot less than, 10^{21} atoms in it. If you've taken chemistry, you know that a mole is something like 10^{24} atoms, and a mole of air would only be on the order of 10 grams. That means your breath has a lot more than 10^{21} atoms in it, so it's probably true that it contains many of the atoms from Caesar's last breath. If you haven't taken chemistry, you might not have as good a feel for this, but you could simply look up the density of air and the mass of an oxygen molecule on the internet or at the library.

(One implicit assumption that could be wrong here is that air stays in the atmosphere indefinitely. Some air could spend a lot of its time in plants, or dissolved in the oceans or in rocks.)

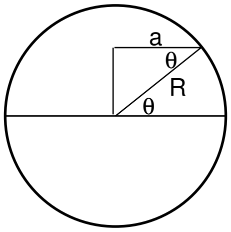
1-25 The relationship between volume and linear dimensions is $V \propto L^3$. We're given information about V and asked to find out about L, so we solve this for $L \propto V^{1/3}$. Converting this into a statement about ratios, we have $L_1/L_2 = (V_1/V_2)^{1/3}$. The result is that the balls could have a diameter as big as 4.48 cm.

1-26 A thousand metric tons is 10^9 g of plutonium. To estimate the volume of the earth's oceans, let r be the earth's radius and d the average depth of the ocean. Looking up r, estimating d to be 1 km, and taking the earth to be mostly ocean, we get a volume of $4\pi r^2 d$, or about $10^9\text{ km}^3 = 10^{24}\text{ cm}^3$. The concentration comes out to be $\sim 10^{-15}$ g/cm^3, which is fifty times lower than the government standard. This method of disposal would therefore have no measurable human health effects (and probably none on any part of the biosphere).

2-2 (a) When its velocity is negative it's coming back toward the hive, so at $t = 8.7$ s it is as far from the hive as it will ever get. (b) Between $t = 0$ and $t = 8.7$ s, the area under the curve is about 22 squares. Each square is 1 s wide and 1 m/s tall, so each one represents 1 m. The greatest distance from the hive is 22 m. (c) The area from $t = 8.7$ s to the end is under the axis, so it counts as negative. It's about 2 squares, so at the end the bee is about 20 m from the hive.

2-3 (a) Let a be the radius of the circle made by the point on the earth's surface. $a = R\cos\theta$, and $L = 2\pi a$, so $L = 2\pi R\cos\theta$.

(b) Plugging in $\theta = 34°$ and $R = 6.4 \times 10^6$ m gives $L = 3.3 \times 10^7$ m, so the speed is $L/24\text{ hr} = 1.4 \times 10^6\text{ m/hr} = 890$ mi/hr.



2-4 $v = \mathrm{d}x/\,\mathrm{d}t = 10 - 3t^2 = -17$ m/s.

2-5 (a) Since b is added to t^2, b must have units of s^2. We want x to come out in units of meters, so a must have units of $\mathrm{m}\cdot\mathrm{s}^2$ (meters *times* seconds squared, not meters divided by seconds squared as in acceleration).

(b) From the given information, we can find her speed $v(t)$ by differentiation, and we can then find the maximum by the usual technique of setting the derivative equal to zero. That means taking two derivatives in a row: a second derivative. The second derivative of the position is the acceleration, so what it boils down to is finding the time when her acceleration is zero. (Note that the a in the original equation doesn't stand for acceleration.)

$$\begin{aligned}
\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} &= \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[a\left(b+t^2\right)^{-1}\right] \\
&= a\frac{\mathrm{d}}{\mathrm{d}t}\left[-\left(b+t^2\right)^{-2}(2t)\right] \\
&= a\left[2\left(b+t^2\right)^{-3}(4t^2) - \left(b+t^2\right)^{-2}(2)\right] \\
&= 0
\end{aligned}$$

$$\begin{aligned}
8\left(b+t^2\right)^{-3}(t^2) &= 2\left(b+t^2\right)^{-2} \\
8t^2 &= 2\left(b+t^2\right) \\
6t^2 &= 2b \\
t &= \pm\sqrt{b/3}
\end{aligned}$$

Plugging $t = \pm\sqrt{b/3}$ into the first derivative, we get $|v| = (3^{3/2}/8)ab^{-3/2}$.

(c) The $3^{3/2}/8$ is unitless, so the units come from the a and b factors: $\mathrm{m}\cdot\mathrm{s}^2\cdot(\mathrm{s}^2)^{-3/2} = \mathrm{m/s}$.

2-6 To infer the units of α, we use the fact that it doesn't make sense to add things unless they have the same units. The 1 is unitless, so αT must also be unitless. Since T has units of degrees, α must have units of deg^{-1}, or "per degree."

$$\begin{aligned}
\frac{\mathrm{d}A}{\mathrm{d}T} &= \frac{\mathrm{d}}{\mathrm{d}T}\left[(1+\alpha T)^2\ell_\mathrm{o}^2\right] \\
&= 2\ell_\mathrm{o}^2\alpha(1+\alpha T)
\end{aligned}$$

To check the units of this, we first note that the 2 and the $(1+\alpha T)$ are unitless, so that the units come from $\ell_\mathrm{o}^2\alpha$. Since ℓ_o has units of meters and α has units of deg^{-1}, the result has units of $\mathrm{m}^2/\mathrm{deg}$. This makes sense, because the derivative $\mathrm{d}A/\,\mathrm{d}T$ represents an infinitesimal increase in area divided by an infinitesimal rise in temperature.

2-7 We expect the derivative to have units of m^3/s, since it represents a rate of change of a volume. The Leibniz notation confirms this, since $\mathrm{d}v/\,\mathrm{d}t$ represents an infinitesimal volume divided by an infinitesimal time. Computing the derivative, we get $4\pi c^3t^2$, which has units of $(\mathrm{m/s})^3\mathrm{s}^2 = \mathrm{m}^3/\mathrm{s}$ as expected.

2-9 (a) Let the subscripts A, E, and G refer to the arrow, Eowyn, and the ground. Let the positive x axis point forward.

$$\begin{aligned}
v_{EG} &= 11\ \mathrm{m/s} \\
v_{AE} &= -25\ \mathrm{m/s}
\end{aligned}$$

(b)

$$\begin{aligned}
v_{AG} &= v_{AE} + v_{EG} \\
&= -14\ \mathrm{m/s}
\end{aligned}$$

The answer makes sense: the arrow flies backward, at less than normal speed.

2-12 Take a frame defined by an observer in another car who is in the lane next to yours, who is initially moving at the same speed as you, and who continues at the speed even while you slow down. In that observer's frame, you started out with zero speed and then gained some (backward) speed when you stepped on the brakes.

2-14 (a) She says her own velocity is zero.

Time dilation happens because of relative motion. We can conclude that A and B are not in motion relative to one another, but that there is relative motion between them and C.

(b) Alice says B is at rest, but C is moving. (c) Betty says A is at rest, but C is moving. (d) Cathy says both A and B are moving, and she sees them moving in the same direction at the same speed.

2-15 The equation for gamma is $\gamma = 1/\sqrt{1-v^2}$. If v is negative, v^2 still comes out positive. Negative and positive values of v give the same results for gamma.

This makes sense, because positive and negative values of v indicate motion in opposite directions. Relativistic effects like time dilation and length contraction shouldn't come out different if you travel west rather than east, or north rather than south. Gamma indicates how strong these relativistic effects are, so gamma shouldn't depend on the direction of motion. This relates directly to the second postulate stated in section 7.2, that all directions in space have the same properties.

Note that it *is* OK for a directional (vector) quantity like velocity to be connected to some other direction quantity, such as acceleration, by a relation such as $a = dv/dt$. Then an observer who chooses the opposite orientation for their coordinate system will simply flip all the signs of the velocities and accelerations, and there is no way to tell in experiments which choice is "special." That's different from the situation with γ, which is not directional and can be measured without even choosing a coordinate system.

2-16 (a)

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-(v/c)^2}} \\ &= \frac{1}{\sqrt{1-\left(\frac{17000\ \text{m/s}}{3\times10^8\ \text{m/s}}\right)^2}} \\ &= 1.0000000016\end{aligned}$$

(b) Gamma gives us a comparison of the rate at which time flows on earth and on the probe. A gamma of one would indicate equal rates. Since the gamma is a tiny bit greater than one, the ratio is a tiny bit different from a one-to-one ratio. Gamma differs from one by 1.6×10^{-9}, i.e., 1.6 parts per billion. The disagreement between clocks on the probe and on earth that accumulates over each year is therefore

$$(31\times10^6\ \text{s})(1.6\times10^{-9}) = 0.050\ \text{s} \qquad .$$

This is a fairly big discrepancy. You might even be able to detect it with a consumer-grade clock. (The way NASA really notices the effect is that when the probe signals back to earth by radio, the radio waves vibrate a little more slowly than they should, i.e., the signal's frequency is shifted a tiny bit.)

2-17 Let $r = 6.4\times10^6$ m be the radius of the earth, $R = 1.5\times10^{11}$ m the radius of its orbit, and $T = 1$ year $= 3.2\times10^7$ s the period of its orbit. Then the velocity is $v = (2\pi R)/T$, and we have $\gamma = 1/\sqrt{1-v^2/c^2} = 1 + 4.8\times10^{-9}$. The resulting contraction of the diameter is about $(4.8\times10^{-9})(2r) = 6$ cm.

2-18 (a) v = (slope) = (rise)/(run) = x/γ, so $x = v\gamma$. (b) Because we're using units in which $c = 1$, the diagram has symmetry with respect to reflection across the 45-degree diagonal. Therefore the (t, x) coordinates of P and Q are simply related by a swap of t and x, and Q has coordinates $(v\gamma, \gamma)$. (c) Points P and Q are separated by $|\Delta t| = (1-v)\gamma$, and $|\Delta x| = (1-v)\gamma$ as well. The Pythagorean theorem results in $\text{PQ} = \sqrt{2}(1-v)\gamma$. (d) Averaging the coordinates of points P and Q, we find that C has coordinates $x = t = (1+v)\gamma/2$. The Pythagorean theorem then gives $\text{OC} = (1+v)\gamma/\sqrt{2}$. (e) The area of triangle PQO is (1/2)(base)(height) = $(1/2)\text{PQ}\cdot\text{OC} = (1/2)(1-v)(1+v)\gamma^2$, which simplifies to $(1/2)(1-v^2)\gamma^2$. Doubling this to find the area of the quadrilateral, we obtain $(1-v^2)\gamma^2$, and setting this equal to 1 and solving for γ gives the desired result.

2-19 (a)

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-v^2/c^2}}\\ &= \frac{1}{\sqrt{1-9/25}}\\ &= \frac{1}{\sqrt{16/25}}\\ &= \frac{5}{4}\end{aligned}$$

(b) The $(3/5)c$ example came out simple because 3, 4, and 5 are integers that have the relationship $3^2+4^2=5^2$. This is just like a 3-4-5 right triangle, but there's no triangle involved here! We need to find another set of numbers like this. The simplest way is to reverse the role of the 3 and 4, in which case we get

$$v=\frac{4}{5} \qquad , \qquad \gamma=\frac{5}{3}$$

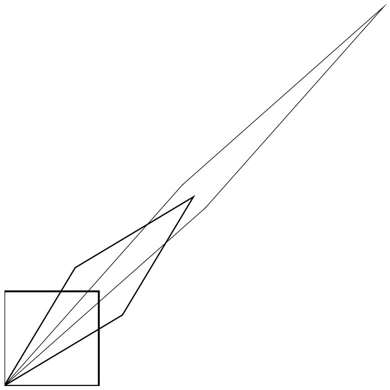
If that seems like a cheap shot, then you need to find another set of integers that form a right triangle; I was lazy, and just did a Google search for "Pythagorean triple," which turned up $11^2+60^2=61^2$ as an example. This provides the examples

$$v=\frac{11}{61} \qquad , \qquad \gamma=\frac{61}{60}$$

and

$$v=\frac{60}{61} \qquad , \qquad \gamma=\frac{61}{11} \qquad .$$

2-21 To get good precision, you need to do this on a full-size printout, as suggested by the problem. To keep from using up a whole page on my solutions handouts, I've shown the idea below at about 25% of the actual size.



First I drew the long diagonal twice as long again, meaning about 276 mm. To keep the area the same, I had to make its "cross-bar" half as long as on the preceding parallelogram, or about 17.5 mm. The speed represented by this new parallelogram is equal to the slope of its bottom edge. To get this slope, which is the rise divided by the run, we have to draw the rise and run accurately, i.e., lined up straight with the edges of the paper. I did this by using a second piece of paper as a right angle. The result is that the slope is (rise)/(run) = 91 mm/105 mm = .86, representing 86% of the speed of light.

2-23 (a) The input to the exponential function has to be unitless, so b must have units of s^{-1} (inverse seconds). The output of the exponential is also unitless, so if the right-hand side is to have units of meters, c must have units of m/s.
(b) Differentiating, we find that the velocity is

$$v=c(1-bt)e^{-bt} \qquad .$$

We want to maximize this as a function of time, which we can do by taking its derivative and setting it equal to zero:

$$0=bc(bt-2)e^{-bt}$$

The maximum occurs at $t=2/b$, and the velocity at this time ia $-ce^{-2}$, so the maximum speed is the absolute value of this, ce^{-2}. (Higher speeds occur at earlier times, but we're only finding the maximum speed as it comes back to the closed position.)

(c) We already know that c has units of m/s, so this does have the right units to be a velocity.

2-24 Draining the can completely isn't optimal, because then its center of mass will be that of the empty can, at its own center. The center of mass is a kind of average. Any beer below the center will lower the average relative to the empty can, while any beer above it will raise it. You should drink half the beer before setting it down.

2-25 (a) Let the length of the race be 2ℓ. The time required for the first half is ℓ/u, and for the second ℓ/v, so the total time is $\ell/u+\ell/v$. Dividing the total distance by the total time gives an over-all speed of $2/(1/u+1/v)$.

(b) The units of $1/u$ are seconds per meter, and so are the units of $1/v$. Since they have the same units, it makes sense to add them. The final result has units that are the inverse of these, meters per second, and that makes sense, because what we're trying to calculate is a velocity.

(c) When $u = v$, we have $2/(1/u + 1/u) = u$, which makes sense.

(d) Mathematically, making u bigger makes $1/u$ smaller, but $1/u$ is in the denominator, so the final result gets bigger. This makes sense physically: if you do the first half of the race faster, your over-all speed is higher. The analysis for v is identical.

2-26 (a) The circumference of a circle is 2π times the radius, so we have $v = 2\pi R/T$.

(b) The factor of 2π is unitless. The radius R has units of meters, and T has units of seconds. Therefore the right-hand side has units of m/s, which makes sense, because we're supposed to be calculating a velocity.

(c) When T is very large, we'd be dividing by a very large number in our expression from part a. Dividing by a very large number gives a very small result. This makes sense, because if the earth stopped rotating, the velocity should equal zero.

(d) Plugging in numbers gives 460 m/s. This is faster than the speed of sound, over a thousand miles an hour! (I rounded the result to 2 sig figs, since a really precise answer would have to take into account other factors, such as how far Nairobi is from the equator, how much the earth deviates from being a perfect sphere, and the distinction between a solar day and a sideral day.)

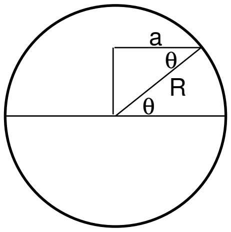
2-27 (a) Let a be the radius of the circle made by the point on the earth's surface. $a = R\cos\theta$, and $L = 2\pi a$, so $L = 2\pi R\cos\theta$.

(b) Dividing by T gives $v = L/T = 2\pi R\cos\theta/T$.

(c) The factor of $2\pi\cos\theta$ is unitless. The radius R has units of meters, and T has units of seconds. Therefore the right-hand side has units of m/s, which makes sense, because we're supposed to be calculating a velocity.

(d) At the north pole, $\theta = 90°$ and $\cos\theta = 0$, so $v = 0$, which makes sense. At the equator, $\theta = 0$ and $\cos\theta = 1$, so we recover the result of the previous problem.

(b) Plugging in $\theta = 34°$ and $R = 6.4\times10^6$ m gives $v = 380$ m/s.



2-28 (a) Split the isoceles triangle into two right triangles, and let the long sides of one of the right triangles be L_p and L, where the subscript p refers to the runner who is passing. Applying the definition of the cosine gives $L/L_P = \cos\theta$. Since the runners cover these distances in equal times, we have $v_p = L_P/t$ and $v = L/t$, so that $v_p = (L_P/L)v = v/\cos\theta$. Therefore $\Delta v = v_p - v = (1/\cos\theta - 1)v$.

(b) The cosine is unitless, so the entire factor $(1/\cos\theta - 1)$ in parentheses is also unitless. The velocity v has units of m/s, so multiplying by the quantity in parentheses still gives m/s. This makes sense, because the thing we're calculating, on the left side of the equals sign, is supposed to be a velocity, with units of m/s.

(c) Since the cosine of 0 is 1, the result in the case of $\theta = 0$ is $\Delta v = 0$. This makes sense. If there is a hundred-mile straightaway available in which to pass, the extra speed required is going to be nearly zero.

(d) Plugging in numbers gives $\Delta v = 1.1 \times 10^{-3}$ m/s. This is not just small (as expected from part c) but insanely small — in fact too small to matter — basically because L_p does not differ from L by as much as we would intuitively expect. It appears that in reality, the extra speed required by the passing runner is not the speed required to run the extra distance, but simply the same extra speed that would have been required if the runners had been in parallel lanes. That is, our geometrical model turned out to include the factor that was unimportant, and ignore the one that would really be important.

2-29 The speed of light is constant, so the equation $v = \Delta x/\Delta t$ holds. The distance traveled by the ray of light is $\Delta x = 2L$. The spacing between teeth is $1/n$ of a revolution, so during the time Δt the wheel moves through $1/2n$ of a revolution. Setting up a ratio between a whole revolution and this small part of a revolution, we have

$$\frac{\Delta t}{\text{time for one revolution}} = \frac{1}{2n}$$

The time for one revolution is $1/f$, so $\Delta t = 1/2nf$. Substituting into the equation for the velocity, we have $c = \Delta x/\Delta t = (2L)/(1/2nf) = 4Lnf$.

(b) The factor of $4n$ on the right is unitless. The units of Lf are $(\text{m})(\text{s}^{-1}) = \text{m/s}$, which are the units we expect for a velocity.

(c) All three numbers are being multiplied on the right, so an increase in any one of these corresponds to an *increase* in c. Since c is big, Fizeau wants all three numbers to be big: a long distance to the mirror, a large number of teeth, and a fast rotation of the wheel.

(d) The result is 3.13×10^8 m/s.

3-2 Zero. $\Delta v = 0$, because the velocity didn't change.

3-3 $s = \frac{1}{2}at^2$, and $a = g = 10\ \text{m/s}^2$, so $s = 45$ m, or call it 40 m (1 sig fig).

3-5 They both have accelerations equal to g. Let's say that the bullet leaves the gun with a velocity of 100 m/s, and for convenience, let's just say g is approximately 10 m/s^2. Then the rock's speed at one-second intervals goes like 0, 10, 20, ... m/s, while the bullet goes 100, 110, 120, ... m/s.

3-9 The speed is always increasing, because without friction there are only forces trying to speed it up, no forces trying to slow it down. A ball rolling without friction across a flat surface would neither speed up or slow down, i.e. would have zero acceleration, so the acceleration is greatest when the slope is greatest. The acceleration is increasing at first, then constant, then decreasing to a final acceleration of zero as it rolls away to the right at constant speed.

3-10 We're given distance and acceleration, and we want to find time. Of the constant acceleration equations, the one that has the right variables in it is $\Delta x = v_o \Delta t + \frac{1}{2}a\Delta t^2$. The initial velocity is zero, and solving for Δt, we have $\Delta t = \sqrt{2\Delta x/a}$. Converting the given distance to SI (meter-kilogram-second) units, we have 4000 mi $= 6.4 \times 10^6$ m, which results in $\Delta t = 1100$ s, or about 20 minutes (1 sig fig).

3-12 Of the time t he spent aloft, he spent $t/2$ rising and $t/2$ falling. The acceleration is $g = (9.8\ \text{m/s}^2)/6$. $\Delta x = (1/2)g(t/2)^2 = 0.43$ m.

3-14 We have $v^2 = 2a\Delta x$, and the factor of $2a$ is the same in both cases, so we can rewrite this as a proportionality $v^2 \propto \Delta x$. Solving for v gives $v \propto \sqrt{\Delta x}$, and converting this into the language of ratios results in $v_1/v_2 = \sqrt{\Delta x_1/\Delta x_2} = \sqrt{2}$.

3-16 No. The acceleration is greatest at the ends, where $v = 0$. Just because the velocity is passing through zero on a graph doesn't mean the acceleration is zero. The velocity is changing rapidly (large Δv), even though $v = 0$.

3-17 The units don't work out: $\text{m/s}^2 \times \text{s} = \text{m/s}$, not m.

3-18 The calculation of $\Delta v = a\Delta t$ is correct, because the equation it's based on, $a = \Delta v/\Delta t$, is valid for constant acceleration. The calculation of $\Delta x = v\Delta t$ is wrong, because the equation it's based on, $v = \Delta x/\Delta t$, is only valid for constant velocity. We know the velocity is increasing steadily, not staying constant. The speed of

10 m/s is only correct for the very end of the time period, not the whole time.

3-19 They're not equivalent. The slope of the tangent line will only be the same as $\Delta v/\Delta t$ if the $v-t$ graph is linear, i.e., if the acceleration is constant. For instance, suppose I drive my car for one minute at 50 km/hr, and then over the course of the second minute I speed up from 50 to 60 km/hr. I could use the $\Delta v/\Delta t$ definition to calculate my acceleration using $\Delta v = 10$ km/hr and $\Delta t = 2$ minutes, but that would give me a misleading result. My actual acceleration (slope of the tangent line) was zero for the first minute, and during the second minute, my acceleration was twice as much as $\Delta v/\Delta t$.

3-20 For example, suppose you accelerate away from a stop sign to 10 km/hr, then 20 km/hr, then 30 km/hr. The equation is telling you that the distance you cover increases faster than your speed. The distance from the stop sign to where your speedometer reads 20 km/hr is four times the distance from the stop sign to where it read 10 km/hr, and likewise the total distance from the stop sign to where it reads 30 km/hr is nine times the distance at 10 km/hr. This makes sense, because later in the motion, you're covering ground more quickly.

3-21 The area under the curve gives the change in velocity. If I just estimate fractions of a box by eye, I get about 36 boxes under the curve, but it's possible to do better than that, because you're given an exact geometric description of the curve. The area under the curve from 0 s to 4 s fits perfectly on top of the part from 5 s to 9 s, forming a complete rectangle with a width of 4 boxes and a height of 7 boxes, an area of 28 boxes. The part from 4 s to 5 s consists of a triangle with an area of 1.5 boxes sitting on top of a 1×2 rectangle, for a total area of 3.5 boxes, and similarly from 9 s to 10 s we have an area of 6 boxes. The total area is exactly $28.0+3.5+6.0 = 37.5$ boxes Each box has a width of 1 s, and a height of 2 m/s^2, so its area is 2 m/s. The change in velocity is 37.5×2 m/s $= 75.0$m/s, so the chipmunk's final velocity is -3.1 m/s $+$ 75.0 m/s $=$ 71.9 m/s. The number of significant figures on the final result reflects the fact that its accuracy is limited by the accuracy of the initial velocity, which is given to a tenth of a meter per second.

3-22 (a) The trip consists of four parts of equal duration. The duration, $T/4$, of each part is related to half the distance, d, to Proxima Centauri, $d/2 = \frac{1}{2}a(T/4)^2$, so $T = 4\sqrt{d/a}$.

(b) Plugging in to the formula from (a), we get 2.6×10^8 s $=$ 7 years.

(c) The maximum speed is $(a)(T/4) = 6\times 10^8$ m/s. This is more than twice the speed of light, which is not actually possible.

3-23 We're given the distance it takes the flea to slow back down to a stop, $\Delta x = 30$ cm, so it's natural to use the equation $v^2 = 2a\Delta x$, which tells us the distance required in order to slow down from v to rest, or speed up from rest to v. The flea first accelerates rapidly, then, once its feet have left the ground, it begins decelerating. The acceleration probably isn't constant, but this is just an estimate anyway, so we'll assume it is. We then have $2g\Delta x = 2ah$, where a is the acceleration as it's taking off, and h is the distance its center of mass travels upward while its legs are straightening. The result is $a = g\Delta x/h$. Estimating $h = 0.1$ cm, we find $a \sim 300g$.

3-24 (a)

$$v_f^2 = v_\text{o}^2 + 2a\Delta x$$

The initial velocity v_o, is the velocity of the spinning earth's surface, which at the equator is $(2\pi r)/(1 \text{ day}) \approx 500$ m/s. This is an order of magnitude less than the orbital velocity given in this problem. Since v_o is much less than v_f, v_o^2 is even more negligible compared to v_f^2, and we're justified in ignoring it for the purposes of this approximate calculation, whose accuracy is really limited by the assumed acceleration.

$$\begin{aligned} v_f^2 &= 2a\Delta x \\ \Delta x &= \frac{v_f^2}{2a} \\ &= 1\times 10^6 \text{ m} \\ &= 1000 \text{ km} \end{aligned}$$

(b) It's not very practical to build a railgun that would stretch halfway across a continent. The railgun idea is definitely not going to be practical for crewed launches, but it might work for lofting payloads that could handle more violent accelerations, such as raw materials. For instance, a payload that could handle $300g$ of acceleration would only need a 10-kilometer railgun.

3-25 (a) b: meters, c: meters/s (because you multiply it by t to get something with units of meters), k: units of s (because it doesn't make sense to take $e^{-t/k}$ unless the units of $-t/k$ cancel out)

(b) $v = \mathrm{d}y/\,\mathrm{d}t = \mathrm{d}(-ct)/\,\mathrm{d}t + \mathrm{d}(-cke^{-t/k})/\,\mathrm{d}t = -c + ce^{-t/k}$

(c) For large values of t, $e^{-t/k}$ is nearly zero, so v approaches c. Thus, c can be interpreted as the maximum velocity she will ever reach (known as her terminal velocity).

(d) Differentiating again, we get $a = \mathrm{d}v/\,\mathrm{d}t = \mathrm{d}(-c + ce^{-t/k})/\,\mathrm{d}t = -(c/k)e^{-t/k}$.

(e) For large t, the exponential becomes very small, so her acceleration is nearly zero.

3-26 (a) The variables involved are distance, acceleration, and time, so we want the constant-acceleration equation that relates them, which is $\Delta x = \frac{1}{2}g\Delta t^2$. The balls are two separate objects, so we need to have two separate equations of this form.

$$\begin{aligned}
\Delta x_1 &= \frac{1}{2}g\Delta t_1^2 \\
\Delta x_2 &= \frac{1}{2}g\Delta t_2^2 \\
\Delta t_1 &= \sqrt{\frac{2\Delta x_1}{g}} \\
\Delta t_2 &= \sqrt{\frac{2\Delta x_2}{g}} \\
T &= \Delta t_2 - \Delta t_1 \\
&= \sqrt{\frac{2\Delta x_2}{g}} - \sqrt{\frac{2\Delta x_1}{g}} \\
g &= \left(\frac{\sqrt{2\Delta x_2} - \sqrt{2\Delta x_1}}{T}\right)^2
\end{aligned}$$

(b) The units inside the parentheses are $\mathrm{m}^{1/2}/\mathrm{s}$, so squaring this quantity, we have m/s^2.

(c) When $\Delta x_1 = 0$, T is the same as Δt_2, and we have simply $g = (\sqrt{2\Delta x_2}/\Delta t_2)^2$, which is what we would have gotten if we had just solved $\Delta x_2 = \frac{1}{2}g\Delta t_2^2$ for g. This case reassures us that the result is correct, but it is useless in practical terms: we have to make Δx_1 great enough so that the ball will make a sound we can record.

(d) Mathematically, we get zero over zero, which is undefined. Physically, we cannot measure all the quantities in the equation with perfect precision, so we will not get exactly zero on the top or on the bottom; the result will be not an error on our calculator but rather a wildly unreliable answer! Nature is giving us a hint that the most accurate results will be obtained when the two heights are as different as possible, and will get less and less reliable as the heights are made closer to one another.

3-27 (a) We want to use a constant-acceleration equation to relate distance and time. We don't know the velocity, and we don't want to know it. The equation that has the right variables in it is $\ell = (1/2)at^2$ (using ℓ instead of x and taking $v_o = 0$). The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $a/2$, giving $\ell \propto t^2$. Solving for t gives $t \propto \ell^{1/2}$. Change this into a statement about ratios:

$$\frac{t_d}{t_h} = \left(\frac{\ell_d}{\ell_h}\right)^{1/2} = \sqrt{4} = 2 \qquad ,$$

where "d" refers to the dinosaur and "h" to a human.

(b) The distance each animal moves in a quarter stride is proportional to ℓ, so $v \propto \ell/t \propto \ell/\ell^{1/2} = \ell^{1/2}$. In terms of ratios,

$$\frac{v_d}{v_h} = \left(\frac{\ell_d}{\ell_h}\right)^{1/2} = \sqrt{4} = 2 \qquad .$$

In other words, the dinosaur's stride is longer by a factor of four, but it loses half of that advantage because its steps are slower by a factor of two.

3-28 We want to use a constant-acceleration equation to relate acceleration and time for a fixed distance. We don't know the velocity, and we don't want to know it. The equation that has the right variables in it is $x = (1/2)at^2$ (setting $v_o = 0$). Solving for the acceleration gives $a = 2x/t^2$. The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $2x$, giving $a \propto t^{-2}$. Change this into a statement about ratios:

$$\frac{a_H}{a_r} = \left(\frac{t_H}{t_r}\right)^{-2} = (0.47)^{-2} = 4.5 \qquad ,$$

where "H" refers to Hatton and "r" to the reporter.

3-29 We want to use a constant-acceleration equation to relate acceleration, speed, and distance. We don't know the time, and we don't want to know it. The equation that has the right variables in it is $v_f^2 = v_o^2 + 2a\Delta x$. The final velocity is zero. Solving for the acceleration gives $a = -v_o^2/2\Delta x$. The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $-1/2$, giving $a \propto v_o^2 \Delta x^{-1}$. Change this into a statement about ratios:

$$\frac{a_F}{a_S} = \left(\frac{v_F}{v_S}\right)^2 \left(\frac{\Delta x_F}{\Delta x_S}\right)^{-1} = (4.8)^2(1.9)^{-1} = 12$$

where "F" refers to the F-14 and "S" to the Sopwith.

3-30 We want to use a constant-acceleration equation to relate acceleration and distance for a fixed final speed. We don't know the time, and we don't want to know it. The equation that has the right variables in it is $v_f^2 = 2a\Delta x$ (setting $v_o = 0$). Solving for the acceleration gives $a = v_f^2/2\Delta x$. The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $v_f^2/2$, giving $a \propto \Delta x^{-1}$. Change this into a statement about ratios:

$$\frac{a_H}{a_m} = \left(\frac{\Delta x_H}{\Delta x_m}\right)^{-1} = (1.62\,\mathrm{m}/1.75\,\mathrm{m})^{-1} = 1.08 \qquad ,$$

where "H" refers to Harrell and "m" to a pitcher of the nominal minimum height.

3-31 We want to use a constant-acceleration equation to relate acceleration and speed for a fixed distance. We don't know the time, and we don't want to know it. The equation that has the right variables in it is $v_f^2 = v_o^2 + 2a\Delta x$. The final velocity is zero. Solving for the initial speed gives $v_o = (-2a\Delta x)^{1/2}$. The rest of the method is the same as the standard one demonstrated in the appendix "Three essential mathematical skills" in the back of the book. Convert the equation to a proportionality by throwing out the constant factor of $\sqrt{-2\Delta x}$, giving $v_o \propto a^{1/2}$. Change this into a statement about ratios:

$$\frac{v_C}{v_D} = \left(\frac{a_C}{a_D}\right)^{1/2} = (1.09)^{1/2} = 1.04 \qquad ,$$

where "C" refers to the Chevy cop car and "D" to the Dodge.

3-33 Let ℓ be the length of one of the chords and t the time the bead takes. If θ is the angle of the chord relative to the horizontal, then the bead's acceleration is $a = g\sin\theta$, and we have $\ell = (1/2)g\sin\theta t^2$, so that t depends on the ratio $\ell/\sin\theta$. But a little trigonometry shows that $\ell = 2r\sin\theta$, so the race is a tie. This fact was proved by Galileo.

3-34 The constant-acceleration acceleration that has the right variables in it is $v_f^2 = v_0^2 + 2a\Delta x$. The final velocity is zero, so we have $a = -v_0^2/2\Delta x$. If we wanted to worry about the sign, then we would have to pick a coordinate so that we could define a sign for Δx, and then we would get some sign for the acceleration, expressed in those coordinates. However, we're only trying to find the magnitude of the deceleration, so ignoring signs, the result is 1.5 m/s^2.

3-35 The constant-acceleration equation with the right variables in it is $\Delta x = v_0\Delta t + (1/2)a\Delta t^2$. The initial velocity is zero. Solving for the acceleration gives $a = 2\Delta x/\Delta t^2 = 16.3\ \text{m/s}^2$.

3-36

$$\begin{aligned} H &= v_0T + \frac{1}{2}gT^2 \\ v_0 &= H/T - \frac{1}{2}gT \end{aligned}$$

4-2 (a) We are supposed to find an equation for mass based on information about forces and accelerations. Newton's second law, $a = F_{total}/m$, is what relates these three quantities. There are two forces acting on the elevator: a downward force from gravity and an upward force from the cable. Let's use positive numbers for upward forces and negative numbers for downward ones. The force of gravity will then be $-mg$, and the maximum force the cable can exert will be $+T$, giving $F_{total} = -mg + T$. Substituting this into Newton's second law gives

$$a = \frac{-mg+T}{m} \qquad .$$

Solving for m gives

$$m = \frac{T}{a+g} \qquad .$$

(b) If $a = 0$, then the equation becomes $m = T/g$, i.e. $T = mg$. Interpretation: if the elevator is not accelerating, then the maximum mass is the mass that results in a gravitational force as great as the maximum tension in the cable. This is intuitively obvious if the elevator is at rest, but perhaps a little surprising for the case where the elevator is, say, moving upward at constant speed. Most people would not expect that the tension in the cable would be the same whether the car was at rest or moving upward at constant speed.

If the car is accelerating downward at 9.8 m/s^2, that would be $a = -g$, since we are using positive for up and negative for down. Plugging this in to the equation gives $m = \infty$ (division by zero). Interpretation: this is the case where the elevator is simply dropping like a rock. The cable does not need to exert any force in this situation, and so there is no limit on the mass of the car.

4-3 No, we can only conclude that if there are forces acting on it, their sum is zero.

4-5 Let's use a coordinate system in which positive x is down. Then the downward force of gravity on Sally is positive, and the blaster's upward force on Sally is negative. (a)

$$\begin{aligned} a &= \frac{F_{total}}{m} \\ &= \frac{mg - F_B}{m} \\ &= g - F_B/m \\ h &= \frac{1}{2}at^2 \\ t &= \sqrt{\frac{2h}{a}} \\ &= \sqrt{\frac{2h}{g - F_B/m}} \end{aligned}$$

(b) The units of g are m/s^2. F_B has units of newtons, and dividing newtons by kilograms gives m/s^2 (as in Newton's second law). In the denominator, we're subtracting m/s^2 from m/s^2. That's good, because you can't subtract things that have different units. Dividing h, with units of m, by m/s^2 gives units of s^2, and taking the square root of that gives units of seconds, which matches up correctly with the units of the variable t on the left side.

(c) For large values of F_B, the quantity inside the square root becomes negative, and the result for the time becomes an imaginary number. The point where this bogus behavior sets in is the point at which the denominator goes from positive to negative, i.e., when the denominator is zero: $g - F_B/m = 0$, or $F_B = mg$. In other words, if F_B is greater than the force of gravity, she'll fly up, and will never even hit the ground.

4-6 The mass is increased by a factor of 1.5, so the acceleration is reduced by a factor of 1.5, to 2 m/s^2.

4-7 (a) Newton's second law says $F_{air} - F_{weight} = ma$, or $F_{air} - mg = ma$. Since the forces are constant, this will be motion with constant acceleration, so $v = at = (F_{air} - mg)t/m$.
(b) 7.8 m/s

4-8 (a)

$$\begin{aligned} v_f^2 &= v_o^2 + 2a\Delta x \\ 0 &= v^2 + 2ad \\ a &= -\frac{v^2}{2d} \\ F &= ma \\ &= -\frac{mv^2}{2d} \end{aligned}$$

(b) The quantity $-\frac{mv^2}{2d}$ has units of

$$\frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{m}} = \text{kg}\cdot\text{m/s}^2 \qquad ,$$

which are the same as the units of force (mass multiplied by acceleration).
(c) If m gets bigger, F gets bigger. That makes sense, because more force would be required in order to stop a more massive object in the same distance. If v increases, so does F, and that also makes sense. Dividing by a bigger number gives a smaller result, so if d gets bigger, F gets smaller. That also makes sense, because a smaller force would take more distance to stop a moving object.
(d) The only thing to be careful about here is that if we want an answer in SI (meter-kilogram-second) units, we have to make sure to convert all the raw data to SI before plugging in:

$$\begin{aligned} F &= -\frac{mv^2}{2d} \\ &= -\frac{\left(6.7\times10^{-27}\ \text{kg}\right)\left(2.0\times10^{7}\ \text{m/s}\right)^2}{2\left(7.1\times10^{-4}\ \text{m}\right))} \\ &= -1.9\times10^{-9}\ \text{N} \end{aligned}$$

4-9 $F = ma = kt$, so $a = (k/m)t$. Integrating once gives its velocity, $v = (k/2m)t^2 + v_o$. The constant of integration, v_o, equals zero, because the initial speed is zero, so $v = (k/2m)t^2$. Integrating again gives $x = (k/6m)t^3 + x_o$.

4-10 Their claim doesn't make sense. The branch isn't accelerating, so evidently the total force on it is zero. The two sides of the shears must be making equal forces on the two sides of the branch, in opposite directions. This has to be true regardless of the type of shears.

4-11 (a) $F_W = mg$, so reducing g by 0.01 m/s^2 reduces his weight by

$$72\ \text{kg} \times 0.01\ \text{m/s}^2 = 0.72\ \text{N}$$

His mass stays the same.

(b) For motion with constant acceleration, $v_f^2 = v_i^2 + 2ax$. At the top of his jump, his vertical velocity is zero, so $v_f = 0$, and $0 = v_i^2 + 2ax$. His acceleration equals $-g$. Solving for x, the height to which he can jump is, $x = v_i^2/2g$. Since the height to which he can jump is inversely proportional to g, a 0.1% decrease in g will result in a 0.1% increase in height, or 2 mm.

4-12 The trickiest thing about this program is that in the python idiom `for i in range(30)`, `i` actually varies from 0 to 29, not from 1 to 30, so we have to add one to `i` to get the thing we're supposed to multiply by.

```
f=1
for i in range(30):
  f=f*(i+1)
print(f)
```

The result is 265252859812191058636308480000000.

4-13 The precision of the calculation of the time can be no better than the size of the time increment, so we need to make n big and make the time increment small. The effect on the time is claimed to be about one part per thousand, and we would like to calculate it to a couple of sig figs, so we need an n of at least 10^4 or 10^5. On a reasonably modern computer, $n = 10^6$ doesn't even take very long.

```
import math
def ball(rho,n):
  # rho=density of air, kg/m3
  r = .01
  m=.021             # mass in kg
  x = 1.             # start at a height of 1 m
  v = 0
  g = 9.8            # m/s2
  t_vac=math.sqrt(2.*x/g) # time it would take in vacuum
  t_max = t_vac*1.1  # guess the longest time it could take
  dt = t_max/n       # Divide t into n equal parts.
  b = 0.25*math.pi*rho*r**2 # calculate this only once
  t = 0.
  for i in range(n):
    dx = v*dt
    x = x+dx
    t = t+dt
    if x<0.:
      return t
    F = -m*g+b*v**2
    a = F/m
    dv = a*dt
    v = v+dv
  return -999.       # If we get here, t_max was too short.
n=1000000
t1 = ball(0.,n)
t2 = ball(1.2,n)
print t1
print t2
print (t2-t1)*1000. # milliseconds
```

The output consists of the time taken in vacuum, the time in air, and the difference between them, in milliseconds:

```
0.451754493563
0.452092405519
0.337911955699
```

The difference is 0.34 ms, as claimed. As a consistency check, we can compare the *in vacuo* time calculated numerically to the exact result, $\sqrt{2h/g} = 0.45175395$. They agree to far better than the precision required in order to find the time difference to two sig figs, which reassures us that even though the program has done several million arithmetic operations, the accumulated rounding error hasn't been enough to cause a problem.

4-14 For motion with constant acceleration, starting from rest, we have $\Delta x = (1/2)at^2$. For a fixed Δx, this becomes $t \propto a^{-1/2}$. Newton's second law gives $a \propto F$, so $t \propto F^{-1/2}$. The result is

$$\frac{t_1}{t_2} = \left(\frac{F_1}{F_2}\right)^{-1/2} = (1/1.9)^{-1/2} = 1.4$$

4-15 For motion with constant acceleration, starting from rest, we have $v^2 = 2a\Delta x$, or $v \propto a^{1/2}$. Newton's second law gives $a \propto m^{-1}$, so $v \propto m^{-1/2}$. The result is

$$\frac{v_2}{v_1} = \left(\frac{340\ \text{kg} + 60\ \text{kg}}{340\ \text{kg}}\right)^{-1/2} = 0.92$$

4-16 (a) We have $v^2 = 2aL$ for the average acceleration, so the average force is $F = ma = mv^2/2L$.

(b) The units on the right-hand side are $\text{kg} \cdot (\text{m/s})^2/\text{m} = \text{kg} {\cdot} \text{m/s}^2$, which is the same as newtons.

(c) Mathematically, increasing m increases F. That makes sense, because more force would be required in order to accelerate a more massive bullet. The dependence on v also makes sense, because accelerating the bullet up to a higher speed should take more force. Raising L in our answer would lower F, and that makes sense, because if we have a greater distance over which to accelerate the bullet, we can do it more gradually.

4-17 (a) Let, e.g., F_{23} mean 2's force on 3. By Newton's first law, the total force on block 3 is zero, so $F_{23} - M_3 g = 0$, so $F_{23} = M_3 g$. Newton's third law then gives $F_{32} = -F_{23} = -M_3 g$.

(b) Applying Newton's first law to block 2, we have $F_{12} + F_{32} - M_2 g = 0$, so $F_{12} = (M_2 + M_3)g$. This makes sense because 1 is supporting the entire weight of 2 and 3.

5-4

5-5

5-6

5-7

5-8

5-9 Newton's third law says their forces on each other are equal. Since $a = F/m$, the less massive old lady has a greater acceleration.

Problem ??:

force acting on magnet			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
magnetism	up	car	magnetism	down	magnet
gravity	down	planet earth	gravity	up	magnet
normal	down	car	normal	up	magnet

Problem ??:
(a) A monkey climbing a palm tree.

force acting on monkey			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	right	tree	normal	left	monkey (right hand)
normal	left	tree	normal	right	monkey (left hand)
gravity	down	planet Earth	gravity	up	monkey
static friction	up	tree	static friction	down	monkey (right hand)
static friction	up	tree	static friction	down	monkey (left hand)

Problem ??:
(b) A piece of tape stuck on the ceiling.

force acting on tape			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	tape
sticky	up	ceiling	sticky	down	tape

Problem ??:

force acting on rower			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	↑	seat	normal	↓	rower
gravity	↓	Earth	gravity	↑	rower
normal	→	oar	normal	←	rower
static friction	←	seat	static friction	→	rower
normal	↖	footrest	normal	↘	rower

Problem ??:

force acting on farmer			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
static friction	up	cow	static friction	down	farmer
static friction	up	wall	static friction	down	farmer
gravity	down	planet earth	gravity	up	farmer
normal	left	cow	normal	right	farmer
normal	right	wall	normal	left	farmer

Problem ??:
(The "lift" force below is really not so different from a normal force.)

force acting on plane			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal?	east	air	normal?	west	plane (propeller)
friction	west	air	friction	east	plane
lift	up	air	lift	down	plane (wing)
gravity	down	earth	gravity	up	plane

5-10 When you drop an object, the earth does accelerate up to meet it, but the acceleration is very small because the earth's mass is so big.

5-11 Yes, they are equal and opposite, because otherwise the total force on you would not be zero, and you would accelerate vertically. No, Newton's third law does not apply, because there are three objects involved, not just two: the earth, your feet, and the floor. Newton's third law says that two objects' forces on each other are equal and opposite, so it never applies to two forces both acting on the same object.

5-12 (a) The two static frictional forces must together cancel out the elevator's weight $F_W = Mg$, so

$$F_s = \frac{Mg}{2}$$

The maximum amount of static friction is given by $F_s < \mu_s F_N$, so

$$F_N > \frac{Mg}{2\mu_s}$$

(b) If friction is weaker, we'd need to press more firmly. The equation demonstrates this behavior, since dividing by a smaller value of μ_s will give a greater result.

5-13 (a) See table.

(b) The vertical forces cancel, so $F_N = mg$. Picking a coordinate system in which positive is forward, the horizontal force of friction produces an acceleration $a = F_k/m = -\mu_k F_N/m = -\mu_k g$. Applying the constant-acceleration equation $v_f^2 = v_i^2 + 2ax$, we find

$$x = \frac{v_i^2}{2\mu_k g} \quad .$$

(c) The coefficient of friction is unitless, because it's defined as the ratio of two forces. The units of v_i^2 are $\mathrm{m^2/s^2}$, and dividing by $\mathrm{m/s^2}$ gives meters.

(d) Mathematically, a higher v_i gives a higher result, and this makes sense physically, because you can go farther if you start out going faster. A greater value of μ_k gives a smaller final result, and this also makes sense; increasing friction (e.g., by roughening the ice) would make the skater stop sooner. Increasing g would

Problem ??:

force acting on skater			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	Earth	gravity	up	skater
normal	up	ice	normal	down	skater
kin. fr.	back	ice	kin. fr.	forward	skater

also decrease the final result; this is because a stronger gravitational field would press the skater more firmly against the ice, increasing F_N, and thereby indirectly increasing friction.

(e) The result is 2.4 km.

(f) This is clearly unrealistic. Probably the biggest reason for the unrealistic result is that we've neglected air resistance. Air resistance increases rapidly with speed, so that could be a reasonable approximation at lower speeds, but at these speeds (about 50% faster than an olympic sprinter), it's probably a poor approximation. When you see pictures of olympic-level speed skaters, they're always wearing special suits that are designed to cut down on air resistance; this shows that air resistance is a very important effect at the speeds they get up to.

5-14

$$\begin{aligned} v &= \sqrt{2aL} \\ &= \sqrt{2\frac{F_k}{m}L} \\ &= \sqrt{2\frac{\mu_k F_N}{m}L} \\ &= \sqrt{2\frac{\mu_k mg}{m}L} \\ &= \sqrt{2\mu_k gL} \end{aligned}$$

Note that the cop doesn't even need to know anything about the car other than the coefficient of kinetic friction between its tires and the asphalt μ_k, and μ_k probably doesn't vary all that much from one car to another.

5-15 If kinetic friction were stronger than static friction, then we could have, for example, a situation in which 4 N was the force necessary to break static friction and make an object slide across a horizontal surface, but 6 N was necessary in order to produce a force greater than kinetic friction in order to produce an acceleration. This wouldn't make sense, because what would happen if you applied 5 N? It would break static friction, but then there would be no acceleration because the force would be weaker than kinetic friction. The way you would actually measure a static frictional force experimentally would be to measure how much force you had to apply in order to get the object to accelerate, so there's no way you could have determined the 4 N force for static friction in the first place.

5-16 (a) Friction would be parallel to the ceiling, but all the motion is vertical. (b) If the rice were to stay stuck onto the ceiling of the cup, it would have to accelerate along with the cup. When you're banging on something like this the acceleration can be huge, perhaps thousands of meters per second squared. This is much more than anything gravity could supply. (c) The cup is the only object touching a grain of rice that could be exerting a force on it, and Newton's second law would then require that the this force equal ma. As the cup decelerates, we therefore need a large upward force from the cup on the rice if the rice is to decelerate along with it. This would be some kind of sticky force, and there's a limit on how strong such a force can be. If it isn't strong enough, then the grain's acceleration can't match that of the cup, and it will come loose.

5-17 The scale reads too high when the backpack is on the way down and decelerating, and also when it's on the way up and accelerating. It reads too low when the backpack is on the way up and decelerating, and also when it's on the way down and accelerating. In other words, when I'm making an upward force on the backpack that's greater than the force required to support its weight, Newton's third law says the backpack must be making a correspondingly large force downward on me, which registers on the scale. Conversely, when I make an unusually small upward force on the pack, it makes an unusually small downward force on me.

5-18 The acceleration a in Newton's second law is the acceleration of the center of mass. The atoms at the point of contact between the ball and the bat move together and have equal accelerations, but that doesn't mean the centers of mass have equal accelerations. The ball and the bat both start squashing on impact.

5-19 (a) (b) (c) Let's choose a coordinate system in which positive is forward. The forces acting on the tugboat give

$$a = \frac{F_f - T}{m}$$

and the ones acting on the ship result in

$$a = \frac{T}{m} \qquad .$$

Note that we use the same symbol a in both cases: they're tied together, so their accelerations must be the same. To solve for T, we need to eliminate a, which we can do by setting the two equations equal:

$$\frac{F-T}{m} = \frac{T}{M}$$

Bringing all the T stuff to one side,

$$\frac{F}{m} = \left(\frac{1}{m} + \frac{1}{M}\right) T \qquad ,$$

so

$$\begin{aligned} T &= \frac{F}{m(\frac{1}{m}+\frac{1}{M})} \\ &= \frac{F}{1+m/M} \\ &= \frac{M}{M+m} F \qquad . \end{aligned}$$

(d) For $M = 0$, the tension is zero, which makes sense. For $M = \infty$, we have $M/(M+m) = 1$, since the bottom becomes essentially the same as M, so $T = F$. The interpretation is that the tugboat is so light that essentially none of the force from the water on its propellers is required in order to accelerate it.

5-20 (a) See table on last page.

(b) Let up be positive, and let a be the (positive) acceleration of mass m. Newton's second law applied to mass m gives

$$a = \frac{T - mg}{m} \qquad ,$$

and for mass M,

$$-a = \frac{T - Mg}{M} \qquad .$$

We have two equations in two unknowns, T and a. To solve them, we need to eliminate one variable. Let's eliminate a.

$$\begin{aligned} \frac{T-mg}{m} &= -\frac{T-Mg}{M} \\ \frac{T}{m} - g &= -\frac{T}{M} + g \\ \left(\frac{1}{m}+\frac{1}{M}\right) T &= 2g \\ (M+m)T &= 2gMm \\ T &= \frac{2gMm}{M+m} \end{aligned}$$

Now we can find a.

$$\begin{aligned} a &= \frac{T-mg}{m} \\ &= \frac{T}{m} - g \\ &= \frac{2gM}{M+m} - g \\ &= \left(\frac{2M}{M+m} - 1\right) g \\ &= \left(\frac{2M}{M+m} - \frac{M+m}{M+m}\right) g \\ &= \left(\frac{M-m}{M+m}\right) g \end{aligned}$$

Problem ??:
Part a

force acting on tugboat			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
fluid fr.	forward	water	fluid fr.	backward	tugboat
tension	back	ship	tension	forward	tugboat

Problem ??:
Part b

force acting on ship			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
tension	forward	tugboat	tension	back	ship

(c) No. If the masses are equal, the acceleration will be zero, not the velocity. The equal masses could be left at rest, and would stay at rest, regardless of their relative heights. They could also be left in motion, and would stay in motion, without speeding up or slowing down.
(d) (found above)
(e) Mathematically, the expression for T vanishes when either mass is zero. Physically this makes sense, because if either mass is not present, the rope will go slack.

5-21 (a) The normal force at the top surface is Mg, and the one at the bottom surface is $(M+m)g$. Since there is slipping at both surfaces, the coefficient of kinetic friction is the one we care about, and the two frictional forces are $\mu_k Mg$ at the top surface (acting on both blocks) and $\mu_k(M+m)g$ at the bottom. The middle block decelerates, while the top one accelerates. The velocity of the middle block is $v-\mu_k(2M/m+1)gt$, and the velocity of the top block is $\mu_k gt$. Setting these equal to each other, we find

$$t=\frac{v}{2\mu_k g(M/m+1)} \quad .$$

(b) The units of the right-hand side are $(\mathrm{m/s})/(\mathrm{m/s^2}) = \mathrm{s}$.
(c) If m gets bigger, M/m gets smaller, and t gets bigger. This makes sense physically. The acceleration of the top block doesn't depend on the mass of the middle block at all. The acceleration of the middle block depends on its own mass for two reasons: (1) if it has more mass it has more inertia; (2) if it has more mass then there will be more friction at the bottom surface. Although these two effects are in the opposite direction, they're not of equal strength. Doubling the mass of the middle block, for example, doubles its inertia, but doesn't double the frictional force at the bottom surface, and has even less of an effect on the total force felt by the block.
(d) If M gets bigger, its own acceleration of $\mu_k g$ is unchanged, because the greater frictional force cancels out the effect of the greater inertia. On the other hand, the frictional force on the middle block is increased, so the time should get shorter. Mathematically, this is the behavior we see.
(e) Making g bigger makes the time shorter. That makes sense, because if gravity presses the surfaces together more firmly, there will be more friction.

5-22 (a) See the tables for an analysis of the forces involved.

Applying Newton's second law to the climber of mass M, and letting the positive direction be

Problem ??:

force acting on mass m			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
normal	up	string	normal	down	mass m
gravity	down	planet Earth	gravity	up	mass m

Problem ??:

force acting on climber of mass m			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	climber
kinetic friction	left	snow	kinetic friction	right	climber
normal	up	snow	normal	down	climber

downward, we have

$$Mg - T = Ma \qquad .$$

The climber of mass m doesn't sink into the snow or fly up in the air, so

$$F_N - mg = 0 \qquad .$$

Newton's second law for m, with positive to the right, gives

$$T - \mu_k F_N = ma \qquad .$$

These are three equations in three unknowns, a, T, and F_N. Eliminating $F_N = mg$ reduces it two a system of two equations in two unknowns:

$$Mg - T = Ma$$
$$T - \mu_k mg = ma$$

Next we solve the first equation for $T = M(g-a)$ and substitute into the other equation, giving one equation with one unknown:

$$M(g - a) - \mu_k mg = ma$$

Solving this for a, we find

$$a = \frac{M - \mu_k m}{M + m} g \qquad .$$

(b) The definition of μ_k is $F_k = \mu_k F_N$, so μ_k is unitless. That means that it makes sense to do the subtraction $M - \mu_k m$, since both terms have units of kg. The quotient $(M - \mu_k m)/(M + m)$ is unitless, and multiplying it by g gives units of acceleration, which checks out.

(c) Mathematically, increasing g increases a. That makes sense because more gravity should make them accelerate faster. Mathematically, increasing μ_k decreases a, and this checks out because increasing friction should keep them from accelerating as fast. Increasing m decreases the numerator and increases the denominator, and both of these effects will reduce a. This makes sense because adding extra weight to m will help both by increasing friction and by increasing the inertia of the whole system, which is measured by $M + m$. M is the hardest variable to analyze mathematically. Increasing M increases both the numerator and the denominator, so there are two countervailing effects, and it's not obvious which is the greater. But since the numerator is smaller than the denominator, increasing M by a certain amount will cause a greater percent change in the numerator, so the net effect will be to increase a. This makes sense physically, because if M was miniature French poodle, we wouldn't expect the acceleration to be as fast.

5-23 (a) See table on last page.

Problem ??:

force acting on climber of mass M			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	climber
normal	up	rope	normal	down	climber

(b) See table on last page.

(c) The vertical forces on Ginny cancel out, so the normal force between Ginny's pants and the sled has magnitude $N_2 = Mg$. Similarly, the normal force between the sled and the snow is $N_1 = (M+m)g$, since the snow supports the total mass of both Ginny and the sled. In the situation where she can just barely accomplish what she wants to do, the initial acceleration is vanishingly small; to find the limiting case, we set it to zero. That means there must be zero total horizontal force on her, and zero total horizontal force on the sled. There are two frictional forces on the sled, F_2 from Giny and F_1 from the snow. If they're canceling, their magnitudes must be equal, $|F_2| = |F_1|$. If the sled just barely starts slipping over the snow before Ginny's pants start slipping over the sled, then both these forces are at their maximum value for static friction,

$$\begin{aligned}\mu_2 N_2 &= \mu_1 N_1\\ \mu_2 Mg &= \mu_1 (M+m)g\\ \mu_2 &= \mu_1(1+m/M)\\ M &= \frac{m}{\mu_2/\mu_1 - 1}\end{aligned}$$

(d) For $\mu_2 = \mu_1$, M becomes infinite. For smaller values of μ_2, we have a negative value for M, which is also impossible. In other words, for $\mu_2 \le \mu_1$, it wouldn't matter if Ginny carried a backpack full of lead bricks; she'd still slip off the sled.

5-24 Let the tension in the rope connected to the right-hand weight be T. By arguments similar to the ones in the example in section 5.6, we know that the tension in the rope connected to the left-hand weight is $2T$. The accelerations of the two weights are also related. For every centimeter that the weight on the left moves up, the pulley in the middle will move one centimeter to the right. This causes one centimeter of rope to come around the middle pulley, lengthening the part the goes over the right-hand pulley and down to the weight. The right-hand weight therefore drops two centimeters: one centimeter because an extra centimeter of rope is being fed around the middle pulley, and another centimeter because the middle pulley is moving a centimeter to the right. Therefore the acceleration of the right-hand weight is twice as much as the acceleration of the left-hand one, and is in the opposite direction, $a_2 = -2a_1$. Applying Newton's second law to each weight (with positive being up), we have the following set of three equations in three unknowns:

$$\begin{aligned}a_2 &= -2a_1\\ a_1 &= \frac{2T-mg}{m}\\ a_2 &= \frac{T-mg}{m}\end{aligned}$$

Substituting the first equation into the third one reduces this to two equations in two unknowns:

$$\begin{aligned}a_1 &= 2T/m - g\\ -2a_1 &= T/m - g\end{aligned}$$

Now we solve the second equation for $T/m = -2a_1 + g$ and substitute into the first one, which makes it one equation in one unknown:

Problem ??:
Part a:

force acting on Ginny			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	Ginny
static friction	left	ropes	static friction	right	Ginny
normal	up	sled	normal	down	Ginny
static friction	right	sled	static friction	left	Ginny

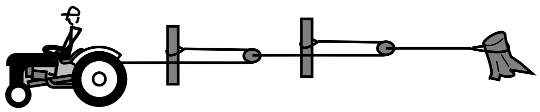
Problem ??:
Part b:

force acting on sled			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	sled
normal	down	Ginny	normal	up	sled
static friction	left	Ginny	static friction	right	sled
normal	up	snow	normal	down	sled
friction	right	snow	friction	left	sled

$$a_1 = 2(-2a_1 + g) - g$$

Solving this, we find $a_1 = g/5$. The positive sign indicates that this mass (the one on the left) goes up.

5-25 The simplest thing to do is to hook up two of the pulley setups in a row, so that the second one doubles the force output from the first one.



5-27 The parallel case is trivial: $k = k_1 + k_2$ by the same reasoning as before. The series case requires more delicacy, because the strings don't get elongated by the same amount. For simplicity of notation, let $x_o = 0$ for both springs. Since their forces on each other are equal by Newton's third law, the elongations are related by $F = k_1x_1 = k_2x_2$. The equivalent spring constant is $F/(x_1 + x_2) = k_1x_1/(x_1 + x_2) = k_1x_1/(x_1 + x_1k_1/k_2) = k_1k_2/(k_1 + k_2)$. Another way of writing this is $1/(1/k_1 + 1/k_2)$.

5-29 (a) From problem 16 we have $k_{whole} = (A/L)E$. The simplest way to work this is to imagine a fiber consisting of only two atoms. Then we have $A = b^2$ and $L = b$, and $k = k_{whole}$, resulting in $k = bE$. (b) The result is on the order of 1 N/m. (c) The stretching of the bond is $x = F/k = mg/k$. A typical mass of an atom would be on the order of 10^{-26} kg, as you can find, for example, by looking up the masses of subatomic particles in the back of the book. (Another method would be to multiply the atomic volume b^3 by a typical density of solid matter.) The resulting extension of the length of the bond is on the order of 10^{-25} m, which is negligible, even compared to b. In other words, individual molecules won't be distorted significantly by gravitational stresses.

5-30 (a) At position x, the fraction of his weight on the snow is $(L - x)/L$, and the fraction on the dirt x/L. Because the snow is frictionless, all that matters is the frictional force with the dirt. The normal force between the dirt and the skis is mgx/L, so the frictional force is $-\mu_k mgx/L$. The acceleration is $-\mu_k gx/L$.

(b) We want a function whose second derivative is basically the same function, but with a negative constant out in front. The functions that have this properties are the sine waves, and we can choose $f = \cos\omega t$ and $g = \sin\omega t$. In other words, the motion will be identical to part of one cycle of simple harmonic motion. Substituting into the differential equation, we find $\omega = \sqrt{g\mu_k/L}$.

(c) The motion ends when the velocity is zero, but what's a little less obvious is at what point in the cycle the motion *starts*. At $x = 0$, the acceleration is zero, because the skis aren't on the dirt yet. For a sine wave, the zero of velocity occurs a quarter cycle after the zero of acceleration, so the motion is identical to one quarter of a period of simple harmonic motion. The time required to stop is $T/4 = \pi/2\omega = (\pi/2)\sqrt{L/g\mu_k}$. Ordinarily you would expect that it would take more time to stop if the initial speed was higher, but our answer doesn't depend on the intial speed at all. The reason is that if he's going faster, his skis will encounter more friction sooner, and the larger force will compensate for the larger speed. This also shows that the acceleration is higher if the initial velocity is higher, so running into a patch of dirt is a lot more dangerous at high speed.

5-31 Let a be the acceleration of the mass on the right and a' the acceleration of the mass on the left. For every centimeter that the mass on the left moves down, the left-hand segment of the rope attached to it lengthens by 1 cm, while the three other segments all shorten by 1/3 cm. Therefore

$$a' = -3a \qquad .$$

Let the tension in this piece of rope be T. Since the middle pulley is massless, the total force acting on it must be zero, so the tension in the piece of rope connecting it to the crossbar must be $2T$. Since the crossbar is massless, the total force on it must be zero, so the rope connecting to the right-hand mass must have tension $3T$. Newton's second law gives

$$\begin{aligned} T - mg &= ma' \\ 3T - mg &= ma \qquad . \end{aligned}$$

This is a set of three equations in three unknowns. The result is $a = g/5$.

5-32 Applying Newton's second law to each of the three masses, we have

$$\begin{aligned} 2T - m_3 g &= m_3 a_3 \\ T - m_1 g &= m_1 a_1 \\ T - m_2 g &= m_2 a_2 \qquad . \end{aligned}$$

Mass 1's acceleration relative to the earth equals the sum of its acceleration a_{1P} relative to the lower pulley and that pulley's acceleration relative to the earth. The pulley's acceleration relative to the earth equals $-a_3$. Doing this for mass 2 as well, we have

$$\begin{aligned} a_1 &= a_{1P} - a3 \\ a_2 &= a_{2P} - a3 \\ a_{1P} &= -a_{2P} \qquad . \end{aligned}$$

These are six equations in six unknowns. Eliminating a_{1P} and a_{2P} reduces the second block of equations to simply

$$a_3 = -(a_1 + a_2)/2 \qquad ,$$

i.e., mass 3's acceleration relates to the average of the other two masses'. Eliminating $T = m_1(a_1 + g)$, and then eliminating a_2, we find

$$\frac{a_1}{g} = \frac{(3/2)m_2 m_3/m_1 - (2m_2 + m_3/2)}{(1/2)m_2 m_3/m_1 + (2m_2 + m_3/2)} \qquad .$$

This can be simplified considerably by setting

$$\beta = m_1\left(\frac{4}{m_3} + \frac{1}{m_2}\right) \qquad ,$$

which yields

$$\frac{a_1}{g} = \frac{3 - \beta}{1 + \beta} \qquad .$$

We can verify that this gives correct results in some simple cases:

$m_1 = m_2 = 1$, $m_3 = 2$, $\beta = 3$, $a_1 = 0$
$m_1 = m_2 = 1$, $m_3 = 0$, $\beta = \infty$, $a_1 = -g$
$m_1 = m_2 = 1$, $m_3 = \infty$, $\beta = 1$, $a_1 = g$
$m_1 = m_3 = 1$, $m_2 = 0$, $\beta = \infty$, $a_1 = -g$

5-33 (a) Let the mass of the block be M. The downward force Mg of gravity on it is canceled by the upward normal force from the ledge, so the strength of the normal force is $F_N = Mg$. The maximum force of static friction is $\mu_s F_N = \mu_s Mg = \mu_s \rho V g$. Since we assume there is no friction between the rope and the lip of the cliff, this is the maximum tension that can be sustained in the rope. Equating this to the climber's body weight gives $\mu_s = m/\rho V$.
(b) The units are mass over mass, which cancels out. That makes sense, because the definition of μ_s implies that it is unitless.
(c) If m goes up, our equation says that μ_s also goes up. That makes sense, because a heavier climber would need more friction. If ρ or V gets bigger, the result gets smaller. That also makes sense, because increasing either of these would make the block heavier, which would allow it to hold without such a high coefficient of friction. (d) The result is 0.04, which is smaller by an order of magnitude than the reference value for granite on granite. This is a large margin of safety.

5-34 (a) We have the following three equations to start off with:

$$\begin{aligned} v^2 &= 2ax \\ F &= \mu_k mg \\ a &= F/m \end{aligned}$$

Eliminating F and a gives

$$\mu_k = \frac{v^2}{2gx} = 0.037.$$

5-35 (a) See tables.

(b) Let N_1 be the normal force between the ground and block 1, and N_2 the normal force between the two blocks. Then because block 2's acceleration is zero, the total force on it is zero, and $T - \mu_k N_2 = 0$, so $T = \mu_k M_2 g$.

(c) Newton's second law gives

$$\begin{aligned} M_1 a &= F - \mu_k N_1 - \mu_k N_2 \\ &= F - \mu_k (M_1 + M_2) g - \mu_k M_2 g \\ &= F - \mu_k (M_1 + 2M_2) g \\ a &= \frac{F - \mu_k (M_1 + 2M_2) g}{M_1} \end{aligned}$$

5-36 Because the pulleys are frictionless, the rope that the person is grabbing has equal tension F throughout. Therefore the pulley immediately above the load has an upward force $2F$ acting on it from the two strands of rope above it. This tells us that the system has a mechanical advantage of 2, and the person can lift a weight $2F$, which corresponds to a mass $2F/g$.

6-1 (a) The horizontal and vertical motions are independent, so they hit simultaneously. (b) While she's in the air, the only force on her is gravity. There is no horizontal force on acting on her, so her horizontal acceleration is zero, meaning she keeps a constant speed. Her speed on impact is the same as when she went over the edge, 30 km/hr. (c) The same as when he went over the edge, 40 km/hr. (d) They have the same vertical velocities, but different horizontal velocities, so Bill's speed is greater.

6-2 In Miss Lettuce's frame of reference, the float is at rest and the sidewalk is moving. The bouquet is initially at rest, so when she drops it, it falls straight down and lands at her feet.

In her fans' frame, the float is moving and the sidewalk is at rest. The bouquet is initially moving to the right, and even after she drops it, it retains that horizontal motion, because there is no horizontal force that would change it. They see a parabolic arc going to the right.

6-3 (a) The horizontal and vertical motions are independent, so we can write down separate equations for them:

$$x = v_x t \qquad \text{[motion with constant velocity]}$$

$$y = y_o - \frac{1}{2} g t^2 \qquad \text{[motion with constant acceleration]}$$

We're not given information about time, and we don't want to find it either, so we need to elimi-

Problem ??:

force acting on block 1			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	↓	Earth	gravity	↑	block 1
normal	↑	ground	normal	↓	block 1
kin. fr.	←	ground	kin. fr.	→	block 1
normal	↓	block 2	normal	↑	block 1
kin. fr.	←	block 2	kin. fr.	→	block 1
not given	→	not given	not given	←	block 1

Problem ??:

force acting on block 2			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	↓	Earth	gravity	↑	block 2
normal	↑	block 1	normal	↓	block 2
kin. fr.	→	block 1	kin. fr.	←	block 2
normal?	←	rope	normal?	→	block 2

nate it. Solving the first equation for t and substituting into the second equation gives

$$y = y_o - \frac{1}{2}g\left(\frac{x}{v_x}\right)^2 \qquad .$$

We're interested in $x = L$, and the variable d corresponds to $y_o - y$, so we have

$$d = \frac{1}{2}g\left(\frac{L}{v_x}\right)^2 \qquad .$$

(b) Convert everything to mks to start with. 73.3 mi/hr=32.8 m/s, 60.0 ft=18.3 m. Plugging in, we get $d = 1.53$ m=5.00 ft. I calculated this using the two-sig-fig value $g = 9.8\ \text{m/s}^2$, so that ends up being the limiting factor for accuracy, and the result should be rounded off to two sig figs, $d = 5.0$ ft.

6-4 The acceleration is mainly vertical. Each car's v_y starts out zero, increases as the bump's slope gets steeper, reaches a maximum, and then decreases back to zero as the car reaches the top of the bump. Riding back down the bump is similar.

The equation $a = \Delta v/\Delta t$ isn't strictly true here, since a isn't constant. However, we can still use it to get a comparison between the two cars. We then have $a_P/a_M = (\Delta v_P/\Delta v_M)(\Delta t_M/\Delta t_P)$. The first factor is clearly equal to 37/25, but so is the second one, since the Maserati's lower speed causes it to take more time to go through the same motion. Therefore $a_P/a_M = (37/25)^2 = 2.2$.

6-5 (a) The ball initially has $v_x = v\cos\theta$ and $v_y = v\sin\theta$. While it's in the air, it has $a = -g$, and its vertical velocity changes from v_y to $-v_y$, so it has $\Delta v = -2v_y$. The time it spends in the air is $\Delta t = -2v_y/a_y = (2v/g)\sin\theta$. The range is

$$R = \Delta x = v_x\Delta t = \frac{2v^2}{g}\sin\theta\cos\theta \qquad .$$

(b) It comes out to be zero for $\theta = 0°$, and that makes sense because it doesn't spend any time in the air. (Remember, we're neglecting the height above the ground at which the bat strikes the ball.) It also comes out to be zero for $\theta = 90°$, and that also makes sense, because the ball just pops up and falls back down to hit home plate.

(c) The angle at which the maximum occurs isn't affected by the constant factor out in front, so we just need to maximize $\sin\theta\cos\theta$. Setting its derivative equal to zero, we have $0 = \cos^2\theta - \sin^2\theta$, $\cos\theta = \pm\sin\theta$, which gives $\theta = \pm 45°$. The positive solution is the physically meaningful one.

6-6 (a) For its motion from the hand to the top of its arc, it has a constant acceleration equal to g, so $v^2 = 2gy$, and $y = v^2/2g$. (b) Only the vertical component, $v_y = v\sin\theta$, of the initial velocity has any effect on its vertical motion, so the equation becomes $y = v^2\sin^2\theta/2g$.

6-7 (a) For the vertical motion, we have $h = (1/2)gt^2$, which gives the air time $t = \sqrt{2h/g}$. You need to travel a horizontal distance $h\tan\theta$ in that time, so $v = h\tan\theta/t = \sqrt{gh/2}\tan\theta$.
(b) The units are $\sqrt{\text{m}^2/\text{s}^2} = \text{m/s}$, which makes sense.
(c) A higher g means you have to run faster, which makes sense physically. A higher h also means you have to run faster, which makes sense, e.g., you can't jump off of the top of a mountain and land at the base, but you could easily jump from a small scale model of the mountain. A bigger θ requires a higher speed, which makes sense. In the case of $\theta = 0$, we have $v = 0$, which makes sense because you can just step off the cliff. For $\theta = 90°$, the velocity becomes infinite, and that also makes sense: the "cliff" is now just flat ground, and you can't jump off of Kansas.
(d) Plugging in numbers gives 1.8 m/s.

6-8 (a) The vertical and horizontal motions are independent of each other. Since the balls reach the same height, their vertical motion is the same, so the initial y velocity of the green ball is also v_0.
(b) The times are the same, for the same reasons as in part a.
(c) The x velocity is $v_x = \sqrt{(2v_0)^2 - v_0^2} = \sqrt{3}v_0$. The time in the air is $2v_0/g$. Multiplying these gives the range, $2\sqrt{3}v_0^2/g$, and since $v_0^2 = 2gH$, this range can be expressed in terms of H as $4\sqrt{3}H$.

6-10 The problem is stated for a cylinder, but the minimum velocity will be one that is possible only in the case where the bullet flies in the plane of the figure, so we can reduce the problem to two dimensions.

The relevant physical conditions are the radius r of the cylinder and the strength g of the gravitational field. Based on units, the solution must be of the form $\sqrt{gr}$ multiplied by a unitless constant. Therefore we can simplify the math by taking $g = 1$ and $r = 1$, and waiting until the end of the problem to put the factor of $\sqrt{gr}$ back in.

There are two qualitatively different ways in which the bullet could fail to clear the cylinder. It could spend a finite time in the air and then hit, or it could simply fail to lift off. If we only consider the second condition, then there are several methods that allow a very short (even one-line) solution, but they involve tricks (such as approximating the bullet's motion as circular motion) or fancy math (such as matching the curvatures of the circle and parabola by setting the second-order terms of their Taylor series equal). We will instead take an approach that uses straightforward algebra and no tricks, and that addresses both possible modes of failure.

Let v be the initial speed of the bullet, and choose Cartesian coordinates with the usual orientation and the origin at the center of the circle. Then its motion is given by $x = vt$, $y = 1 - (1/2)t^2$, and eliminating t gives $y = 1 - (1/2)(x/v)^2$, the equation of the bullet's parabolic trajectory. For big enough values of v, this parabola will only intersect the circle at $x = 0$. For too-small values, it will intersect it in at least one other place. By symmetry, any such intersections will occur in pairs on the left and the right, but we consider only $x > 0$. If the bullet achieves lift-off but then hits the circle, then there should be two intersections for $x > 0$, one where the bullet enters the circle and one where it exits again at the bottom. If it never achieves lift-off, then there should be only one, where it exits.

To locate and count these intersections, we equate the equations of the circle and the parabola, $1 - (1/2)(x/v)^2 = \pm\sqrt{1 - x^2}$. The only positive solution is $x = 2v\sqrt{1-v^2}$, which exists if the square root is real. Since there is never more than one positive solution, we have shown rigorously that the only way in which we can fail is if the bullet never lifts off.

The solution fails to exist if $v > 1$, so the minimum speed is 1. Reinserting the factors that contain units, we have $v = \sqrt{gr}$ for the minimum speed.

6-11 We have $v_{o,y} = u \sin\theta$, so ignoring signs, $\Delta v = 2_v o, y$, and the time spent aloft is $t = (2u/g)\sin\theta$. During this time, the horizontal distance traveled is $L = v_x t = (2u^2/g)\sin\theta\cos\theta$. Since we want to solve for θ, it is convenient to use a double-angle identity to rewrite this as $L = (u^2/g)\sin 2\theta$, which results in

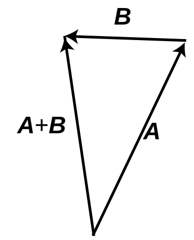
$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{Lg}{u^2}\right).$$

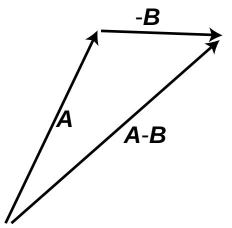
Since the physically admissible values of θ must be between 0 and 90 degrees, the arcsine has to be between 0 and 180 degrees. In a typical case, there are two such arcsines, one in the first quadrant and the other in the second quadrant. The values of θ are such that $\theta_1 = 90° - \theta_2$.

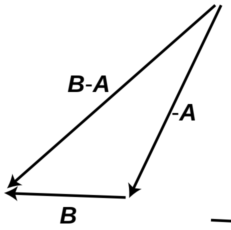
When $L > u^2/g$, there are no solutions: the target is out of range. When $L = u^2/g$, there is exactly one solution, and $\theta = 45°$.

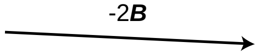
Regarding units, the quantity inside the arcsine is unitless. Call this x. From the given variables L, g, and u, the only unitless quantities we can form are x and functions of x. Since θ is unitless (radians are not really units), we can tell from units that θ can depend only on x, but we can't tell what form this dependence will take.

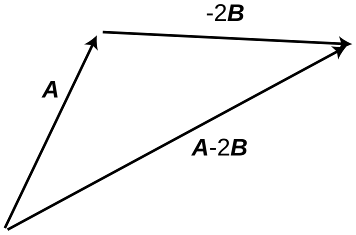
7-1











7-2 (a) Let's use a coordinate system with positive x being east and positive y being north. In these coordinates, the vector from Bangkok to Phnom Penh has

$$\Delta x = 470 \text{ km} \qquad \Delta y = -250 \text{ km}$$

and the one from Phnom Penh to Hanoi has

$$\Delta x = 60 \text{ km} \qquad \Delta y = 1030 \text{ km} \qquad .$$

(b) The components of the vector from Bangkok to Hanoi equal the sum of the components of the two vectors above,

$$\Delta x = 530 \text{ km} \qquad \Delta y = 780 \text{ km} \qquad .$$

7-3 Since the two given angles are both measured counterclockwise from east, we can just use a coordinate system with positive x being east and positive y north, and the positive and negative signs of the components will come out right if we just use cosines for x components and sines for y components. We find the components of the total Δr vector by adding the components of the two individual vectors:

$$\begin{aligned}\Delta x &= \Delta x_1 + \Delta x_2\\ &= (35\text{ km})\cos\ 25^\circ + (22\text{ km})\cos 230^\circ\\ &= 17.6\text{ km}\end{aligned}$$

(temporarily keeping 1 extra sig fig).

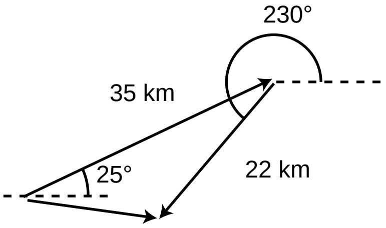
$$\begin{aligned}\Delta y &= \Delta y_1 + \Delta y_2\\ &= (35\text{ km})\sin 25^\circ + (22\text{ km})\sin 230^\circ\\ &= -2.1\text{ km}\end{aligned}$$

(temporarily keeping 1 extra sig fig).

The magnitude of the total Δr vector is 18 km, and the direction is

$$\tan^{-1}(-2.1\text{ km}/17.6\text{ km}) = -7^\circ \text{ or } 173^\circ \qquad .$$

Checking against the diagram, the correct arctangent is -7°.



7-4 This problem has to be done analytically, not graphically, because it requires such high precision. The two given angles are both measured counterclockwise from the right, which is convenient; for angles defined this way, we can always find x components using a cosine, and y components using a sine, and the plus and minus signs will be come out automatically. The first vector's components are

$$\begin{aligned}\Delta x_1 &= (35.24\text{ mm})\cos 217.3^\circ\\ &= -28.03\text{ mm}\\ \Delta y_1 &= (35.24\text{ mm})\sin 217.3^\circ\\ &= -21.36\text{ mm} \qquad ,\end{aligned}$$

and the second one's are

$$\begin{aligned}\Delta x_2 &= (48.01\text{ mm})\cos 11.7^\circ\\ &= 47.01\text{ mm}\\ \Delta y_2 &= (48.01\text{ mm})\sin 11.7^\circ\\ &= 9.74\text{ mm} \qquad .\end{aligned}$$

Checking these against the figure, their sizes and signs look reasonable. Adding these would give the vector from the top hole to the right-hand hole, but she wants to go the opposite way, so the components we want are

$$\begin{aligned}\Delta x_3 &= -(\Delta x_1 + \Delta x_2)\\ &= -18.98\text{ mm}\end{aligned}$$

and

$$\begin{aligned}\Delta y_3 &= -(\Delta y_1 + \Delta y_2)\\ &= 11.62\text{ mm} \qquad .\end{aligned}$$

Again, the signs and sizes look about right. The distance she has to move is given by the Pythagorean theorem,

$$\begin{aligned}|\Delta \mathbf{r}_3| &= \sqrt{\Delta x_3^2 + \Delta y_3^2}\\ &= 22.25\text{ mm} \qquad .\end{aligned}$$

The direction is

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\Delta y_3}{\Delta x_3}\right)\\ &= -31.4^\circ \text{ or } 148.5^\circ\end{aligned}$$

(measured counterclockwise from the right). We can tell from looking at the figure that the second arctangent is the correct one. (I've given the final answers with four sig figs of accuracy, although you could argue that it should be three, since the least accurate piece of given data is the 11.7° angle. In reality, however, that angle isn't any less accurate than the 217.3° one, i.e., changing either one by a tenth of a degree will have about the same effect on the final result.)

7-5 To prove that this operation is not rotationally invariant, it suffices to find one example in which converting the inputs **A** and B into a different, rotated coordinate system causes the output **C** to be a completely different vector, not just the same vector represented in a different system of coordinates. Almost any example will show this, unless we really manage to shoot ourselves in the foot with an unusual choice like $\mathbf{A} = 0$, $B = 0$. As an example, let's take $B = 1$, and $\mathbf{A} = \hat{\mathbf{x}} + \hat{\mathbf{y}} - \hat{\mathbf{z}}$ in the original coordinate system, resulting in $\mathbf{C} = 2\hat{\mathbf{x}} + 2\hat{\mathbf{y}}$. Now rotate the coordinate system by 180 degrees about the z axis. In this new coordinate system, $\mathbf{A} = -\hat{\mathbf{x}} - \hat{\mathbf{y}} - \hat{\mathbf{z}}$, but the scalar B stays the same, giving $\mathbf{C} = 0$. This new **C** is clearly not just the same vector expressed in a differently oriented coordinate system, since its magnitude has changed from a nonzero value to zero.

7-6 Here's the program I came up with:

```
import math
def r(theta,k,dt):
  # theta = angle at which it's hit, in deg
  # k     = constant for air friction
  # dt    = amount of time for each step
  y0 = 1.  # height from which it's hit
  v = 45.  # speed of hit
  m = .146 # mass
  g=9.8 # gravitational field
  t=0
  # Built-in trig functions want radians:
  radians = theta*math.pi/180.
  vx = v*math.cos(radians)
  vy = v*math.sin(radians)
  x = 0
  y = y0
  for i in range(1000000):
    # ... actually we hit the ground
    #     before 1000000
    t = t + dt
    x = x + vx*dt
    y = y + vy*dt
    if y<0 :  # hit the ground
      return x
    v = math.sqrt(vx*vx+vy*vy)
    radians = math.atan(vy/vx)
    f_air = k*v*v
    f_grav = m*g
    fx = -f_air*math.cos(radians)
    fy = -f_air*math.sin(radians)-f_grav
    ax = fx/m
    ay = fy/m
    vx = vx + ax*dt
    vy = vy + ay*dt
```

Running the program:

```
1   >>> r(45,0,0.01)
2   208.101525703
3   >>> r(45,0.0007,.01)
4   122.955242954
5   >>> r(40,0.0007,.01)
6   123.93693697
7   >>> r(40,0.00057,.01)
8   133.058781684
```

Lines 1-2 approximately reproduce the result of problem 51, with air resistance turned off; the small disagreement is due to a combination of the errors in the approximation made by the numerical simulation and the use of a starting height of 1 meter.

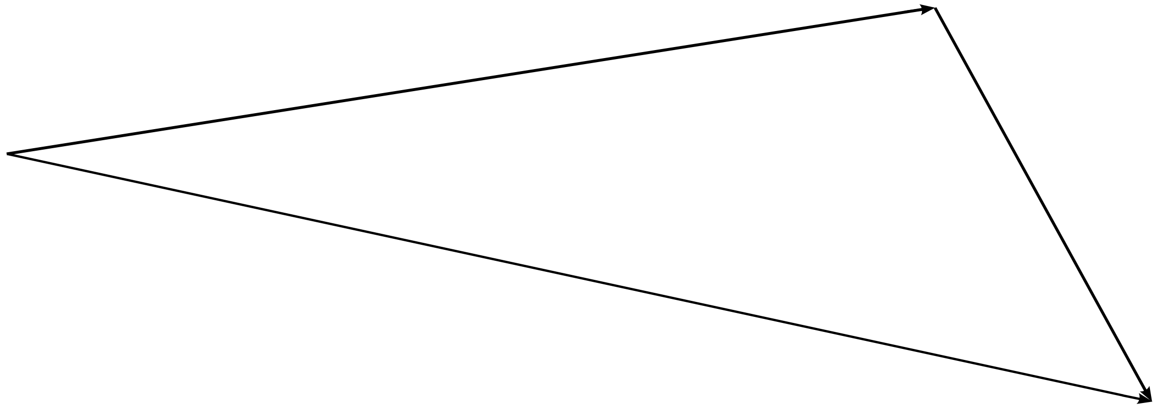
Lines 3-4 show the effect of air resistance, while keeping the angle at the value of 45 that would have been optimal in vacuum. Air resistance decreases the range by almost a factor of 2!

Lines 5-6 show the best range, which is obtained at around 40 degrees. (Finding this optimal angle actually required some trial and error, not shown here.)

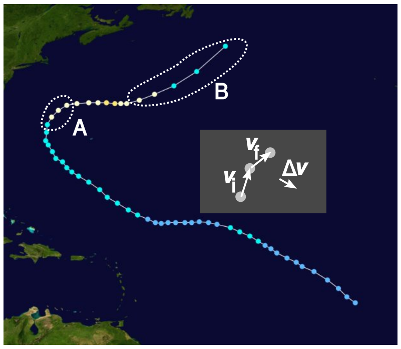
Lines 7-8 show that the ball flies about 9 m farther at Coors Stadium.

8-1 Yes, it would be slowing down.

8-2 The velocity vector points in the direction of motion. There are two time periods during which the direction of motion is northeast:



Problem 8-4.



During period A, around Aug. 15, the path is curving to the right. The $\Delta\mathbf{r}$ vectors connecting the points in dot-to-dot fashion can be thought of as synonyms for the $\mathbf{v}$ vectors, since $\mathbf{v} \approx \Delta\mathbf{r}/\Delta t$, and Δt is just a constant (6 hours). As shown in the inset figure, vector subtraction of these $\mathbf{v}$ vectors gives a $\Delta\mathbf{v}$ that points south-east, so the acceleration is to the southeast.

During period B, a similar vector subtraction gives an acceleration to the northwest, which is not what is required by the question.

8-4 See the figure. I chose a scale of 0.5 cm ↔ 10^{-7} m/s. The vector sum is 14.3 cm in length, which converts to 2.9×10^{-7} m/s. The direction, measured with a protractor, is about 12° south of east.

8-5 The original velocity vector had $v_x = 21.1$ m/s, $v_y = 0$. The later one has $v_x = (21.1\text{ m/s})\times\cos 7^\circ = 20.94$ m/s and $v_y = (21.1\text{ m/s})\times\sin 7^\circ = 2.57$ m/s. The change in the velocity vector is

$$\Delta v_x = -0.16\text{ m/s} \qquad \text{and}$$
$$\Delta v_y = 2.57\text{ m/s} \qquad ,$$

so the acceleration vector has components

$$a_x = \Delta v_x/\Delta t = -0.16\text{ m/s}^2 \qquad \text{and}$$
$$a_y = \Delta v_y/\Delta t = 2.57\text{ m/s}^2 \qquad ,$$

giving a magnitude of

$$\sqrt{(0.16\text{ m/s}^2)^2 + (2.57\text{ m/s}^2)^2} = 2.6\text{ m/s}^2 \qquad .$$

Since the x component is negative and the y component is positive, it has to be pointing northwest, and the angle measured counterclockwise from east is $\tan^{-1}(2.57/-0.16) = 92^\circ$. (Your calculator probably gives you the other arctangent, -88°, which is not the appropriate one.)

8-6 (a) The units of the first term are just the units of b, so b must have units of meters. The units of c have to be m/s, so that the seconds will cancel and give meters. Similarly d must have units of m/s^2, i.e., acceleration.

(b)

$$\begin{aligned}\mathbf{v} &= \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\\ &= c\hat{\mathbf{y}} + 2dt\hat{\mathbf{z}}\\ \mathbf{a} &= \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\\ &= 2d\hat{\mathbf{z}}\end{aligned}$$

(c) The only force acting on the bullet is gravity, so its acceleration $2d$ equals g, and $\hat{\mathbf{z}}$ points down. The bullet's velocity at $t = 0$ is $c\hat{\mathbf{y}}$, so c is its initial velocity, and $\hat{\mathbf{y}}$ points to the west. If the coordinate system is right-handed, then $\hat{\mathbf{x}}$ must point south, and b is simply the bullet's location in the north-south direction, measured from some arbitrary origin.

8-7 (a) $v_f = \Delta x/\Delta t = w/t_b$.

(b) We know its final velocity, and the distance it traveled, and we want to find its acceleration. The constant acceleration equation that has the right variables in it is $v_f^2 = v_o^2 + 2a\Delta x$, which in our case translates into $v_f^2 = 2ax$. Solving for the unknown and expressing it in terms of the things we're actually able to measure directly, we have

$$\begin{aligned}a &= \frac{v_f^2}{2x}\\ &= \frac{w^2}{2xt_b^2}\end{aligned}$$

(c) See table on last page.

(d) This is easiest if we adopt a coordinate system where x is along the track, and y is perpendicular to the track. The cart only has an acceleration along the x axis, and the only force acting on the cart that has an x component is gravity.

$$\begin{aligned}a_x &= \frac{F_x}{m}\\ &= \frac{F\sin\theta}{m}\\ &= \frac{mg\sin\theta}{m}\\ &= g\sin\theta\end{aligned}$$

You might wonder how I knew it was a sine rather than a cosine. Well, the sine of zero is zero, and I know that the acceleration is zero if the angle of the track is zero.

8-8 (a) There are three forces acting on the part of the rope right under the person's feet: the two tensions and the normal force of the person's feet on the rope, which equals the person's weight. The vector sum of the three forces must be zero, i.e. their x components must add up to zero and so must their y components. The x components will just naturally cancel out by symmetry, so that's not helpful information. The y components of the two tension forces are both equal to each other, so double the y component of the tension in one side must equal the person's weight. The y component of the tension in one side is $T\sin\theta$, where θ is the angle of the rope above horizontal. Therefore, we must have $Mg = 2T\sin\theta$. The length of each half of the tightrope is given by the Pythagorean theorem, $\sqrt{h^2 + (L/2)^2}$, and using trig, $\sin\theta = h/\sqrt{h^2 + (L/2)^2} = 1/\sqrt{1 + (L/2h)^2}$. Solving for T, $T = Mg/(2\sin\theta) = \frac{1}{2}Mg\sqrt{1 + (L/2h)^2}$.

(b) Examining the equation $T = Mg/(2\sin\theta) = \frac{1}{2}Mg\sqrt{1 + (L/2h)^2}$, if h was zero, $L/2h$ would be infinite, so the tension would have to be infinite. Physically, the rope would break at some nonzero value of h.

8-10 (a) There are four forces acting on the skier, a normal force F_N from the snow, a gravity force F_W, a kinetic friction force $F_k = \mu_k F_N$, and an air friction force bv^2. If the skier is to approach a constant velocity, the total force on her must become nearly zero. Laying the vectors tip-to-tail results in the right triangle shown in the figure. At first the bv^2 force will not be big enough to close the gap in the triagle, and the skier will be accelerating, but eventually as her velocity grows and the bv^2 force gets stronger the total force will approach zero and she will stop accelerating. Trigonometry gives

$$\begin{aligned}bv^2 + F_k &= F_W\sin\theta \qquad \text{and}\\ F_N &= F_W\cos\theta \qquad ,\end{aligned}$$

and using these two equations plus $F_W = mg$

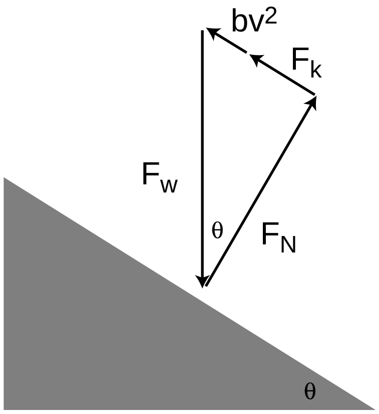
Problem ??:

force acting on cart			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	earth	gravity	up	cart
normal	up and right	track	normal	down and left	cart

and $F_k = \mu_k F_N$ we find

$$v = \sqrt{\frac{mg}{b}\left(\sin\theta - \mu_k \cos\theta\right)} \qquad .$$

(b) For θ less than $\tan^{-1}\mu_k$, we'd be taking the square root of a negative number. These are angles so shallow that the skier would slow to a halt.



8-11 The main reason that this problem is difficult is that you have to realize there are two separate cases. Pressing the block and the wall together can result in either an upward or a downward frictional force on the block. If you're applying almost enough force to slide the block up the wall, then the frictional force is downward, but if you're applying just barely enough to keep it from sliding down, then the frictional force is upward. Because the vertical component of the frictional force has an opposite signs in the two cases, it is necessary to do a separate version of the algebra in each case.

Case A: upward friction on the block

Let the positive x axis be to the right and the positive y axis up. Then both the x and y components of the force on the block must equal zero. Let $\mathbf{F}_N$ be the normal force of the wall on the block. The maximum force of static friction is given by $|\mathbf{F}_{s,max}| = \mu_s|\mathbf{F}_N|$, and it is the maximum in which we're interested, since we're looking for the largest possible value of $|\mathbf{F}_H|$.

zero total force in the x direction:

$$|\mathbf{F}_H|\cos\theta - |\mathbf{F}_N| = 0$$

zero total force in the y direction:

$$|\mathbf{F}_H|\sin\theta - mg + \mu_s|\mathbf{F}_N| = 0$$

We solve the first equation for $|\mathbf{F}_N| = |\mathbf{F}_H|\cos\theta$, then substitute into the second equation and solve to find

$$|\mathbf{F}_H| = \frac{mg}{\sin\theta + \mu_s\cos\theta}$$

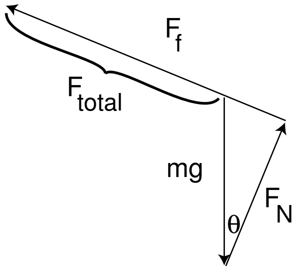
Case B: downward friction on the block

The solution of this part is exactly analogous, except that the total force in the y direction is $|\mathbf{F}_H|\sin\theta - mg - \mu_s|\mathbf{F}_N| = 0$, with a minus sign in front of the frictional force. The result is

$$|\mathbf{F}_H| = \frac{mg}{\sin\theta - \mu_s\cos\theta}$$

In the case of $\theta = 90°$, both the maximum and the minimum are mg, which makes sense. The maximum force blows up to infinity for $\theta = \tan^{-1}\mu_s$. With this angle, no amount of force will make the block slip, because the greater normal force leads to a greater frictional force, which is great enough to counteract the greater vertical force from the hand. For angles smaller than this, the equation for the maximum force gives a negative result, which is not physically meaningful because it was supposed to represent the magnitude of the force.

8-12 (a) See table on last page.



(b) From the figure, we see that the total force is equal to the force of static friction, minus the length of the short leg of the right triangle, $F_s - mg\sin\theta$. The maximum possible deceleration will be obtained if static friction has its maximum value $F_s = \mu_s F_N = \mu_s mg\cos\theta$. The result for the total force is $\mu_s mg\cos\theta - mg\sin\theta = mg(\mu_s\cos\theta - \sin\theta)$. By Newton's second law, the maximum deceleration is the total force divided by the mass, or $g(\mu_s\cos\theta - \sin\theta)$.

(c) Increasing the car's mass increases its inertia, which makes it harder to stop, but it also increases the gravitational force on it, which causes a greater normal force, and therefore a greater maximum value for the static friction force.

(d) At $\theta = 0$, we find that the maximum deceleration of a car on a flat road is $\mu_s g$. For $\theta < 0$, the $\sin\theta$ is negative, which means the result for the deceleration is *greater* than on a flat road. That makes sense: it's easier to make a panic stop when you're going uphill.

(e) As θ increases, the $\cos\theta$ term gets smaller and the $\sin\theta$ term gets bigger. At some angle, the result for the deceleration will become zero. This is the steepest slope the car can be on without slipping. For angles greater than this, static friction must give way to kinetic friction, and our result from part b becomes incorrect, since we derived it assuming static friction.

8-15 In each case, the y component, T_y, of the tension has to be enough to support half the boy's weight. The y component of the tension is going to be given by either the sine or the cosine, and we can see that it must be the cosine because that gives the right answer for the first positoin, $T_y = T\cos\theta = T$. Since T_y is always the same, the tension must be proportional to $1/\cos\theta$. The tensions are in the ratios of 1 to $1/\cos 45°$ to $1/\cos 60°$, or 1 to 1.41 to 2.

8-16 (a) Let the positive x axis be to the right and positive y up. The x components of the forces cancel by symmetry, so they don't give us any useful information. The y components of the three forces are $S\cos\theta/2$, $S\cos\theta/2$, and $-L$. Setting the sum of these equal to zero, we find $S = L/(2\cos\theta/2)$. (b) For $\theta = 0$, we find $S = L/2$, which makes sense physically because two forces S are cooperating perfectly to cancel L. (c) For $\theta = 180°$, we have $S = \infty$, which is not physically possible, but does make sense. If we were to grab the two anchor lines in our hands and pull them to the right and left away from each other, it would take infinite force to make $\theta = 180°$ exactly. (c) This occurs when $2\cos\theta/2 = 1$, i.e., $\theta = 2\cos^{-1}(1/2) = 120°$.

8-17 (a) In the case where the block just barely sticks, the static frictional force is at the maximum possible value,

$$F_s = \mu_s F_N \qquad .$$

(This is static friction because there is no slipping.) Let positive x be to the right and positive y be up. Applying Newton's second law in the x direction gives

$$-F_N\sin\theta + F_s\cos\theta = ma \qquad ,$$

and in the y

$$F_N\cos\theta + F_s\sin\theta - mg = 0 \qquad .$$

These are three equations in the three unknowns F_s, F_N, and a. Eliminating F_s and F_N and solving for a, we find

$$a = \left(\frac{\mu_s\cos\theta - \sin\theta}{\mu_s\sin\theta + \cos\theta}\right)g \qquad .$$

(b) Coefficients of friction are unitless, so the whole expression on the right consists of unitless

Problem ??:

force acting on car			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet Earth	gravity	up	car
static friction	uphill	road	static friction	downhill	car
normal	up and forward	road	normal	down and backward	car

stuff, except for the factor of g. That makes the right-hand-side come out to have units of acceleration, which makes sense.

(c) Mathematically, increasing θ decreases $\cos\theta$ and increases $\sin\theta$. That means that it decreases the numerator of our expression for a — until the numerator reaches zero, at the critical angle $\theta_c = \tan^{-1}\mu_s$. This is the angle at which it becomes impossible to accelerate the wedge at all without making the block slip, because the block is barely able to stick even when the wedge is sitting still.

The denominator is a little trickier. For small values of θ, $\cos\theta$ is nearly constant, while $\sin\theta$ increases approximately in proportion to θ, so the denominator is increasing. But is it increasing for all θ of interest? If it stops increasing at some θ, then at that θ $\mathrm{d}(\mu\sin\theta + \cos\theta) = 0$, but this occurs at θ_c. Therefore the denominator is increasing for all physically relevant values of θ.

Since the numerator is decreasing and the denominator is increasing, the result is decreasing. This makes sense physically, because if the wedge is steeper, it should be harder to keep the block from slipping.

8-18 (a) Each block has three forces acting on it: gravity, a normal force from the ramp, and a force from the rope. The components perpendicular to the slope cancel out, while the ones parallel to it cause an acceleration a, which will be the same for both blocks provided that the rope doesn't go slack. We therefore have $ma = mg\sin\theta + T$ and $ma = mg\sin\phi - T$. Eliminating a and solving for T, we find $T = \frac{1}{2}mg(\sin\phi - \sin\theta)$.

(b) The units of mg are newtons, and the sines are unitless, so the result does have units of newtons, as expected.

(c) For $\phi = \theta$, the tension is zero, i.e., the rope goes slack. This makes sense. For $\phi < \theta$, the tension becomes negative; this is unphysical, but that makes sense, because the assumption we made (that the rope would stay taut) has failed. For $\theta = 0$ and $\phi = 90°$, the result is $T = mg/2$, which we can readily verify is a correct solution. It accelerates the top block across its horizontal surface with $a = g/2$, and it cancels out half of the gravitational force on the bottom block, also causing $a = g/2$.

8-19 Let a small section of the rope, extending from $-\,\mathrm{d}\theta/2$ to $+\,\mathrm{d}\theta/2$, have tension $T - \mathrm{d}T/2$ and $T + \mathrm{d}T/2$ applied at its ends. Within this section, there are normal forces that point in different directions, but their directions are so similar that we can consider them as a single force of magnitude $\mathrm{d}N$ acting in the direction normal to the post at $\theta = 0$. Since the total force in the normal direction must be zero, we have $\mathrm{d}N = (T - \mathrm{d}T/2)\sin(\mathrm{d}\theta/2) + (T + \mathrm{d}T/2)\sin(\mathrm{d}\theta/2)$. Applying the small-angle approximation $\sin x \approx x$, we have $\mathrm{d}N = T\,\mathrm{d}\theta$. The total force in the tangential direction must also be zero, so $\mu_s\,\mathrm{d}N = \mathrm{d}T$. Combining the two results, we find $\mu_s T\,\mathrm{d}\theta = \mathrm{d}T$. Separating variables, we have $\mathrm{d}T/T = \mu_s\,\mathrm{d}\theta$. Integration of both sides gives $\ln T = \mu_s\theta + c$, which upon exponentiation of both sides gives $T_2/T_1 = e^{\mu_s\theta}$

8-20 Let m be the mass of the board, x the displacement of the board relative to the center, and N_1 and N_2 the two normal forces.

Suppose that (1) the board is not rotating, and (2) kinetic friction obtains on both sides. Then the total torque is zero, $mgx + r(N_1 - N_2) = 0$, and the total horizontal force on the board is $\mu_k(N_1 - N_2) = -mg\mu_k x/r$. This is a restoring force proportional to x, so we have simple harmonic motion, provided that the assumptions hold: no rotation, and purely kinetic friction. The frequency is $\omega = \sqrt{g\mu_k/r}$. This is a funny kind of simple harmonic motion, because the kinetic frictional forces are continually dissipating energy as heat, and yet the motion is undamped.

Under what conditions can assumptions 1 and 2 be violated? Let the amplitude of the simple harmonic motion be A.

For $A > r$, the board will tip as its center moves beyond the top of one of the wheels, violating assumption 1. The board might either fall on the ground or make a bobble and come back.

Assumption 2, purely kinetic friction, can also be violated. The maximum velocity in simple harmonic motion is $A\omega$. If this velocity ever becomes as high as the tangential velocity of the wheels, Ωr, then kinetic friction would be replaced by static friction on one side, and we would no longer have a self-consistent solution in simple harmonic motion. One possibility is that under these conditions, the board can accelerate up to velocity Ωr, then stick at that velocity for a while before its weight shifts onto the other wheel sufficiently, at which point it will start to decelerate again; the motion is periodic but not simple harmonic.

We can also have possibilities that combine the features of the tipping solutions with those of the static-frictional solutions.

8-21 (a) Most of the forces on the box are either parallel to or perpendicular to the slope, so it's convenient to pick coordinate axes aligned with the slope. Let positive x be uphill, and positive y perpendicular to that (up and to the left). Four forces act on the box: a normal force F_N in the positive y direction, a kinetic frictional force F_k in the $-x$ direction, a force from the rope with magnitude T in the $+x$ direction, and a downward force of gravity with magnitude Mg. The gravitational force is the only one with both an x and a y component; it has x component $-Mg\sin\theta$ and y component $Mg\cos\theta$.

Since the box doesn't hop off of or sink into the slope, the total force on the box in the y direction must be zero:

$$0 = F_N - Mg\cos\theta$$

To predict the box's acceleration, we apply Newton's second law in the x direction:

$$a = (T - Mg\sin\theta - \mu_k F_N)/M$$

These are two equations in the three unknowns F_N, a, and T. Since we don't have enough equations to solve for the unknowns, we apply Newton's second law to the person:

$$a = (mg - T)/m \qquad ,$$

where we make use of the facts that the tension in a rope is unaffected by going around a frictionless pulley and that the person's acceleration must match that of the box, since they're tied together. Solving these equations for a results in

$$a = (m - M\sin\theta - \mu_k M\cos\theta)g/(M+m) \qquad .$$

(b) All the terms in the factor $m - M\sin\theta - \mu_k M\cos\theta$ have units of kilograms, and these cancel with the units of the $M + m$, giving nothing but acceleration units from the factor of g.

(c) When $M = 0$, the rope should be slack and the person should simply free-fall with acceleration g. This is indeed what happens when we plug in $M = 0$.

When $\theta = -90°$, the slope is rotated clockwise so that it becomes a vertical wall above the person's head. There is no normal force between the box and the slope, so there is no friction. The person and the box should both simply free-fall with acceleration g. Plugging in $\theta = -90°$ does produce this result.

8-22

$F_{F1,x}$	small, −	$-F_{F1}\sin\theta$
$F_{F1,y}$	large, +	$F_{F1}\cos\theta$
$F_{F2,x}$	small, −	$-F_{F2}\sin\theta$
$F_{F2,y}$	large, +	$F_{F2}\cos\theta$
$F_{N1,x}$	large, −	$-F_{N1}\cos\theta$
$F_{N1,y}$	small, −	$-F_{N1}\sin\theta$
$F_{N2,x}$	large, +	$F_{N2}\cos\theta$
$F_{N2,y}$	small, +	$F_{N2}\sin\theta$
$F_{W,x}$		0
$F_{W,y}$		$-F_W$

8-23 (a) Let positive x be to the right and positive y up. Zero total force acts on the carabiner in the y direction,

$$-L+2T\cos(\theta/2)=0 \qquad .$$

Zero total force also acts on the left-hand anchor, so

$$\begin{aligned}S_y-T\cos(\theta/2)&=0 \qquad \text{and}\\ S_x+T+T\sin(\theta/2)&=0 \qquad .\end{aligned}$$

Algebra leads to the result

$$S=L\frac{\sqrt{(1+\sin(\theta/2))/2}}{\cos(\theta/2)} \qquad .$$

(b) When $\theta=0$, we have $S=L/\sqrt{2}$, which makes sense because in this situation $T=L/2$, and each anchor experiences two forces equal in magnitude to T, acting at right angles. This is worse than the correct setup, in which $\theta=0$ gives $S=L/2$.

8-24 (a) The tension at the center describes a force that each half of the wire exerts on the other half. Therefore we should analyze the forces on one half of the wire, say the left half.

(Here "cohesive" is just a name I made up for the force that holds the wire together and keeps it from breaking.)

Let the magnitude normal force be F_N, and let the tension in the middle of the wire be T_m. By Newton's first law, the total force acting on the left half of the wire must be zero. Let's do analytic addition, which means adding components. Let the positive x axis be to the right, and the positive y axis up. Then adding the horizontal components gives

$$T_m-F_N\cos\theta=0 \qquad ,$$

and adding the vertical ones gives

$$F_N\sin\theta-\frac{1}{2}mg=0 \qquad .$$

The result is $T_m=mg/2\tan\theta$.

(b) The tension at the ends equals F_N. Solving the original two equations for this unknown, we have $F_N=mg/2\sin\theta$. Since $\sin\theta$ is always less than $\tan\theta$ for angles between 0 and 90°, we find $F_N>T_m$. The tension at the ends is greater than in the middle. This is easy to verify intuitively by imagining the case of $\theta\approx 90°$.

8-25 (a) Let positive x be to the right, and positive y up. The horizontal motion is at constant velocity $u\cos\alpha$, so

$$x=ut\cos\alpha.$$

The vertical motion has initial velocity $u\sin\theta$ and acceleration $-g$, so

$$y=ut\sin\alpha-\frac{1}{2}gt^2.$$

Setting $x=\ell$ in the first equation and solving for t gives $t=\ell/u\cos\alpha$. Now we set $y=h$ in the second equation and substitute for t, which gives

$$h=\ell\tan\alpha-\frac{1}{2}g(\ell/u\cos\alpha)^2.$$

Solving this for u results in

$$u=\sqrt{\frac{gl}{2\cos^2\alpha(\tan\alpha-h/\ell)}}.$$

(b) The ratio h/ℓ is unitless, so it makes sense to subtract it from $\tan\alpha$. The units of the quantity inside the square root come solely from the factor gl, which has units of $\mathrm{m}^2/\mathrm{s}^2$. Taking the square root gives m/s, which makes sense.

(c) Physically, if gravity is stronger we'll need to roll the ball faster to get it up to the hole. Mathematically, g is on top, so increasing it increases u.

Problem ??:

force acting on left half of wire			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	planet earth	gravity	up	left half
cohesive	right	right half	cohesive	left	left half
normal	up and left	hook	normal	down and right	left half

(d) Physically, if the height is increased we'll need to increase the speed. Mathematically, h/ℓ is subtracted on the bottom, so increasing h reduces the bottom, which increases the result.
(e) If $\alpha = 90°$, then u blows up to infinity. This makes sense, because if the ball is launched straight up, then it doesn't matter how fast we roll it, it will never cover the horizontal distance h.
(f) When $\tan\alpha = h/\ell$, the ramp is pointed straight at the hole. If we were playing the game in zero gravity, this would be just fine, but when there is gravity, the path of the ball can't be straight, so there is no way it can get to the hole when it's aimed this way. Mathematically, our result for u blows up under these conditions, which makes sense.
(g) Plugging in gives $u = 4.1$ m/s.

8-26 Let positive x be the direction parallel to the ramp and uphill, and $+y$ perpendicular to the ramp and upward. Let F_N be the normal force between the ramp and the box, and F the force you apply. Applying Newton's second law in the x and y directions gives

$$\begin{aligned} F_x &= -mg\sin\theta + F\cos(\varphi-\theta) - \mu F_N \\ 0 &= F_N - mg\cos\theta + F\sin(\varphi-\theta). \end{aligned}$$

Eliminating F_N gives

$$\begin{aligned} F_x = -mg\sin\theta + F\cos(\varphi-\theta) - \mu mg\cos\theta \\ + \mu F\sin(\varphi-\theta). \end{aligned}$$

We want to choose φ so as to maximize F_x. This is made simpler if we change the independent variable to $\alpha = \varphi - \theta$ and recognize that the only part of F_x that is relevant is proportional to

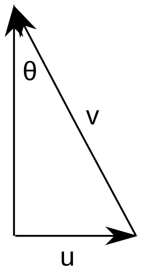
$$A = \cos\alpha + \mu\sin\alpha.$$

Taking the derivative and setting it equal to zero gives $\alpha = \tan^{-1}\mu$, or

$$\varphi = \theta + \tan^{-1}\mu.$$

One way of interpreting this is as follows. If there is no friction, then we simply set $\varphi = \theta$, so that the force in the direction of motion is maximized. If there is friction, then we can win by raising φ slightly higher than θ. For small values of α, this gives almost no reduction in the x component of our force, while producing a significant reduction in the normal force.

8-27 The addition of the velocity vectors looks like this:



(a) We have $\theta = \sin^{-1}(u/v)$.
(b) The velocity of the plane relative to the ground is the left side of the triangle, $w = \sqrt{v^2 - u^2}$, so the time to get to the city is $t = D/w = D/\sqrt{v^2-u^2}$.
(c) The units are $\mathrm{m}/\sqrt{(\mathrm{m}^2/\mathrm{s}^2)}$, which does turn out to be seconds.
(d) If $u = v$ we get an infinite result. This makes sense, because in this situation the plane can only stand still by flying directly into the wind.

8-28 Let the positive x axis be to the right, and positive y up.

(a) The block isn't popping up into the air or sinking into the surface, so the y forces cancel:

$$F_N - Mg - F\sin\theta = 0$$
$$F_N = Mg + F\sin\theta$$

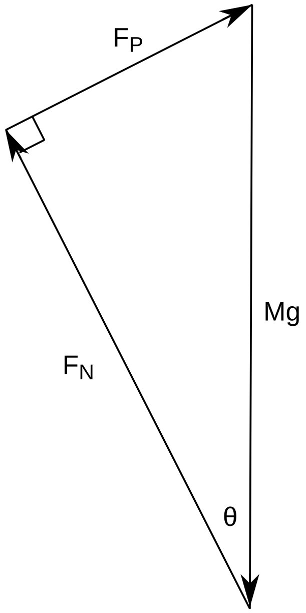
(b) The case where it's just about to slip is the one where it's in equilibrium horizontally and the force of static friction is at its maximum value $\mu_s F_N$, so that $F\cos\theta - \mu_s F_N = 0$. Substituting the answer from part a for F_N and solving for F, we find

$$F = \frac{Mg}{\cos\theta/\mu_s - \sin\theta}.$$

(c) This is the same as part b, but with μ_k.

(d) The answer to part b misbehaves starting at the critical angle for which the denominator is zero, which is $\tan^{-1}(1/\mu_k)$. Beyond this angle, the result for F is negative, which is unphysical.

8-29 (a) Three forces act on the mass: a gravitational force Mg, a normal force from the ramp, and a force from the rope that equals the reading F_P on the force probe. As shown in the figure, these three forces, when assembled tip-to-tail for vector addition, make a right triangle with Mg on the hypotenuse and F_P on the leg opposite to the angle θ. The result is that $F_P = Mg\sin\theta$.



(b) The expression found in part a gives 0 when $\theta = 0$ and Mg when $\theta = 90°$. Both of these make sense.

8-31 The horizontal motion is at the constant velocity v_0, while the vertical motion is with constant acceleration g. The vertical component of the impact velocity is given by $v_y^2 = 2gh$. Given the x and y components of the vector, we can use the inverse tangent function to find its direction. The result is

$$\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{v_0}\right)$$

8-32 (a) Let positive x be to the right and positive y up. The block is acted on by four forces: the two external forces of magnitude F, along with a normal force and a frictional force from the surface. If the block is to remain at rest, then the forces in the x direction must cancel, $F - F\cos\theta - \mu_s F_N = 0$, and similarly in the y direction, $F_N - F\sin\theta = 0$. Eliminating F_N and solving for the coefficient of friction results in

$$\mu_s = \frac{1-\cos\theta}{\sin\theta}.$$

(b) At $\theta = 0$ we expect that the block should be able to remain at rest without friction, since the horizontal forces cancel by symmetry. Our expression for μ_s is an indeterminate form when $\theta = 0$, but application of l'Hôpital's rule gives

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta} = \lim\frac{\sin\theta}{\cos\theta} = 0,$$

as required.

9-1 $(\mathrm{m/s})^2/\mathrm{m} = \mathrm{m}^2/\mathrm{s}^2/\mathrm{m} = \mathrm{m/s}^2$

9-2 If neither the seat nor the seatbelt is applying a force to the pilot, then the only force on him is the earth's gravity, i.e. his weight. When the only force on an object is gravity, its acceleration is g. The speed needs to be such that the acceleration equals g. The acceleration is v^2/r, so $v^2/r = g$, or $v = \sqrt{gr} = 99$ m/s.

9-3 (a)

$$\begin{aligned} a &= F/m \\ v^2/r &= F_N/m \\ F_s &= mg \\ \mu_s F_N &\geq mg \\ \mu_s m v^2/r &\geq mg \\ v &\geq \sqrt{gr/\mu_s} \end{aligned}$$

(b) The velocity is inversely proportional to the square root of the coefficient of static friction, so if her friction is three times weaker, she has to wait until the speed is greater by a factor of $\sqrt{3} = 1.7$.

9-4 The force is $F = ma = m\omega^2 r = m(2\pi T)^2 r = 1.3\times 10^{47}$ newtons. This is an absurd amount of force, and it clearly isn't being exerted by any physical object. There isn't actually any force, and Sirius isn't actually accelerating. It just appears to accelerate in this noninertial frame of reference.

9-6 The "radius" concept is a bit fuzzy here, because the train has a finite width to it. Although the inside of the outer rail is where the force has to be applied when the train is starting to tip, its radius from the center of the curve isn't quite the same as the radius of the circle traveled by the train's center of mass. Then again, maybe the train's center of mass would be above the outer rail if it was just on the verge of flipping over. Anyhow, ignoring these niceties, we assume that there is a maximum acceleration, which is the same in both cases. Converting $a = v^2/r$ into a proportionality by throwing out the factor of a, we have $v^2 \propto r$, or $v \propto \sqrt{r}$. Rewriting this in terms of ratios gives $v_1/v_2 = \sqrt{r_1/r_2}$. Since we're doing a ratio, it doesn't matter whether we do diameter or radius. The result is $(0.95\ \mathrm{m/s})\sqrt{662\ \mathrm{mm}/1067\ \mathrm{mm}} = 0.75$ m/s. We know we set up the ratio the right way, since it makes sense that the speed comes out slower on the tighter curve.

9-7 (a) The force on the hamster is $k(L-b)$, where L is the length of the spring (which is the same as the radius of the circle). Newton's second law gives $v^2/L = k(L-b)/m$. We can eliminate $v = 2\pi L/T$, and straightforward algebra then gives

$$L = \frac{b}{1-\frac{4\pi^2 m}{kT^2}}$$

(b) For a certain value of T, the denominator becomes zero. If we were to speed up the motor gradually, L would grow without bound as we approached this value of T. Physically, the spring would have to break. The reason for this surprising behavior will become clearer after you learn about the phenomenon of resonance.

9-8 The apparent paradox arises if you assume, incorrectly, that v and T are the same even though r is different. You can't change r while leaving v and T the same. A should have said that *for objects moving at the same speed*, the smaller circle gives a greater acceleration. B should have said that *for objects that take the same amount of time per cycle*, the smaller circle gives a smaller acceleration.

9-9 (a) Only an inward force, i.e., a force perpendicular to the surface of contact, can have an effect on whether the batter separates from the beater. Friction is always parallel to the surface of contact.

(b) See table on last page.

(c) An inward force is required to keep the batter from following Newton's first law and flying off straight, on a tangent to the circle. The amount of force required is given by Newton's second law, $F = ma = mv^2/r$. If the speed in-

Problem ??:

Part b:

force acting on tape			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
sticky	left	beater	sticky	right	tape

creases, so does amount of inward sticky force needed to continue deflecting the batter off of its desired straight-line path. There's a maximum amount of force that the stickum can supply, beyond which it becomes unstuck.

9-11 (a)

equal and opposite force

force acting on ball involved in Newton's 3rd law

type of object exerting object

force direction the force type direction exerting it

tension in and up string tension down and out ball

gravity down earth gravity up ball

(b)

Newton's second law, horizontal component:

$$v^2/r = F_T \sin\theta/m \qquad (1)$$

Newton's second law, vertical component:

$$0 = (F_T \cos\theta - mg)/m \qquad (2)$$

geometry:

$$r = L\sin\theta \qquad (3)$$

$$v = 2\pi r/P \qquad (4)$$

The variable v is not given and not desired, so we eliminate it using equation (4):

$$4\pi^2 mr/P^2 = F_T \sin\theta (5)$$

We next use equation (3) to eliminate r:

$$4\pi^2 mL/P^2 \sin\theta = F_T \sin\theta \qquad (6)$$

$$4\pi^2 mL/P^2 = F_T \qquad , \qquad (7)$$

which has the unintended side-effect of making θ disappear. The equation is still useful, however, because it tells us F_T in terms of variables we know. We can use it to eliminate F_T in equation (2):

$$0 = 4\pi^2 L/P^2 \cos\theta - g \qquad , \qquad (8)$$

which can be solved for θ:

$$\theta = \cos^{-1}\left(\frac{gP^2}{4\pi^2 L}\right) (9)$$

(c) We can't take the arccosine of a number that's greater than one, so there would be no solution for sufficiently large values of P. Physically, if P approached infinity, the ball would be standing still, in which case θ would equal zero. In the case where the angle is zero, step (6) $\rightarrow$ (7) is invalid, since we can't divide both sides of the equation by zero.

9-12 Applying Newton's second law and $a = v^2/r$ to each block independently, we have

$$v_1^2/L_1 = (T_1 - T_2)/m_1$$

$$v_2^2/(L_1 + L_2) = T_2/m_2$$

We can eliminate $v_1 = 2\pi L_1/P$ and $v_2 = 2\pi(L_1 + L_2)/P$, and straightforward algebra then yields

$$T_1 = \left(\frac{4\pi^2}{P^2}\right)[m_1 L_1 + m_2(L_1 + L_2)]$$

$$T_2 = \left(\frac{4\pi^2}{P^2}\right) m_2(L_1 + L_2)$$

9-14 (a) Let the tension in each of the top bars be T, and in each bottom one U. The vertical components of the forces on the mass M must cancel, so

$$2U\cos\theta = Mg \qquad ,$$

and similarly for the vertical forces on each mass m,

$$(T-U)\cos\theta = mg \qquad .$$

The horizontal force of the bars on each mass m must satisfy Newton's second law,

$$(T+U)\sin\theta = \frac{4\pi^2 mL\sin\theta}{P^2} \qquad .$$

We now have three equations in the three unknowns U, T, and θ. We use the first equation to eliminate U, which reduces this to two equations in two unknowns:

$$T + \frac{Mg}{2\cos\theta} = \frac{4\pi^2 mL}{P^2}$$
$$T\cos\theta - \frac{Mg}{2} = mg$$

Solving the second equation for $T = (mg + Mg/2)/\cos\theta$, substituting into the first equation, and solving for θ, we find the result claimed for θ.

(b) Low speeds correspond to large values of the period, P. At low enough speeds, the equation derived in part a ends up being the arccosine of a number greater than one, i.e., there is no solution of that form. What happens at these speeds is the same things that happens at zero speed: everything hangs straight down. In this solution, the radius of the circle traveled by each mass m is zero. (The mathematical loophole here was that we canceled a $\sin\theta$ on both sides of one of the equations, but you can't divide both sides of an equation by zero.) By making a nonzero, we make such a zero-radius solution impossible, because even when the engine isn't spinning, each mass m is already at some distance from the axis.

9-15 Because the bead slides frictionlessly on the string, there can't be any difference in tension between the top and bottom segments. The vector sum of the two forces acting on the ball lies in the direction that bisects the two strings, and this direction is the direction of a simple conical pendulum. As found in another problem, this angle lies below the horizontal by an angle $\beta = \tan^{-1}(g/\omega^2 r)$. Geometry then gives $\phi = \theta + 2\beta = \theta + 2\tan^{-1}(g/\omega^2 r)$.

9-16 (a) As the bead speeds up, a greater force toward the axis is required to keep it from flying off straight. This means that the normal force has to get stronger, and therefore the kinetic friction force will get stronger.

(b) Define a coordinate system with axes t, r, and p. Axis r points radially outward away from the central axis; t downhill and tangent to the wire; and p perpendicular to t and r. The velocity has only a component $v_t = v$. The motion, projected into the horizontal plane, is circular motion with speed $v\cos\theta$, where θ is the angle of the downhill slope, $\tan\theta = \alpha$, where $\alpha = d/2\pi r$. The acceleration is purely in the r direction, with $a_r = -v^2\cos^2\theta/r$.

The frictional force has only a t component. The normal force has both an r and a p component. The weight has both t and p components.

Newton's second law in the r direction gives $F_{n,r} = -mv^2/r\cos^2\theta$. Cancellation of the forces in the p direction gives $F_{n,p} = -F_{w,p} = mg\cos\theta$. We therefore have

$$F_{f,t} = -\mu_k F_n = -m\mu_k\cos\theta\sqrt{g^2 + \left(\frac{v^2\cos\theta}{r}\right)^2} \qquad .$$

This has to cancel with $F_{w,t} = mg\sin\theta$, and solving for v we find

$$v = \sqrt{gr\sqrt{(1+\alpha^2)(1-\frac{\alpha^2}{\mu_k^2})}} \qquad .$$

(c) In the limit of r approaching infinity, the problem is the same as that of an object sliding down an inclined plane. Under those circumstances, the only possible way of getting a constant velocity would be to have the slope chosen to have some particular value that would match properly with the coefficient of friction. A quick calculation shows that the necessary condition is $\mu_k = \tan\theta$. This matches up properly with our expression for v in the case of the helical wire, because the only way to get a finite limiting value for v as r approached infinity would be to let the factor $1 - \alpha^2/\mu_k^2$ approach zero.

9-17 (a) Let positive x be inward and positive y up. Then $a_x = F_x/m$ gives $v^2/r = F_N/m$, and $a_y = F_y/m$ results in $0 = \mu_s F_N - mg$. Solving for r, we find $r = \mu_s v^2/g$. (b) $(\mathrm{m/s})^2/(\mathrm{m/s}^2) = \mathrm{m}$. (c) Mathematically, increasing v increases r; this makes sense because it should be harder to do if r is bigger, so you should have to run faster. Increasing g decreases r, and that makes sense because, e.g., if g were 0, you could do this with an unlimited r. Increasing μ_s increases r; this makes sense because, e.g., in the absence of friction, it should be impossible to do the stunt, and the maximum r should be 0, i.e., it shouldn't be possible for any positive r. (d) The result is 50 meters, which is counterintuitively huge.

9-18 Application of Newton's laws gives a normal force mv^2/r and a tangential acceleration $(\mu/r)v^2$ due to friction. If we let x be the distance traveled around the circumference, then the motion satisfies the differential equation

$$\left(\frac{r}{\mu}\right) x'' = \pm(x')^2,$$

where the sign depends on the direction of motion. Since only the derivatives of x occur, not x itself, we can substitute $v = x'$ and rewrite this as a first-order equation

$$A\frac{\mathrm{d}v}{\mathrm{d}t} = \pm v^2,$$

where $A = r/\mu$. Separating variables and integrating twice, we find

$$x = \mp A \ln \ell(t),$$

where ℓ is any linear function.

9-19 (a) Newton's second law applied to the driver's body gives

$$a = \frac{F_{\text{total}}}{m}.$$

Their acceleration is

$$a = \frac{v^2}{R}.$$

The total force acting on the driver's body is

$$\begin{aligned} F_{\text{total}} &= mg - N \\ &= \frac{2}{3}mg. \end{aligned}$$

Eliminating a and F_{total} gives

$$v = \sqrt{\frac{2}{3}gR}.$$

(b)

$$\frac{\mathrm{m}}{\mathrm{s}} = \sqrt{\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)\cdot \mathrm{m}}$$

(c) If g is bigger, v gets bigger. That makes sense, because if gravity is stronger, the car has to be going faster to get the same feeling of partial lift-off. If R is bigger, v gets bigger. That also makes sense, because a very large R indicates a nearly flat road, and you can't get any lift-off on a flat road.

9-20 (a) Newton's second law in the vertical direction:

$$T\sin\theta - Mg = 0.$$

Newton's second law in the horizontal direction:

$$T\cos\theta = Mv^2/R.$$

Eliminating T and solving for v gives

$$v = \sqrt{\frac{gR}{\tan\theta}}$$

(b)

$$\frac{\mathrm{m}}{\mathrm{s}} = \sqrt{\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)\cdot \mathrm{m}}$$

(c) If g is bigger, v gets bigger. That makes sense, because if gravity is stronger, Tommy has to whirl the brick faster to get it to rise as high. If R is bigger, v gets bigger. That also makes sense, because intuitively this would be very hard to do with a long rope. If θ gets bigger, v gets smaller. That makes sense, because the faster he whirls the brick, the higher it will rise. As $\theta \to 0$, $v \to \infty$, so Tommy can never get the brick to spin in a perfectly horizontal plane.

9-21 Let r be the distance of the heart from the axis, and let f be the rate of rotation in revolutions per second, $f = 296/360\ \mathrm{s}^{-1}$. The time for one rotation is $1/f$. The speed at which the heart is circling the axis is (circumference)/(time) = $2\pi rf$. We then have $a/g = v^2/r$, or

$$\frac{a}{g} = \frac{4\pi^2 f^2 r}{g} = 1.8.$$

10-2 (a) Since he's on the earth's surface, either $F = mg$ or $F = GMm/r^2$ will work. $mg = 590$ N.
(b) Same as in part a (Newton's third law).
(c) $F = Gm_{Laurie}m_{Roy}/r^2 = 1.2 \times 10^{-7}$ N.
(d) Her distance from the sun is essentially the same as the center-to-center distance between the sun and the earth, 1.5×10^{11} m, so $F = Gm_{sun}m_{Laurie}/r^2 = 0.38$ N.

10-4 (a)

$$\begin{aligned}\frac{g_U}{g_E} &= \frac{m_U/r_U^2}{m_E/r_E^2} \\ &= \left(\frac{m_U}{m_E}\right)\left(\frac{r_E}{r_U}\right)^2 \\ &= 0.91\end{aligned}$$

(b) It's surprising that Uranus's gravitational field is weaker than Earth's, since Uranus is a giant planet. However, "giant" means two things: large in size and large in mass. The large size means that if you were at the visible surface of Uranus (the cloud-tops), you'd be very far away from most of its mass. The exact result of this competition between mass and distance depends on how dense the planet it. Uranus is far less dense than Earth.

10-5 First we take Newton's law of gravity, $F = Gm_1m_2/r^2$, and throw out the constant factor of Gm_1m_2, turning it into the proportionality $F \propto 1/r^2$, or $r \propto 1/\sqrt{F}$. Changing this into a statement about ratios, we have $r_1/r_2 = \sqrt{F_2/F_1} = 10$. At the earth's surface, r equals the earth's radius. To make it ten times bigger, we have to put the rocket nine earth radii above the surface.

10-7 Kepler's law of periods says $T \propto R^{3/2}$, and solving for R gives $R \propto T^{2/3}$. Converting this into a statement about ratios gives $R_1/R_2 = (T_1/T_2)^{2/3} = 2.9$, or 3 if we round off to one sig fig.

10-8 Your path to Mars is half of an elliptical orbit. Kepler's third law says that that the time required for an elliptical orbit is proportional to its long axis to the power 3/2. It is convenient to work the problem in units of astronomical units (a.u.) and years, where 1 a.u. is the average distance from the earth to the sun. The long axis of our ellipse equals the sum of the radii of the earth's and Mars' orbits, which is 1.00 a.u.+1.53 a.u.=2.53 a.u. Comparing with the earth's orbit, which takes one year, we have

$$\frac{\text{time for half of your orbit}}{\text{time for half the earth's orbit}} = \left(\frac{\text{long axis of your orbit}}{\text{diameter of earth's orbit}}\right)^{3/2}$$

The quantities in the denominators are 0.5 years and 2.0 a.u., and solving for the unknown we find that the outward leg of the trip takes 0.70 years.

10-9 Looking up the masses of the earth and moon in the back of the book, we find that the earth is 81.2 times more massive than the moon. The cancellation point must therefore lie much closer to the moon than to the earth. Not 81.2 times closer, though, because gravity depends on the square of the distance. The distance to the moon only has to be $\sqrt{81.2} = 9.01$ times smaller than the distance to the earth, i.e., almost exactly 9/10 of the way from the earth to the moon.

10-10 Not if they are well above the planet's atmosphere, where there is no air friction. They do not need any force other than the planet's gravity to keep them in orbit, any more than our moon needs a power plant to keep it going around the earth. As Kepler figured out, the only possible orbits are circles, parabolas, ellipses and hyperbolas. No spirals! If, however, they were low enough to get some air friction, it would be possible for their orbit to decay in a spiral.

The people who wrote these plots were probably reasoning based on a kind of intuitive Aristotelianism: the incorrect belief that things "naturally" slow down if there is no forward force on them.

In the special case where the orbit is a circle (not an ellipse), we can also relate this to what we learned in chapter 9 about circular motion. Only an inward force is required in order to cause circular motion, and in this situation the inward force is being supplied by the planet's gravity. If the ship is initially moving at the right speed to be in a circular orbit, there is no reason why it would slow down, and "the right speed" means the speed at which the planet's gravity is just the right amount of inward force to make the ship go in a circle.

10-11 Let M be the mass of the mysterious object, and m the mass of one of the stars orbiting around it. The relevant equations are Newton's second law,

$$a = \frac{F}{m} \quad ,$$

Newton's law of gravity,

$$F = \frac{GMm}{r^2} \quad ,$$

and the circular-motion equation

$$a = \frac{v^2}{r} \quad .$$

The acceleration a isn't given, and we don't want to know it either, so let's eliminate it by setting the first and third equations equal to each other. We're now down to two equations:

$$\begin{aligned} \frac{F}{m} &= \frac{v^2}{r} \\ F &= \frac{GMm}{r^2} \end{aligned}$$

The force is also something we don't know and don't care about, so we'll get rid of it by substituting the second equation into the first:

$$\begin{aligned} \frac{GMm/r^2}{m} &= \frac{v^2}{r} \\ \frac{GM}{r^2} &= \frac{v^2}{r} \\ M &= \frac{v^2 r}{G} \\ &\sim 10^{36}\ \text{kg} \quad , \end{aligned}$$

which is something like a million times the mass of the sun!

10-12 The relevant data are in the back of the book. Shortcut: for parts a, d and e, it's accurate enough to assume the distance between the moon and the sun equals the distance from the earth to the sun.

(a) sun on moon: -4.35×10^{20} N;
(b) earth on moon: 1.98×10^{20} N;
(c) sun on earth: -3.52×10^{22} N;
(d) total force on sun: $|\text{part a}|+|\text{part c}| = 3.56\times 10^{22}$ N;
(e) total force on moon: $\text{a} + |\text{b}| = -2.37 \times 10^{20}$ N;
(f) total force on earth: $-|\text{b}|-|\text{c}| = -3.54\times 10^{22}$ N

10-13 The earth's force on the moon points up and to the right, and has magnitude 1.99×10^{20} N, as calculated in problem #2. The sun's distance from the moon is slightly different than in problem #2, but not enough to cause a significant change in the force, which points to the left, and has magnitude 4.36×10^{20} N.

For vector addition, we need to find the x and y components. Let x be to the right, and let y be up.

$$\begin{aligned} F_{sun,x} &= -4.36 \times 10^{20}\ \text{N} \\ F_{earth,x} &= (1.99 \times 10^{20}\ \text{N}) \cos 45^\circ \\ F_{total,x} &= -2.95 \times 10^{20}\ \text{N} \\ F_{sun,y} &= 0 \\ F_{earth,y} &= (1.99 \times 10^{20}\ \text{N}) \sin 45^\circ \\ F_{total,y} &= 1.41 \times 10^{20}\ \text{N} \end{aligned}$$

The magnitude of the total force is $\sqrt{F_{total,x}^2 + F_{total,y}^2} = 3.27 \times 10^{20}$ N.

10-14 Let the positive x axis be toward the sun, and the positive y axis be toward the top of the figure. The sun's force then has a positive x component and a zero y component. Using Newton's law of gravity, we find

$$\begin{aligned} F_{s,x} &= 0.096\ \text{N} \\ F_{s,y} &= 0 \qquad . \end{aligned}$$

For Jupiter's force,

$$\begin{aligned} F_{J,x} &= (GMm/r^2)\cos 95.2529^\circ = -1.015\ \text{N} \\ F_{J,y} &= (GMm/r^2)\sin 95.2529^\circ = 11.044\ \text{N} \end{aligned}$$

The sun's force is about a thousand times smaller than Jupiter's. That makes sense, because although the sun's mass is greater than Jupiter's, the sun is also hundreds of times farther away, and the $1/r^2$ in Newton's law of gravity means that it depends very strongly on distance. As a further check, we also observe that Jupiter's force has a small negative x component and a large positive y; this makes sense geometrically.

The total force has components

$$\begin{aligned} F_x &= F_{s,x} + F_{J,x} = -0.919\ \text{N} \\ F_y &= F_{s,y} + F_{J,y} = 11.044\ \text{N} \end{aligned}$$

Its magnitude is 11.082 N, which is just slightly less than the 11.091 N magnitude of Jupiter's force alone. It makes sense that it's less. If the two forces had been in the same direction, the magnitude of the total would have been more than the magnitude of the individual forces. If they'd been perpendicular, the total would have been given by the Pythagorean theorem applied to the individual forces, and it would have been very slightly greater than Jupiter's force. In reality, the angle between the two forces is not 90 degrees, it's somewhat more than 90 degrees. This causes some cancellation in the x components.

10-15 It's tempting to say that the gravity on the space station is zero, since we see the astronauts floating around on TV. But that can't be true, because Newton's law of gravity implies $g \propto 1/r^2$, which could only be zero if r were infinite. (By the shell theorem, this holds as long as we're on or above the surface of the Earth.) The astronauts just seem weightless because they're orbiting the earth in the same orbit as the space station that surrounds them, so they move along with it. Let r be the radius of the Earth, which, by the shell theorem, is also the distance that appears in Newton's law of gravity for people living on the Earth's surface. Let primed variables refer to the space station, so that in particular r' is its distance from the center of the earth.

$$\begin{aligned} g' &= g\left(\frac{r}{r'}\right)^2 \\ &= g\left(\frac{r}{r+h}\right)^2 \\ &= g\left(\frac{1}{1+h/r}\right)^2 \qquad , \end{aligned}$$

where h is the space station's height above the earth's surface. Since h/r is small, the quantity in parentheses won't be too different from one, and g' won't be very different from g. Looking up r, we find g' to be about 8.8 m/s^2.

10-16 (a) At the center of the earth, your weight is zero by symmetry, since all the outward forces cancel out. That makes one suspect weight would decrease steadily with depth. More quantitatively, the shell theorem says that the part of the earth at radii greater than yours exerts no net force on you, while the part at smaller radii exerts a force as if it was concentrated at the earth's center. If you assume constant density for the earth, then the amount of the earth's mass pulling on you is proportional to r^3, where r is your distance from the earth's center. Your weight is proportional to this mass divided by r^2, so under the assumption of constant density, your weight varies as r^3/r^2, i.e. it is proportional to r.

(b) Since this observation contradicts the logical conslusion of part a, based on the assumption of constant density, the assumption must have been wrong. The earth's density must be greater as you go deeper.

10-17 (a) We're given the period, T, and we know that the condition for a circular orbit is

$$v = \sqrt{gr} \qquad .$$

The gravitational field is

$$g = GM/r^2 \qquad .$$

To get the given period, the speed of the satellite must be related to the radius of its orbit by $v =$ circumference$/T$, or

$$v = \frac{2\pi r}{T} \qquad .$$

We have three equations in three unknowns, v, g, and r. We don't care about v or g, so let's eliminate them first. Plugging the first equation into the third one, we eliminate v. The result is two equations in two unknowns:

$$\begin{aligned} g &= GM/r^2 \\ \sqrt{gr} &= \frac{2\pi r}{T} \end{aligned}$$

Now we substitute the first equation into the second one, giving one equation in one unknown:

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

Solving this for r, the result is

$$\begin{aligned} r &= \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} \\ &= 4.2 \times 10^4 \text{ km} \end{aligned}$$

This is the distance from the center of the earth, not the altitude. Subtracting the radius of the earth, we get an altitude of 3.6×10^4 km.

(b) The gravitational field at this altitude can either be calculated directly from $g = GM/r^2$ or, more easily, by using the inverse-square nature of the field and computing it by comparison with the field at the earth's surface:

$$\begin{aligned} \frac{g}{g_{surface}} &= \left(\frac{\text{radius of earth}}{\text{radius of orbit}}\right)^2 \\ &= \left(\frac{6.4 \times 10^3 \text{ km}}{4.2 \times 10^4 \text{ km}}\right)^2 \\ &= 0.023 \\ &= 2.3\% \end{aligned}$$

10-18 (a) Differentiating twice, we find

$$\begin{aligned} v &= kpt^{p-1} \\ a &= kp(p-1)t^{p-2} \end{aligned}$$

This acceleration is given by Newton's second law:

$$\begin{aligned} a &= F/m \\ &= GMr^{-2} \end{aligned}$$

To relate the two expressions for the acceleration, we need to get them both in terms of the same variable, say r:

$$\begin{aligned} t &= (r/k)^{1/p} \\ a &= kp(p-1)(r/k)^{\frac{p-2}{p}} \\ &= GMr^{-2} \end{aligned}$$

If the powers of r are to be the same, then $(p-2)/p = -2$, or $p = 2/3$. Setting the constants out in front equal to each other, we find $k = (9GM/2)^{1/3}$.

(b) It approaches zero. If you shoot the bullet straight up at exactly escape velocity, then it slows down more and more, but never quite stops and falls back down.

(c) This is a little tricky, because we have an equation for v, but it's in terms of time. We want the initial velocity, i.e., the velocity at the earth's surface, so we need to find the time t_o at which the bullet's distance from the center of the earth is r_o, the same as the earth's radius. The initial time is *not* zero! We already have the relation between t and r, so with a little algebra we find

$$\begin{aligned} t_o &= (r_o/k)^{3/2} \\ v_o &= \frac{2k^{3/2}}{3r_o^{1/2}} \\ &= 1.1 \times 10^4 \text{ m/s} \quad . \end{aligned}$$

10-19 (a) Let M be the mass of the whole planet, and m the mass of some object on the surface. Then

$$\begin{aligned} g &= F/m - v^2/r \\ &= (GMm/r^2)/m - (2\pi r/T)^2/r \\ &= GM/r^2 - 4\pi^2 r/T^2 \quad . \end{aligned}$$

The mass, m, of the object has canceled out, which is good because otherwise we would not be able to solve the problem. Eliminating $M = \rho V = 4/3\pi r^3 \rho$, we get

$$g = \frac{4}{3}\pi G r \rho - \frac{4\pi^2 r}{T^2} \quad .$$

(b) Plugging in, the second term is 3.4×10^{-2} m/s^2. We know that the first term is 9.8 m/s^2, so the fractional change is 0.35%.

(c) $g = 0 = (4/3)\pi G r \rho - 4\pi^2 r/T^2$, so $T = \sqrt{\frac{3\pi}{G\rho}}$, which does not depend on r.

(d) Plug in.

(e) Solving for ρ, $\rho = 3\pi/GT^2 = 1.4 \times 10^{11}$ kg/m^3 for $T = 1$ s.

(f) Plugging $\rho = 10^{17}$ kg/m^3 in to the equation in part c, we get $T \approx 1$ ms. If they went faster, they'd fly apart.

10-23 The sun's gravitational field is

$$g = \frac{GM}{r^2} \quad ,$$

where M is the sun's mass. For the Earth's circular orbit, we have

$$v = \sqrt{gr} = \frac{2\pi r}{T} \quad ,$$

where r is one astronomical unit, and T is one year. Eliminating g and solving for G, we find

$$G = \frac{4\pi^2 r^3}{MT^2} \quad ,$$

which equals $4\pi^2$ in this system of units, since r, M, and T are all defined as 1.

10-24 (a) Yes. The two masses are treated in a symmetric manner in the equation. It doesn't matter which one you label 1 and which one you label 2. (b) The force on the greater mass would be greater, but not greater in proportion to its mass. For example, if the more massive object had four times the mass, the force on it would only be double. According to $a = F/m$, a heavier object would therefore fall with a smaller acceleration. (c) The raindrop's mass is less by a factor of 10000, but the force on it would be smaller by a factor of only 100, so its acceleration would be 100 times greater, i.e. 1000 m/s^2. Raindrops falling from any significant height would be as deadly as bullets! (d) Both halves would suddenly start accelerating more rapidly, and the lighter one would pull away from the heavier one. (e) Instead of raising both masses to the 1/2 power, we'd have to raise them to some power greater than 1, e.g. make the force proportional to the product of the cubes of the masses.

10-25 (a) The cloud's mass M is proportional to r^3. The gravitational force acting on an atom at the cloud's outskirts is proportional to M/r^2, or to r. The acceleration is therefore proportional to r to the first power. (b) Although the acceleration is not constant, we can approximate it as being constant for the purposes of comparing one cloud with another of a different size. For constant acceleration, the distance traveled is proportional to at^2, so the time required to collapse to some fraction of the cloud's original size is proportional to $\sqrt{r/a}$. But since a is proportional to r, the result is independent of r.

10-26 S should have axial symmetry, for if not, then some mass could be moved from a location farther from the line of the vector $\mathbf{g}_S$ to a location closer to that line, and also closer to the point of maximum field strength. This would increase both the magnitude of its contribution to the field and the component of that contribution along the line.

Now that we've established axial symmetry, consider a particular thin ring of mass lying at the surface of S and in a plane perpendicular to the axis. This has some mass $\mathrm{d}m$. Take the point of maximum field to be the origin of a system of spherical coordinates, in which r is the distance from the origin and θ is the angle of the $\mathbf{r}$ vector away from the axis of symmetry. The ring is a curve of constant r and θ. Its contribution to the total field is $(G\,\mathrm{d}m/r^2)\cos\theta$. Since S is the optimal shape, moving this ring to some other point on the surface cannot improve the result. Therefore the surface has an equation of the form $r^2 \propto \cos\theta$. This turns out to be an egg shape with its flatter end at the point of maximum field.

10-27 All that is required is a modification of the limits of integration so that they run from $b-s$ to $b+s$. We then have

$$\begin{aligned} F &= \left(\frac{1}{4b}\right) A \int_{b-s}^{b+s} \frac{\mathrm{d}r}{r}\left(r+\frac{s^2-b^2}{r}\right) \\ &= (\ldots)\left[2s+(s^2-b^2)\left(-\frac{1}{b+s}+\frac{1}{b-s}\right)\right] \\ &= 0 \qquad . \end{aligned}$$

10-28 Let Newton's law of gravity have an exponent r^{-2+c}, where $c \neq 0$. Let's continue to define A using the exponent s^{-2}. For the case calculated explicitly in the book, where m is outside the shell, the integral becomes

$$\begin{aligned} F &= \left(\frac{1}{4b}\right) A \int_{s-b}^{s+b} \frac{\mathrm{d}r}{r^{1-c}}\left(r+\frac{s^2-b^2}{r}\right) \\ &= \left(\frac{1}{4b}\right) Ar^c \left[\frac{r}{1+c}+\frac{s^2-b^2}{-1+c}r^{-1}\right]_{s-b}^{s+b} \end{aligned}$$

The factor of Ar^c is what this entire expression is supposed to boil down to according to the shell theorem. To prove that the theorem fails, it suffices to show that it fails for some specific choice of s and b. Let's take $s=2b$. Then the expression in square brackets comes out to be $2b[1/(1+c)+1/(1-c)]$. For the shell theorem to hold, we would need this to equal $4b$, but if we require this and solve for c, we find that the only solution is $c=0$.

10-29 In the original derivation for the r^{-2} version of the force law, we have

$$F = \frac{A}{4b}\int_{s-b}^{s+b} \frac{r\,\mathrm{d}r}{r^2}\left(r+\frac{s^2-b^2}{r}\right) \qquad ,$$

where $A = GMm/s^2$ and the r^2 in the denominator comes from the force law. To handle the r^1 case, we just change that exponent, obtaining the integral

$$F = \frac{A}{4b}\int_{s-b}^{s+b} r\,\mathrm{d}r\cdot r\left(r+\frac{s^2-b^2}{r}\right) \qquad .$$

Evaluating the integral gives $F=(A/16b)I$, where $I = Q^4-P^4+2PQ(Q^2-P^2)$, $Q=s+b$, and $P=s-b$. Expanding I by brute force makes a bit of a mess. We can simplify things a litle by observing that flipping the sign of b corresponds to interchanging the limits of integration, which should flip the sign of the result; therefore all even powers of b must cancel, and we can skip evaluating them. The result turns out to be $I=16s^3b$, which gives $F=A$ as claimed.

10-30 (a) Combining Newton's law of gravity $F=GMm/r^2$ with Newton's second law $a=F/m$ gives $a=GM/r^2$, and if the motion is to be circular, then this has to equal v^2/r, so

$$\frac{v^2}{r}=\frac{GM}{r^2} \qquad .$$

We then have $v=\sqrt{GM/r}$, and since the volume of a sphere is $(4/3)\pi r^3$, we have $M=(4/3)\pi r^3\rho$ and

$$v = r\sqrt{(4/3)\pi G\rho} \qquad .$$

(b) Solving for r and plugging in numbers gives about 3 km.

10-31 As the probe enters the interior of B, the shell theorem says that the probe feels zero force from B, but feels the same force from A that it would have felt if A's mass had been concentrated at its center. From this we can predict that the probe will curve to the right and speed up. Furthermore, since the probe's initial velocity is small, we can tell that this segment of the probe's orbit will be an ellipse, not a parabola or hyperbola.

The probe will then enter the region where the two clouds' interiors overlap. In this region, the shell theorem tells us that the probe feels no force from either cloud. It moves in a straight line at constant speed.

It will then emerge into the region that is interior to A but exterior to B. It now feels a force from B but none from A. The force from B is now probably more or less directly backward, which will decelerate the probe somewhat, but without more detailed data there is not much more we can say beyond this point.

10-32 (a) If we divide the earth up into a central sphere that's deeper than P and an outer part that's shallower than P, then the shell theorem tells us that the outer part doesn't contribute anything to the field at P, and that the contribution of the inner part is the same as if it had been concentrated at the center. Because mass is proportional to volume, and volume is proportional to the cube of the linear dimensions, the mass of the inner part is 1/8 the mass of the earth. If this was the only factor, then we would have $g_P/g_S = 1/8$. But P is also closer to the center than S by a factor of 2, and since Newton's law of gravity depends on $1/r^2$, this brings the result back up by a factor of 4. The result is $g_P/g_S = 1/2$.
(b) Generalizing the result of part a, we have $g/g_S = r/r_S$. At $r = 0$, this produces the correct result, $g = 0$, because the field must cancel out by symmetry at the earth's center. At $r = r_S$, it gives $g = g_S$, which also makes sense.

10-33 By the shell theorem, the mantle gives no contribution, and we can take the entire mass of the core (including both its layers) to be concentrated at the center. Let m be the mass of the core and r its outer radius.

$$\begin{aligned}\frac{g_G}{g_s} &= \frac{m}{M}\cdot\left(\frac{R}{r}\right)^2\\ &= (0.307)(1/0.55)^2\\ &= 1.01\end{aligned}$$

Surprisingly, it's almost the same as the field at the surface.

10-34 The acceleration is $a = \omega^2 r = (2\pi/T)^2 r = 4.7\times10^{-5}$ m/s^2. If the astronaut stands on the inside of the outer bulkhead, the force pushing up on her feet is what produces her circular motion. This force is $F = ma = 2.8\times10^{-3}$ N. If she adopts the rotating frame, she interprets this "upward" force as canceling the "downward" fictituous force of apparent gravity. This fictitious force is roughly equivalent to the weight of a raisin.

10-36 For the limit $g_1 = \lim_{y\to0}\lim_{b\to0} g$, we know that the inside limit is proportional to $1/y$ from the result of the previous problem. Therefore $g_1 = \infty$.

In the case of $g_2 = \lim_{b\to0}\lim_{y\to0} g$, the inside limit is zero by symmetry for all nonzero values of b. Thus $g_2 = 0$.

We find that the two ways of defining the field on the filament do not agree, and therefore there is no unambiguous way of saying what is the truth of the matter. The notion of the field at a point on such a filament is an idealization, and the idealization does not cleanly map onto actual observables.

10-37 Let the filament be an arc of the unit circle, and let P be the point $(1,0)$. Then the distance r between P and the point at angle θ on the unit circle is given by $r^2 = (1-\cos\theta)^2+\sin^2\theta = 2(1-\cos\theta)$. The infinitesimal arc from θ to $\theta+\mathrm{d}\theta$ has mass proportional to $\mathrm{d}\theta$ and exerts a force $\mathrm{d}F$ on the mass element at P that is proportional to $\mathrm{d}\theta r^{-2}$. The x component of this force is $\sin(\theta/2)\,\mathrm{d}F$. The resulting force in the x direction is, ignoring positive constant factors,

$$\begin{aligned}F_x &= \int_{\theta_1}^{\theta_2} -\sin(\theta/2)\,\mathrm{d}F\\ &= \int_{\theta_1}^{\theta_2} -\frac{\sin(\theta/2)\,\mathrm{d}\theta}{1-\cos\theta}.\end{aligned}$$

Near $\theta = 0$, small-angle approximations give $\sin(\theta/2)\approx\theta/2$, and $1-\cos\theta\approx\theta^2/2$, so the integrand is proportional to $1/\theta$. Therefore the improper integral diverges logarithmically at $\theta = 0$. The integrand is negative everywhere, so the integral diverges to $F_x = -\infty$, i.e., the force acting on the element of mass at P is infinite and to the left (in the direction of concavity).

Based on this result, it would appear that such a filament would behave as if it had infinite stiffness with respect to being bent away from straightness. This result, however, is not necessarily physically realistic, for the reasons explored in the precednig problem.

11-1 No. The mass can't be negative, and neither can v^2.

11-2 If she can run the 100 m dash in about 10 s, then she's going about 10 m/s. If her mass is 60 kg, then her kinetic energy is about 3000 J (one sig fig).

11-3 Converting the speeds to mks, we have 18 m/s and 36 m/s. (a) $(1/2)mv^2 = 2.4 \times 10^5$ J; (b) Four times greater, 9.6×10^5 J. In this sense, driving twice as fast is four times more dangerous, not twice as dangerous as you might think.

11-4 (a) We have to heat it by 20 degrees, melt it, heat it by 100 degrees, boil it, and then heat it by another 37 degrees. The result is

$$\begin{aligned}
&(20)(2.05\text{ J})\\
+&333\text{ J}\\
+&(100)(4.19\text{ J})\\
+&2500\text{ J}\\
+&(37)(2.01\text{ J})\\
=&3.4 \times 10^3\text{ J} \qquad .
\end{aligned}$$

Surprisingly, the majority of the energy is used in a single step of the process, the boiling.

(b) To minimize the amount required, we want the initial temperature of the water to be the maximum, 100°C, and similarly the ice should be at 0°C. At the end of the process, we'll just have one big sample of water at some intermediate temperature, and to minimize the amount of water, we want this temperature to be as low as possible, zero degrees. Conservation of energy gives

$$\begin{aligned}
\Delta E &= 0\\
0 &= -m(100^\circ\text{C})(4.19\text{ J/g/}^\circ\text{C}) + (1\text{ g})(333\text{ J/g})\\
m &= 0.795\text{ g}
\end{aligned}$$

11-5 The total amount of energy required to kill her is

$$(60\text{ kg})\left(\frac{1\text{ J}}{0.24\text{ g}\cdot{}^\circ\text{C}}\right)\left(\frac{10^3\text{ g}}{1\text{ kg}}\right)(6^\circ\text{C}) = 1.5\times10^6\text{ J}$$

Solving the equation $P = \Delta E/\Delta t$ for $\Delta t = \Delta E/P$, we find that she can survive for 7500 s=120 min.

11-6 Force is a transfer of momentum from one object to another. Equivalently, we can describe force as an interaction between two objects. Whichever way we say it, it takes two to tango — you can't have a force unless two objects are involved. There is no force when the bullet is in the air. There is a force when the bullet is passing through the book. This force will cause the bullet to lose momentum and the book to gain some.

Energy, on the other hand, exists at all times in this story. The bullet has kinetic energy before, during, and after the time when it passes through the book. (While it's passing through the book, some of this energy is converted to heat and sound.)

11-9 $(50/3)^{3/7} \approx 3$

11-10 The earth is 81 times closer to the center of mass. They both orbit this point once a lunar month, so the earth's speed is 81 times smaller. Kinetic energy is proportional to mv^2, so the earth's kinetic energy is $(81)(1/81)^2 = 1/81$ of the moon's.

11-12 We're testing conservation of energy,

$$E_{total,i} \quad =? \quad E_{total,f}.$$

The only form of energy we can determine from the photo is kinetic energy, so this amounts to testing whether

$$KE_{1,i} \quad =? \quad KE_{1,f} + KE_{2,f},$$

where the initial time is before the collision and the final time is after the collision is over. Using $KE = (1/2)mv^2$, and taking advantage of the fact that the masses are equal, this becomes

$$v_{1,i}^2 \quad =? \quad v_{1,f}^2 + v_{2,f}^2.$$

Now $v = d/t$, where t is the time between flashes, and the common factor of t^{-2} can be divided out on both sides, so we end up with

$$d_{1,i}^2 \quad =? \quad d_{1,f}^2 + d_{2,f}^2,$$

where d is the distance between one position and the next. That is, the square of the distance is our measure of energy.

But rather than measuring directly from one position to the next, it's more accurate to measure the distance over several of the time intervals in succession. In four flashes, the incoming ball moves 6.0 cm. In the same amount of time after the collision, it moves 2.8 cm, and the target ball moves 5.0 cm.

Since we're leaving out lots of factors that have units, I'll write the numbers below without units.

$$\begin{aligned} 6.0^2 \quad &=? \quad 2.8^2 + 5.0^2 \\ 36.0 \quad &=? \quad 7.8 + 25.0 \\ 36.0 \quad &=? \quad 32.8 \end{aligned}$$

There appears to have been a slight loss of energy, but it's really at the limit of our ability to measure things from the figure. We also know that there will be some extra energy after the collision in the form of heat and sound. The only thing that would really disprove conservation of energy in this collision would be if the final KE was *greater* than the initial KE, and if the increase was significant given the limited accuracy of the measurements.

11-13 (a) The total initial kinetic energy is 0.5 J+0 J, and the total final kinetic energy is 0 J+0.5 J. (b) Before the collision, we see the balls moving toward each other, each moving at a speed of 0.50 m/s. The total kinetic energy is 0.12 J+0.12 J. After the collision, the balls are moving apart at 0.50 m/s. Their velocity vectors have been swapped (i.e. each one reverses itself), but this has no effect on the energies, so the total is still the same.

11-14 (a) Plugging the data into $KE = (1/2)mv^2$ gives a kinetic energy of 23 kilojoules. (b) The hundred-times-greater mass increases the KE by a factor of 100, but KE depends on the square of the velocity, so reducing the speed by 1/100 cuts the kinetic energy by 1/10,000. The combined effect is a factor of 100/10,000=1/100. The boulder has 100 times less KE than the foam.
(c) Our intuition doesn't do well with things that are squared. We don't expect the foam to have so much kinetic energy, but its big speed, squared, results in a really really big KE.

11-15 As in the example of the water faucet, conservation of mass says that a narrower stream corresponds to a higher velocity. The water in the discharge pipe is therefore moving faster than the water in the drive pipe, in inverse proportion to its smaller cross-sectional area. A kilogram of water with a high velocity has a higher KE than a kilogram of water with a low velocity, and therefore it will be able to rise higher, converting that KE into a lot of PE. Note, however, that we're not really getting anything for free here. The amount of water that gets up to the top reservoir is a lot less than the amount of water that was allowed to flow into the big pipe. Most of the water is wasted into the ditch way down at the bottom. People use this device in areas where there's a free source of water, but no electricity to run a pump.

11-16 This is only an order-of-magnitude estimate, so let's make some simplifying approximations. We'll assume the surface area of the oceans is the same as the surface area of the planet ($4\pi R^2$), and that the density of ice is about the same as that of water, 1000 kg/m^3. Then the amount of energy required is

$$\begin{aligned} E &= (1000\ \text{kg/m}^3)(10\ \text{m})(4\pi R^2)(3\times10^3\ \text{J/kg}) \\ &\sim 10^{23}\ \text{J} \end{aligned}$$

Dividing by 10 years (about 3×10^8 seconds), we find that the power is on the order of 1% of the sun's light output. This is a surprisingly small amount. The equilibrium is extremely delicate.

11-17 As in the examples in ch. 1, we don't try to estimate the mass of the wing directly. Instead, we estimate linear dimensions and get mass from that. Let's approximate a fly's wing as a square 3 mm on a side. I don't know the thickness, but since flies' wings are translucent, it must be much less than the thickness of a piece of paper. I'm going to guess that the thickness is 0.1 mm. This gives a volume of about 1 mm^3, and if we assume the same density as water, we get a mass of 10^{-6} kg.

Let's say that a fly's wing beats 100 times a second, which would be of the right order of magnitude to match the fact that the buzzing is in the audible range of human hearing. Then the speed of a wing would be about $(3\ \mathrm{mm})/(10^{-2}\ \mathrm{s}) \approx 0.3$ m/s.

The result for the kinetic energy is about 10^{-7} J.

11-18 As always (see sec. 1.4), we avoid estimating mass or volume directly and instead estimate linear dimensions. Let's say a blade of grass is 0.1 cm thick, 1 cm wide, and 10 cm tall. Then its volume is about 1 cm^3, and if it has the same density as water, then its mass is about 1 g, or 10^{-3} kg.

For the velocity, let's say it grows 1 cm in 10 days. This gives 10^{-8} m/s.

Putting these factors together, we have $KE = (1/2)mv^2 \sim 10^{-19}$ J.

12-1 Kinetic energy depends on how fast the ball is moving. It's moving fastest at the bottom, so that's where it has the maximum kinetic energy. Energy is conserved, so it's trading KE and gravitational energy back and forth, and it has the most gravitational energy when it has the least KE, which is at the top when it's momentarily at rest.

12-2 Kinetic energy doesn't depend on the direction of the motion, so since both pennies start out at the same speed, they both start out with the same kinetic energy. They also start out at the same height, so they have the same gravitational energy and the same total energy. Energy is conserved, so if they start out with a certain amount of energy, that's the same amount of energy they'll always have; their total energies will always be equal to each other. Although the impacts with the ground will happen at different times, we don't care about that. On impact they have the same gravitational energy and the same total energy, so they must have the same kinetic energy on impact as well. They hit the ground with equal kinetic energies and equal speeds.

12-3 Yes, if it's below the reference level you arbitrarily chose as $PE = 0$.

12-4 (a) Magnetic energy plus kinetic energy stays the same, so if it's speeding up, the KE is increasing and the magnetic energy must be decreasing. (b) By similar reasoning, magnetic energy must be increasing as they approach each other.

12-5 (a)

$$\begin{aligned}
PE_i &= PE_f + \text{(energy of boiling)}\\
PE_i &= PE_f + mE_b\\
\Delta PE_i &= -mE_b\\
F_{grav}\Delta y &= -mE_b\\
mg\Delta y &= -mE_b\\
\Delta y &= -\frac{E_b}{g}
\end{aligned}$$

(b)

$$\begin{aligned}
\mathrm{m} &\overset{?}{=} \frac{\mathrm{J/kg}}{\mathrm{m/s^2}}\\
&\overset{?}{=} \frac{\mathrm{kg{\cdot}m^2/s^2/kg}}{\mathrm{m/s^2}}\\
&\overset{?}{=} \frac{\mathrm{m^2/s^2}}{\mathrm{m/s^2}}\\
&= \mathrm{m}
\end{aligned}$$

12-7 Since $KE = (1/2)mv^2$, we have $v \propto 1/\sqrt{m}$, so the ratio of the rates at which the two gases leak out is $\sqrt{14.01/16.00} = 0.9357$, with the oxygen leaking out more slowly.

12-8 Conservation of energy tells us that the initial energy equals the final energy. What times should we pick as initial and final? Our goal is to find out what height the bike has to start from, so the initial time should be when the bike is starting out, at the top of the ramp. The height is determined by the requirement that the bike not drop off the loop, even at the top, so to connect that to the other information, we should pick the final moment to be when the bike is at the top of the loop, at a height of $2r$.

First let's write conservation of energy in its most general form, and then make it more and more specific:

$$\begin{aligned} E_{total,i} &= E_{total,f} \\ PE_i + KE_i &= PE_f + KE_f \\ mgh + 0 &= mg(2r) + \frac{1}{2}mv^2 \\ h &= 2r + \frac{v^2}{2g} \end{aligned}$$

The height depends on the speed v the bike needs at the top of the loop. But how fast *does* the bike have to be moving when it's at the top? If it was just barely about to lose contact with the loop, then the only force on it would be gravity, so its downward acceleration would be $a = F_{grav}/m = (mg)/m = g$. For circular motion, the inward acceleration is v^2/r, so we have $g = v^2/r$, and $v = \sqrt{gr}$. Plugging this into the equation for h, we have

$$\begin{aligned} h &= 2r + \frac{v^2}{2g} \\ &= 2r + \frac{r}{2} \\ &= \frac{5}{2}r \end{aligned}$$

12-9 (a) Each atom in the hoop travels the full circumference of the hoop in time T, so its speed is $2\pi r/T$. Plugging this into $KE = mv^2/2$ gives the result claimed.
(b) As the hoop rolls down the slope, the distance its center travels in one period equals the circumference. Therefore the hoop's center-of-mass speed is the same as the speed calculated in part a, $2\pi r/T$. That means that the hoop's KE due to center-of-mass motion is the same as its KE due to rotation. It's as though the hoop had double its normal inertia.

12-11 (a) If the water lost potential energy and the energy was just destroyed, then energy wouldn't be conserved. The only reason Joule thought there would be a temperature effect was that he was already convinced that energy was conserved. By making surprising predictions and then proving them to be correct, he could convince people that his theory was right.
(b) A joule is defined as the amount of heat needed to raise 0.24 g of water by 1°C. If 0.24 g falls 50 meters, the amount of potential energy released is $(2.4\times10^{-4}\ \text{kg})(9.8\ \text{m/s}^2)(50\ \text{m}) = 0.1$ J. If all this energy is converted to heat, then it heats that particular amount of water by 0.1°C.
(c) We had to assume all the energy went into heating. There are three main effects I can think of. First, the water at the top and bottom of the falls might be moving, in which case there would be a third form of energy involved, kinetic energy. If the pool at the bottom is still, whereas the water above the top is moving rapidly before it goes over the edge, then there would be extra heating. On the other hand, waterfalls are noisy, so energy is released into sound waves at the bottom; this effect would tend to reduce the amount of heat production. The third effect is that water might evaporate in mid-air, further reducing the heat produced.

12-13 At the atomic level, the helium's heat energy is the random motion of the helium atoms. Cylinder B's atoms are moving at the same average speed as A's, since they're at the same temperature, but B has twice as many atoms, so its total KE is twice as much.

12-14 When the sweat evaporates, electrical potential energy is increasing, because as the sweat changes from a liquid to a gas, its molecules, which are attracting each other electrically, get farther away. By conservation of energy, some other form of energy must be decreasing: heat.

If you wipe the sweat off with a towel, you lose the cooling effect. The water evaporates after it's out of contact with you, so it doesn't cool you off. As a method of cooling yourself, it's no more effective than spitting!

12-15 (a) The distance traveled by each atom in one revolution is $2\pi r$, so every atom is traveling at a speed of $v = (2\pi r)/T = \omega r$. Substituting this into $K = (1/2)mv^2$ gives the claimed result.
(b) The quantity ω has units of s^{-1}, inverse sec-

onds, so $m\omega^2 r^2$ has units of kg·s^{-2}·m^2, equivalent to joules.

(c) Let's use lowercase letters for the hoop, uppercase for the block. After descending by a vertical distance h, conservation of energy gives

$$mgh = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}mv^2$$

and

$$Mgh = \frac{1}{2}MV^2 \qquad ,$$

and dividing out the masses and substituting $\omega = v/r$ simplifies these to

$$gh = v^2$$

and

$$gh = \frac{1}{2}V^2 \qquad .$$

These are similar in form to the kinematic equation $v_f^2 = 2ax$ for the final velocity of an object that accelerates from rest, traveling a distance x. Although one equation has h, the vertical distance, and the other has x, the distance along the slope, the relationship between these is the same for the block and the hoop, so it's clear that inserting a factor of 2 on the v^2 side of the equation corresponds to a change in acceleration by a factor of 2.

12-17 Its kinetic energy is equal to its mass-energy minus its resting value,

$$\begin{aligned} K &= \mathcal{E} - mc^2 \\ &= m(\gamma - 1)c^2 \\ &= (8.0\times 10^7\ \text{kg})\left(\frac{1}{\sqrt{1-(1/2)^2}} - 1\right)c^2 \\ &= 1.1\times 10^{24}\ \text{J} \end{aligned}$$

This is a thousand times greater than the total energy content of the world's nuclear arsenals. In other words, the Enterprise is the ultimate weapon of mass destruction. If it crashed into a planet (as it did, in one of the movies), it would destroy all life on the planet's surface.

12-18 (a) In units of 10^{-27} kg, the amount of mass lost is

$$1.67495 - 1.67265 - 0.00091 = 0.00139 \qquad ,$$

i.e., 0.00139×10^{-27} kg. This mass has been converted into energy:

$$\begin{aligned} E &= mc^2 \\ &= (0.00139\times 10^{-27})(3\times 10^8)^2 \\ &= 1.25\times 10^{-13}\ \text{joules} \qquad . \end{aligned}$$

(In the meter-kilogram-second system of units, kilograms multiplied velocity squared give units of joules.)

(b) For this process, the total mass changes from 1.67265 to 1.67586 (again in units of 10^{-27} kg). This is an *increase* in mass, which means that we would have to have a source of energy to make the reaction happen. A free proton has no such source of energy; it's all by itself, so there's nothing for it to get energy from.

12-19 The bottom of the circle is the hardest place in which to avoid skidding, since the speed is highest there, and the orientation of the car is exactly sideways. By conservation of energy, we find that the speed at the bottom is $2\sqrt{gt\sin\theta}$. Three forces act on the car: gravity, static friction, and the normal force. Their sum must be uphill toward the center of the circle, with magnitude mv^2/r. Vector addition gives $F_s/F_N = (v^2/gr + \sin\theta)/\cos\theta = 5\tan\theta = \mu_s$, so $\theta = \tan^{-1}(\mu_s/5)$ is the steepest angle that will not result in skidding. Surprisingly, the answer turns out to be independent of g and r. With hindsight, we can see that this was to be expected based on units; there is no way to combine g and r to give a unitless expression, so they can't enter into the relation between the unitless variables θ and μ_s.

12-20 (a) First we set up an equation that states conservation of energy:

$$\begin{aligned} &E_{total,i} = E_{total,f} \\ &KE_{1,i}+PE_{1,i}+KE_{2,i}+PE_{2,i} = KE_{1,f}+PE_{1,f}+KE_{2,f}+PE_{2,f} \end{aligned}$$

Subtracting $PE_{1,i}$ and $PE_{2,i}$ from both sides gives:

$$KE_{1,i}+KE_{2,i} = KE_{1,f}+KE_{2,f}+\Delta PE_1+\Delta PE_2$$

Since the masses start from rest, we have:

$$0 = KE_{1,f} + KE_{2,f} + \Delta PE_1 + \Delta PE_2$$

We no longer have any variables that occur in the equation with both i and f subscripts, so we can stop writing them without any risk of confusion.

$$\begin{aligned}
0 &= KE_1 + KE_2 + \Delta PE_1 + \Delta PE_2 \\
0 &= \frac{1}{2}Mv^2 + \frac{1}{2}mv^2 + Mg\Delta y_1 + mg\Delta y_2 \\
0 &= \frac{1}{2}(M+m)v^2 + Mg(-h) + mgh \\
v &= \sqrt{\frac{2(M-m)gh}{M+m}}
\end{aligned}$$

(b) Comparing the result of part a with $v^2 = 2ah$ gives $a = (M-m)g/(M+m)$, which is the result found before using Newton's laws.

12-21 (a) Conservation of energy gives

$$\frac{1}{2}(\dot{x}^2 + \dot{y}^2) + y = 0.$$

By the chain rule, $\dot{x} = (\mathrm{d}x/\,\mathrm{d}y)(\mathrm{d}y/\,\mathrm{d}t)$, and substituting this into the equation gives the result claimed.

(b) Under the approximation $y' \ll 1$, the equation of motion becomes

$$y = -\frac{1}{2}\dot{y}^2 y'^{-2}.$$

By the chain rule, $\dot{y} = y' \cdot 4t^3$. Solving this for y' and eliminating y' in the equation above, we end up with $y = -8x^{3/2}$.

(c) Clearly $x = 0$ (remaining at rest) is a solution to the equations of motion. Furthermore, the equations of motion have no explicit dependence on time, so for any function $x(t)$ that is a solution, so is any other function $x(t - t_0)$.

The only remaining issue (and it's a philosophically important one!) is to check that the solution satisfies the equation of motion at the moment t_0 when the bead decides to start moving. Both of the derivatives appearing in the equation of motion, y' and $\dot{y}$, are defined at this time and equal to zero. Plugging these in to the equation of motion gives an expression involving the square of the indeterminate form $\dot{y}/y'$. But if we go ahead and evaluate this indeterminate form, we get

$$4(t-t_0)^3/(-12x^{1/2}) = -\frac{(t-t_0)^3}{3(t-t_0)^{3/2}} = 0,$$

which does satisfy the equation of motion.

12-22 (a) There are three forms of energy present: gravitational PE, electric PE in the rope, and the climber's KE. The climber's KE is zero at both the initial time and the final time, so we can ignore it. We then have

$$PE_{g,i} + PE_{e,i} = PE_{g,f} + PE_{e,f},$$

or

$$\begin{aligned}
PE_{e,i} &= \Delta PE_g + PE_{e,f} \\
0 &= -mgx\sin\theta + \frac{1}{2}kx^2.
\end{aligned}$$

The result is $k = (2mg/x)\sin\theta$.

(b)

$$\frac{\mathrm{N}}{\mathrm{m}} = \frac{\mathrm{kg{\cdot}m/s^2}}{\mathrm{m}}$$

(c) The result increases with m, g, and θ, and decreases with x. A heavier climber, greater gravity, or higher angle will all make it harder to stop the climber in time, so the dependence on these variables makes sense. A greater x makes it easier to stop the climber in time, so this also makes sense.

13-1 (a) The amount of KE to be converted into heat is the same in both cases, so the amount of work that has to be done is the same in both cases. Since work equals force times distance, and the work and force are the same, the stopping distance is the same.

(b) Since the stopping distances are equal, if you watched videos of the two boats stopping, the videos would look exactly the same except for a rescaling of the time. That is, the slower boat's motion would look like the faster one's, but in slow motion. Therefore the time for the slower boat to stop will be greater.

13-2 (a) +, foot moves same direction as foot's force on ground
(b) +, force and motion are both forward
(c) −, force is backward, motion is forward
(d) −, force is up and motion is down

13-3 (a) The definition of work is a (mechanical) transfer of energy. Changing the direction of the positive coordinate axis has no effect on the work. Energy is a scalar, so directions are irrelevant.
(b) Reversing the direction of the positive coordinate axis reverses the sign of both F and d, so $W = (-F)(-d) = Fd$ comes out the same.

13-5 In a solid, the molecules are packed in place, and aren't free to twist around very much. In a liquid, they can rotate more freely. The microwaves exert the same force F on a molecule, regardless of whether it's in a solid or a liquid, but it's not just the force that matters, since $W = Fd$. In the solid, d is small, so little energy is transferred.

13-6 The mechanical work done by the hand equals the area under the curve between A and B. I counted about 136 squares. Each square is 20 N tall and 0.05 m wide, so it represents an area of 1 N·m = 1 J, and the total area is 135 J. Accounting for the loss of 30% of this energy, and solving for $v = \sqrt{2K/m}$, we find 85 m/s.

13-7 (a) $W = \int F\,\mathrm{d}x = a(\ln x_2 - \ln x_1) = a\ln(x_2/x_1)$
(b) With the given data, the work per power stroke comes out to be 470 J. The engine's power is the work done per unit time, which is $6 \times 2400 \times 470$ J/s $= 1.1 \times 10^5$ W = 150 horsepower.
(c) If the exponent is −1, as assumed above, the work only depends on x_2/x_1. The argument fails for $F \propto x^{-1.4}$, because then the integral would be of the form $x_2^{-0.4} - x_1^{-0.4}$, which doesn't just depend on x_2/x_1.

13-8 (a) We want to find the area under the graph from $x = 0$ to $x = cL$. This area can be split up into a rectangle plus a triangle sitting on top of it. To make all the math easier, let's start by leaving out the factors of L and T_o, since these just set the scale on each axis; we'll start by calculating everything as if we had $L = 1$ and $T_\text{o} = 1$, and then throw those two factors back in at the end. In these fake units, the point at the graph at the corner, where the muscle can't contract anymore, has coordinates $(c, 1-c)$. The rectange has width c, height of $1 - c$, and area $c(1 - c)$. The triangle has base c, height c, and area $(1/2)c^2$. The total area is $c - c^2/2$. Putting the factors of $L = 1$ and $T_\text{o} = 1$ back in, we find that the work is $W = T_\text{o}L(c - c^2/2)$.
(b) T has units of newtons, L meters, and c is unitless, so the result has units of newton-meters, which is equivalent to joules.
(c) The $c = 0$ indicates a muscle that is "frozen" in its extended position and can't flex at all; physically it shouldn't be able to do any work, and mathematically we do get $W = 0$ in this case. In the $c = 1$ case, the area would be that of a complete triangle with width L and height T_o, giving an area of $(1/2)T_\text{o}L$, and this is indeed what we get if we plug in $c = 1$ into the general equation.
(d) For a cylinder, we have $V = LA$, and with $T_\text{o} = kA$ the result for the work becomes $kV(c - c^2/2)$. The whole thing is strictly proportional to the muscle's volume, V, which is why bodybuilders talk about "bulking up."
(e) Converting the given data to mks, 200 cm^3 = 2×10^{-4} m^3, and $k = 10^6$ N/m^2. Plugging in, we get 96 J.

13-9 (a) $KE = \frac{1}{2}mv^2$, so if the average KE is a fixed number, the average velocity is proportional to $m^{-1/2}$. The lighter He atom is moving $\sqrt{8} = 2.8$ times faster.
(b) He atoms are always speeding up and slowing down due to collisions, but because of their small mass, their average speed is very high, and a few unusual ones are moving very very fast — faster than escape velocity, meaning they leave the earth and never fall back.

13-10 (a) As Weiping raises the rock, the sign of the force and the sign of d are the same (both are up), so she does 1 joule of work. On the way down, her force is up and the motion is down, so she does −1 joule of work. The total is zero.

Bubba's force was in the same direction as the motion during both halves of his motion, since, unlike gravity, the force of friction reversed directions on the way back. Bubba did 2 J of work.

(b) Work is defined as a transfer of energy. Weiping added gravitational PE to the rock on the way up, but on the way back down, she took it back out. The rock ends up with the same gravitational PE it started with, so by the definition of work, we verify that her work on the rock is zero.

As Bubba slides the table, he's adding heat energy to it. When he slides the table back in the opposite direction, he's not refrigerating it! Therefore, based on the definition of work, we know that the total work he does is not zero. This agrees with the result from part a.

13-11 $PE = -GMm/r$, where r is the distance from the center of the earth. The earth-moon distance is so much greater than the earth's radius that the initial PE is essentially zero, and the result is pretty much the same as falling from infinitely far away. That is, almost all the speed is gained near the end of the drop. The amount of PE lost is

$$\begin{aligned} PE_i - PE_f &\approx 0 - PE_f \\ &= GMm/r_{earth} \qquad . \end{aligned}$$

Whatever is lost as PE is gained as KE, so for the final speed v, we have $\frac{1}{2}mv^2 = GMm/r_{earth}$. The mass, m, of the grasshopper cancels out, since all objects fall with the same acceleration, regardless of their masses. (The acceleration is not a constant 9.8 m/s^2, however, in this situation.) Solving for v, we have $v = \sqrt{2GM/r_{earth}} = 1.1 \times 10^4$ m/s.

13-12 To apply conservation of energy, we start by writing down the law in its most generic form,

$$E_{total,i} = E_{total,f} \qquad ,$$

and we then make it more specific by putting in the actual forms of energy involved,

$$KE_i + PE_i = KE_f + PE_f \qquad ,$$

and then substituting in the correct expressions for the kinetic and potential energies,

$$\frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f} \qquad ,$$

Now we have to decide what moments in time will be initial and final. Initial should clearly be the beginning of the motion, when the object is at distance r, so in terms of the variables used in posing the question, we have $r_i = r$ and $v_i = v$, the escape velocity, giving

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f} \qquad .$$

But what should we choose as the final moment in time? We're interested in the case where the object just barely has enough energy to keep from falling back. If the object had less than this amount of energy, it would fly away from the planet, slow down, and finally stop at some distance before falling back down. The greater the energy, the greater the distance at which it stops. When the energy is just barely enough (the velocity is exactly equal to escape velocity), we can think of this distance as being infinite, or at least extremely large. Thus, $r_f = \infty$ and $v_f = 0$. Dividing by a very large number gives a very small result, so dividing by infinity gives zero. That means that both terms on the right are zero. In other words, the total energy is zero.

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0 \qquad ,$$

It may sound strange to say that this object, moving at a gazillion miles an hour, has zero energy, but that's because the object has a huge negative potential energy to start with, which is canceled by its huge initial kinetic energy.

Solving for the escape velocity v, we have

$$v = \sqrt{\frac{2GM}{r}} \qquad .$$

The direction of the velocity doesn't matter, because the only requirement was for the object to have a certain amount of energy, and energy is a scalar. This is counterintuitive: most people would expect that it would be easier to make the object escape if it was shot straight up. One way to think about it is that if the object is launched at an angle just barely above the horizontal, it has the disadvantage that at the beginning, it's hardly getting any farther away from the earth, but it has the advantage that the force of gravity is almost perpendicular to its direction of motion, so gravity is doing almost no (negative) work on it, and is therefore hardly slowing it down.

The mass of the object has canceled out, which is why we can speak of a single escape velocity, rather than different escape velocities for different objects.

No, it is not valid to think of the object as escaping because it is moving too fast for gravity to act on it. The force of gravity only depends on the masses and the distance between them, not on their motion.

13-13 (a) If it's moving at exactly escape velocity, then its kinetic energy will approach zero as r approaches infinity. Therefore, the total energy is $U + K = 0$. We then have

$$\begin{aligned}0 &= U+K\\ &= -\frac{GMm}{r}+\frac{1}{2}mv^2\\ &= -\frac{GMm}{r}+\frac{1}{2}m\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2\end{aligned}$$

Now we need to separate the variables r and t.

$$\begin{aligned}\frac{GMm}{r} &= \frac{1}{2}m\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2\\ \frac{2GM}{r} &= \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2\\ \sqrt{\frac{2GM}{r}} &= \frac{\mathrm{d}r}{\mathrm{d}t}\\ \mathrm{d}t &= \sqrt{\frac{r}{2GM}}\ \mathrm{d}r\end{aligned}$$

Integrating both sides, we get

$$t + \text{constant} = \sqrt{\frac{2}{9GM}}\, r^{3/2} \qquad .$$

The constant of integration just indicates when we start our clock, so we're free to pick it to be zero. Solving for r, we then have

$$r = \left(\frac{9GM}{2}\right)^{1/3} t^{2/3} \qquad .$$

The velocity is

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \left(\frac{4GM}{3}\right)^{1/3} t^{-1/3}$$

(b) For $r(t)$, we have

$$\begin{aligned}\left(\frac{\mathrm{J}\cdot\mathrm{m}}{\mathrm{kg}^2}\cdot\mathrm{kg}\right)^{1/3}\mathrm{s}^{2/3} &= \left(\frac{(\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^2)\cdot\mathrm{m}}{\mathrm{kg}^2}\cdot\mathrm{kg}\right)^{1/3}\mathrm{s}^{2/3}\\ &= \mathrm{m} \qquad .\end{aligned}$$

The units of the expression for $v(t)$ differ only by a factor of 1/s, so they work out as well.

(c) As t gets large, the velocity approaches zero, which makes sense.

13-14 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2t^2$.
(b) $P = dW/dt = dK/dt = ma^2t$.
(c) Four times more.

13-15 (a) The distance traveled in time Δt is $d = v\Delta t$, so the work done is $\Delta E = Fd = Fv\Delta t$, and $P = \Delta E/\Delta t = Fv$.
(b) The vertical forces cancel, so $F_N = mg$, and $F_{s,max} = \mu_s F_N = \mu_s mg$. To burn rubber, the engine has to supply $P > F_{s,max}v = \mu_s mgv$. The critical speed is $v = P/(\mu_s mg)$.
(c) W/N = J/N·s = N·m/N·s = m/s.
(d) Physically, a more powerful engine should be able to burn rubber at even higher speeds; mathematically, increasing P increases v. Physically, better traction makes it harder to burn rubber; mathematically, increasing μ_s decreases v. Increasing m or g also has the effect of increasing traction, so the dependence on these two variables also makes sense, for similar reasons.
(e) (Plug in.)
(f) The power consumed by friction is $Fv \propto v^3$. At top speed, 100% of the engine's power is being consumed by fighting air friction. At the lower speed, this is decreased by $(40/176)^3 \approx .01$.

13-16 The force is the derivative of the potential energy with respect to distance. (Actually it's minus the derivative, but all that tells you is the direction of the force, and we know it's attractive anyway.)

$$\begin{aligned}\frac{\mathrm{d}PE}{\mathrm{d}r} &= \frac{\mathrm{d}}{\mathrm{d}r}\left(gr^{-1}e^{-r/a}\right)\\ &= g\frac{\mathrm{d}}{\mathrm{d}r}\left(r^{-1}e^{-r/a}\right)\\ &= g\left[r^{-1}\frac{\mathrm{d}}{\mathrm{d}r}\left(e^{-r/a}\right)+e^{-r/a}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^{-1}\right)\right]\\ &= g\left[r^{-1}\left(-\frac{1}{a}\right)\left(e^{-r/a}\right)+e^{-r/a}\left(-r^{-2}\right)\right]\\ &= -gr^{-2}e^{-r/a}\left(\frac{r}{a}+1\right)\end{aligned}$$

13-17

$$\begin{aligned}E_i &= E_f\\ KE_i+PE_i &= KE_f+PE_f\\ 0+PE_i &= KE_f+PE_f\\ KE_f &= PE_i-PE_f\\ &= -\int_{r_i}^{r_f} kr^{-3}\,\mathrm{d}r\\ &= \frac{k}{2}\left(r_f^{-2}-r_i^{-2}\right)\end{aligned}$$

13-18

$$\begin{aligned}W &= \int_0^\infty F\,\mathrm{d}x\\ &= \int_0^\infty be^{-cx}\,\mathrm{d}x\\ &= b/c\end{aligned}$$

13-19

$$\begin{aligned}E_{total,i} &= E_{total,f}\\ PE_i &= KE_f+PE_f\end{aligned}$$

We can use the equation $PE = -GMm/r$ for the potential energy, and this equation is defined so that $PE = 0$ at $r = \infty$, so actually PE_i is zero. The potential energy is really the sum of the potential energies for the interaction of the comet with the two stars. Writing v for the final velocity, we have

$$\begin{aligned}0 &= \frac{1}{2}mv^2-\frac{Gm_1m}{b/2}-\frac{Gm_2m}{b/2}\\ v &= \sqrt{\frac{4G(m_1+m_2)}{b}}\end{aligned}$$

13-20

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= |\mathbf{a}||\mathbf{b}|\cos\theta_{ab}\\ \theta_{ab} &= \cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)\\ &= 12.9^\circ\end{aligned}$$

13-21

$$\begin{aligned}W_{wp} &= \int_0^x F_\parallel\,\mathrm{d}x'\\ &= \int_0^x (a+bx')\,\mathrm{d}x'\\ &= ax+\frac{1}{2}bx^2\end{aligned}$$

By Newton's third law, the plane's force on the air is of the same strength, but in the opposite direction, and therefore $W_{wp} = -W_{pw}$.

13-22 The dot product was defined as

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB} \qquad ,$$

which only depends on the magnitudes of the two vectors and the angle between them. Rotating everything has no effect on the magnitudes, or on the angle of one vector relative to another, so the dot product comes out the same regardless of rotation.

13-23 The definition $\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB}$ implies immediately that $\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$, since the cosine of zero is 1. So for example if two vectors $\mathbf{A}$ and $\mathbf{B}$ only have nonvanishing x components, their dot product equals A_xB_x. Also, the cosine of 90 degrees is zero, so the other possibilities, such as $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}}$, equal zero. Now suppose we have a general dot product such as

$$\mathbf{A}\cdot\mathbf{B} = (A_x\hat{\mathbf{x}}+A_y\hat{\mathbf{y}}+A_z\hat{\mathbf{z}})\cdot(B_x\hat{\mathbf{x}}+B_y\hat{\mathbf{y}}+B_z\hat{\mathbf{z}}) \qquad .$$

The rule $\mathbf{P}\cdot(\mathbf{Q}+\mathbf{R}) = \mathbf{P}\cdot\mathbf{Q}+\mathbf{P}\cdot\mathbf{R}$ allows us to break this into nine terms, of which the three nonvanishing ones add up to $A_xB_x + A_yB_y + A_zB_z$.

13-24 The tricky thing here is that the shell theorem works for forces, but not necessarily for potential energies. The shell theorem tells us that

$$\begin{aligned} F &= \frac{GmM_{inside}}{r^2} \\ &= \frac{Gm}{r^2}\int_0^{r'} \rho\cdot 4\pi r'^2\,\mathrm{d}r' \qquad , \end{aligned}$$

where the integral can be visualized as dividing the part of the cloud at $r' < r$ into thin spherical shells. This force equals $\mathrm{d}U/\,\mathrm{d}r$, so

$$\frac{1}{4\pi Gm} r^2 \frac{\mathrm{d}U}{\mathrm{d}r} = \int_0^r \rho r'^2\,\mathrm{d}r$$

and by differentiating on both sides we find

$$\frac{1}{4\pi Gm}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}U}{\mathrm{d}r}\right) = \rho r'^2 \qquad .$$

13-26 As suggested in the hint, let $k = 1$, $m = 1$, $g = 1$, and also let the force of kinetic friction equal 1.

When the object is released at a, it travels a distance a while being acted on by a frictional force 1, so the work done by friction is a. This dissipates the entire initial potential energy $(1/2)ka^2 = (1/2)a^2$. This gives $a = 2$.

When the object is released at b, it travels to a position x on the far side of equilibrium at which the force of friction is canceled by the force of the spring. We're given that b is the maximum distance from which this can occur, so the force of friction must be at its maximum. Since we're given that $\mu_s = \mu_k$, this is the same as the force of kinetic friction, which is 1. The magnitude of the spring's force is $|kx| = x$, so $x = -1/$ In summary, the object travels from b to -1.

Applying conservation of energy in this case, we have $PE_i = PE_f + W_{friction}$, or $(1/2)b^2 = (1/2)1^2 + (b+1)\cdot 1$. This gives a quadratic equation, $(1/2)b^2 - b - 3/2 = 0$. The positive solution is $b = 3$.

The result is that $b/a = 3/2$.

13-27 (a) To lift the bag at constant velocity, the person must cancel out the force of gravity mg. The work done is therefore about $mgd =$ 50 kJ. (b) The calculation comes out the same, and this is what we expect based on conservation of energy if the pulley is frictionless: the bag still gains the same amount of energy. (In reality there would be some friction in the pulley, so the force would be slightly more than the given value, the work would be slightly greater, and the extra energy would go into heating the pulley and the rope.)

13-28 Let $\mathbf{v} = c\hat{\mathbf{x}} + d\hat{\mathbf{y}}$, where c and d are to be found. We don't expect to be able to determine both c and d uniquely, because for any $\mathbf{v}$ that works, so will any nonzero scalar multiple of it. That is, we only expect to be able to find the ratio c/d, which expresses how big c is in proportion to d.

We want $\mathbf{v}$ to be perpendicular to $\mathbf{u}$, so $\mathbf{u}\cdot\mathbf{v} = 0$, or $ac + bd = 0$. Solving for the unknown ratio gives $c/d = -b/a$ (assuming $a \neq 0$). Concretely, a choice that works is $c = b$ and $d = -a$, so that

$$\mathbf{v} = b\hat{\mathbf{x}} - a\hat{\mathbf{y}}.$$

(Writing it this way also works in the special case where $a = 0$.) To check that this is a solution, we can compute $\mathbf{u}\cdot\mathbf{v} = ab - ba = 0$, as desired.

13-29 From the work-kinetic energy theorem, the work you do on the ball equals the change in kinetic energy of the ball. $\Delta\mathrm{KE} = (1/2)m(3v)^2 - (1/2)mv^2 = 4mv^2$, and the work done on the ball is $W = Fx$, where x is the distance over which your foot is in contact with the ball. Setting these quantities equal gives $x = 4mv^2/F$.

14-1 $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$

14-2 The people plus the boat are, roughly speaking, a closed system, since the water does not exert much horizontal force on the boat. Momentum is conserved in a closed system. At the beginning, nothing is moving and there is zero momentum. If the two people's momenta didn't add up to zero, then the boat would have to move to conserve momentum. They should make sure their momenta add up to zero, i.e., they should move in opposite directions, and the heavier person has to move more slowly.

14-4 (a) $KE = \frac{1}{2}mv^2$, so $v = \sqrt{2KE/m} =$ 130 m/s.

(b) $p = mv = 1.3$ kg·m/s.

(c) same as part b (conservation of momentum)

(d) $KE = \frac{p^2}{2m} = 0.2$ J. The recoiling gun is harmless, because it would take more energy than this to break bones, sever tissues, etc.

14-5 The momentum transferred to the rocket by the exhaust pushing on it equals the area under the curve. Each rectangle represents a momentum of (0.5 MN) × (1 s) = 5×10^5 N·s, and there are about 54 rectangles under the curve, for a total Δp of $54 \times 5 \times 10^5$ N·s $= 2.7 \times 10^7$ N·s. It starts with zero momentum, so its final velocity equals the momentum transferred divided by its mass, which is 6.8×10^3 m/s.

14-6 We are given that the KE is on the order of that of a well-thrown rock, which might be 10 J. This energy would be dissipated in your body as heat, and you would probably feel it. The momentum would be $p = \sqrt{2\,m\,KE}$, which comes out to be about 10^{-13} kg·m/s if we put in the mass of a proton. This is a tiny amount of momentum, and you definitely wouldn't notice it, i.e. you would not be knocked back perceptibly.

14-7 We start with

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} \qquad .$$

Since each object has the same value of m, m can be factored out of the sums, giving

$$x_{cm} = \frac{m\,(x_1 + x_2 + \ldots)}{m\,(1 + 1 + \ldots)} \qquad .$$

If you add up a series of 1's for each object, you just get the number of objects, N, so this equals

$$x_{cm} = \frac{x_1 + x_2 + \ldots}{N} \qquad ,$$

which is the average of all the x's.

14-9 Horizontal momentum is conserved, since there are no external horizontal forces. Let's pick masses for the planes of 1 kg and 5 kg – although the real masses must be thousands of times bigger, it won't matter as long as the ratio is 5. Let the positive x axis point east and the positive y axis north, and measure angles counterclockwise from the x axis. The magnitudes of their initial momenta are $|p_L| = 300$ kg·mi/hr and $|p_J| = 750$ kg·mi/hr. The x components are

$$\begin{aligned} p_{L,x} &= 300 \text{ kg·mi/hr} \qquad \text{and} \\ p_{J,x} &= (750 \text{ kg·mi/hr})(\cos 225^\circ) \\ &= -530 \text{ kg·mi/hr} \qquad , \end{aligned}$$

giving

$$p_{total,x} = -230 \text{ kg·mi/hr} \qquad .$$

Similarly,

$$\begin{aligned} p_{L,y} &= 0 \qquad , \\ p_{J,y} &= -530 \text{ kg·mi/hr} \qquad \text{, and} \\ p_{total,y} &= -530 \text{ kg·mi/hr} \qquad . \end{aligned}$$

Since momentum is conserved, their common direction of motion after the crash has to be the same as the direction of their total momentum vector before the crash, which was at an angle of

$$\tan^{-1}(-530/-230) = 67^\circ \text{ or } 247^\circ \qquad .$$

(Both angles are arctangents of -530/-230.) Since the x and y components are both negative, 247° is the correct angle. That's the same as 23° west of south.

14-10 The simplest way to do this is by using the center-of-mass frame of reference. If the big object's mass is essentially infinite compared to the little one's ($M >> m$) then the center of mass of the whole system is always located at the big one's center of mass, so the center-of-mass frame is the frame fixed to the big object. (After the collision, the big object recoils a tiny bit from the collision, so you could say that it's no longer at rest in the c.m. frame, but because its mass is assumed to be infinite compared to the little one's mass, its recoil is negligible.)

In this frame, it's not the high-mass object with velocity v coming along and hitting the low-mass object, it's the other way around. The low-mass object, moving at velocity $-v$, hits the high-mass one. This is like hitting a brick wall: the recoil of the wall is negligible. The only thing moving before or after the collision is the small object, and since we're assuming no KE is converted into anything else, the small object must recoil at velocity $+v$, i.e., in the opposite direction but at the same speed. (Reasoning slightly more rigorously, it's not quite true that the big object doesn't recoil at all, but the momentum it absorbs equals $-2mv$, and its KE after the collision is $p^2/2M = 2m^2v^2/M$, which is extremely small because M is so big compared to m.)

In the center-of-mass frame, then, we've proved that the small object enters with velocity $-v$ and exits with $+v$. To convert back to the original frame of reference, we add v to all the velocities. In the original frame, the small object starts with velocity 0, and ends up with $2v$.

14-11 The elegant way to do this is to switch to the center of mass frame. In that frame, the balls are initially moving toward each other at velocities $v/2$ and $-v/2$. After the collision, we can maximize the amount of heat released if we simply make the two masses stick together and stop. That means that *in the center of mass frame*, it's possible to convert all the kinetic energy, $2(1/2)m(v/2)^2 = (1/4)mv^2$, into heat and sound. Back in the original frame of reference, the balls are still moving after the collision, so not all the kinetical energy has been lost, but the amount of heat and sound is still the same amount we found in the c.m. frame: $(1/4)mv^2$, or half the initial kinetic energy of the single moving ball.

As an alternative, a brute force technique would be to require conservation of momentum and energy and crank out the result. Let the final velocities be b and c. Then

$$\begin{aligned} mv &= mb + mc \\ \frac{1}{2}mv^2 &= \frac{1}{2}mb^2 + \frac{1}{2}mc^2 + Q \qquad , \end{aligned}$$

where Q is the amount of heat and sound that comes out. Solving the first equation for $b = v-c$ and substituting into the second, we find $Q = m(vc-c^2)$. Using calculus to find the maximum, we set $0 = \mathrm{d}Q/\,\mathrm{d}c = mv - 2mc$, giving $c = v/2$, and then the rest of the result carries through as in the center-of-mass approach.

14-13 Let's use M for the mass of the ball, U for its initial velocity, and V for its final velocity. Similarly, let m be the mass of the pin, and v its final velocity. Conservation of momentum and energy give

$$MU = MV + mv$$

and

$$\frac{1}{2}MU^2 = \frac{1}{2}MV^2 + \frac{1}{2}mv^2 \qquad ,$$

two equations in the two unknowns V and v. We aren't given V and don't want to find it, so let's eliminate it. Solving the momentum equation for $V = U - (m/M)v$ and substituting into the second equation, we find

$$\begin{aligned} \frac{1}{2}MU^2 &= \frac{1}{2}M[U-(m/M)v]^2 + \frac{1}{2}mv^2 \\ U^2 &= [U-(m/M)v]^2 + \frac{m}{M}v^2 \\ 2\frac{m}{M}U &= \left(\frac{m}{M} + \frac{m^2}{M^2}\right)v \\ v &= \frac{2U}{1+m/M} \\ &= 3.6\ \text{m/s} \qquad . \end{aligned}$$

14-14 Let V be the large mass's initial velocity, V' its velocity after the collision, and v the small one's final velocity. Conservation of momentum gives

$$3mV = 3mV' + mv \qquad .$$

Since no energy is converted into heat and sound,

$$\frac{3}{2}mV^2 = \frac{3}{2}mV'^2 + \frac{1}{2}mv^2 \qquad .$$

Both equations can be simplified by dividing out the factor of m, and let's set $V = 1$ to cut down on the writing. We then have $v = 3(1 - V')$ from conservation of momentum, and substituting this into the energy equation gives $4V'^2 - 6V' + 2 = 0$. This quadratic has solutions 1 and 1/2, meaning V and V/2. The first solution describes the case where the balls miss and fly past one another. The second one is the one we want: $V' = V/2$. Since it's positive, we find that the more massive ball "follows through."

14-17 (a) Suppose that at some time, the rocket has mass m remaining. Now an infinitesimal time passes, and the rocket's mass changes to $m + \mathrm{d}m$ (with $\mathrm{d}m < 0$) because it expels $-\,\mathrm{d}m$ worth of exhaust. This causes its velocity to increase by the infinitesimal amount $\mathrm{d}v$. We can now require conservation of momentum, and the algebra is reduced if we do this in the center of mass frame, in which the rocket is initially at rest, i.e., its velocity is increasing from 0 to $0+\mathrm{d}v$.

$$0 = (-\,\mathrm{d}m)(-u) + (m + \mathrm{d}m)\,\mathrm{d}v$$

Discarding the product of the two infinitesimals $\mathrm{d}m\,\mathrm{d}v$, and separating variables, this becomes

$$-\frac{\mathrm{d}m}{m} = \frac{\mathrm{d}v}{u} \qquad ,$$

and integration gives

$$-\ln m + c = \frac{v}{u} \qquad .$$

The interpretation of the minus sign is that as the mass gets smaller, the logarithmic term gets less negative, the left-hand side gets bigger, and the velocity goes up. If we now switch back to the original frame of reference, in which the velocity is not zero but v, the effect is only to add a constant onto v, and that can be absorbed into the constant of integration. Back in this frame, the only time when we have $v = 0$ is initially, when $m = m_i$, and this can be used to determine the constant of integration $c = \ln m_i$. For the final velocity, we have

$$v = u \ln \frac{m_f}{m_i}$$

(b) Solving for m_i/m_f, we have

$$\begin{aligned} m_i/m_f &= e^{v/u} \\ &= \exp(3 \times 10^7 \text{ m/s / } 4000 \text{ m/s}) \\ &= 2 \times 10^{434} \qquad . \end{aligned}$$

This is a ridiculously huge amount of fuel, vastly more than the mass of the entire galaxy.

14-18 (a) Let T be the tension in the rope at the corner of the table. For the bottom part of the rope with length x that hangs vertically, we let positive be down (in the direction of motion), so

$$\begin{aligned} F &= \frac{\mathrm{d}p}{\mathrm{d}t} \\ m_{bottom} g - T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(m_{bottom}\frac{\mathrm{d}x}{\mathrm{d}t}\right) \\ \frac{x}{L} mg - T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{x}{L} mg \frac{\mathrm{d}x}{\mathrm{d}t}\right) \\ xg - \frac{L}{m} T &= \frac{\mathrm{d}}{\mathrm{d}t}\left(x\frac{\mathrm{d}x}{\mathrm{d}t}\right) \\ xg - \frac{L}{m} T &= \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + x\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \qquad . \end{aligned}$$

For the top part, with length $L - x$, we let positive be toward the edge of the table (again, in the direction of motion).

$$\begin{aligned} F &= \frac{\mathrm{d}p}{\mathrm{d}t} \\ T &= \frac{\mathrm{d}}{\mathrm{d}t}\left[\left(1 - \frac{x}{L}\right) m \frac{\mathrm{d}x}{\mathrm{d}t}\right] \\ \frac{LT}{m} &= -\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + (L - x)\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \qquad . \end{aligned}$$

Adding the results of the two calculations, we have $xg = La$, which is the result that was claimed.

(b) For $x = 0$, nothing is hanging over the edge, so the upward normal force of the table completely cancels the force of gravity. For $x = L$, the whole rope is hanging, and there is no upward normal force from the table at all; the result becomes $a = g$. In between these two extreme cases, the upward normal force of the table only partially cancels gravity.

(c)

$$\begin{aligned}\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} &= \frac{g}{L}x\\ \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(be^{ct}\right) &= \frac{g}{L}be^{ct}\\ bc^2e^{ct} &= \frac{g}{L}be^{ct}\\ c^2 &= \frac{g}{L}\\ c &= \pm\sqrt{\frac{g}{L}}\end{aligned}$$

(d) This is true because the derivative of a sum is the sum of the derivatives.

(e) The most general solution is of the form

$$x = qe^{ct} + re^{-ct} \qquad ,$$

where q and r are constants with units of meters, and we'll say c represents the positive root $\sqrt{g/L}$. At $t = 0$, both exponentials equal one, and we have

$$x_o = q + r$$

and

$$\begin{aligned}0 &= \frac{\mathrm{d}x}{\mathrm{d}t}\\ &= qce^{ct} - rce^{-ct}\\ &= (q-r)c\\ q &= r\end{aligned}$$

The solution is therefore $q = r = x_o/2$, or

$$x = x_o\left(e^{ct} + re^{-ct}\right)/2 \qquad ,$$

which if you like you can write using the hyperbolic cosine.

14-19 (a)

$$\begin{aligned}p &= m\gamma v\\ &= \frac{mv}{\sqrt{1-v^2}}\\ p^2\left(1-v^2\right) &= m^2v^2\\ p^2 &= \left(m^2+p^2\right)v^2\\ v &= \frac{p}{\sqrt{m^2+p^2}}\end{aligned}$$

(b) At low velocities, the second term inside the square root is negligible, and we have

$$\begin{aligned}v &\approx \frac{p}{\sqrt{m^2}}\\ &= \frac{p}{m} \qquad ,\end{aligned}$$

which is the nonrelativistic result.

(c) For very large momenta, the m^2 term is negligible compared to the p^2 term, so

$$\begin{aligned}v &= \frac{p}{\sqrt{p^2}}\\ &= \pm 1,\end{aligned}$$

which is the speed of light in natural units.

(d) Converting to SI units, we have

$$v = \frac{p}{\sqrt{m^2 + p^2/c^2}}.$$

14-20 The change in momentum is $\Delta p = \int_0^T F(t)dt$. Also, since the object is at rest at $t = 0$, $\Delta p = p_f$ (which we'll just call $p(t = T)$):

$$\begin{aligned}p(T) &= \int_0^T At^2 dt\\ &= \frac{1}{3}AT^3\end{aligned}$$

14-21 (a) By drawing a circular arc from where the block was originally hanging to the maximum height it reaches, you should be able to see that the maximum height reached is $H = L(1-\cos\theta)$.

(b) We will have to do this in two parts: first, find the speed of the bullet/block system after the collision by using conservation of energy, and second, to find the initial speed of the bullet by using conservation of momentum. Note that you cannot just set the final potential energy of the bullet/block system equal to the initial kinetic energy of the bullet because some mechanical energy was lost during the collision.

Call the bullet/block system "BB." The final PE of BB is equal to the kinetic energy of BB right after the collision. Therefore,

$$\begin{aligned} KE_{\text{i,BB}} &= PE_{\text{final,BB}} \\ \frac{1}{2}(m+M)v^2 &= (m+M)gH \\ \frac{1}{2}v^2 &= gL(1-\cos\theta) \\ v &= \sqrt{2gL(1-\cos\theta)} \end{aligned}$$

Now, using conservation of momentum, we can solve for the initial speed of the bullet, v_0:

$$\begin{aligned} mv_0 &= (m+M)v \\ v_0 &= \frac{m+M}{m}\sqrt{2gL(1-\cos\theta)} \end{aligned}$$

(c) $PE_{\text{final,BB}} = (m+M)gL(1-\cos\theta)$ and $KE_{\text{bullet}} = mgL(1-\cos\theta)$, so the ratio is $(m+M)/m$. For $M \gg m$, this is basically equal to M/m, which is a number much greater than 1. This means that almost all of the mechanical energy of the bullet is lost in such a collision.

14-22 It's best to do part (c) before parts (a) and (b):

(c) Suppose that, initially, the truck is moving with velocity v and the car with velocity $-v$. Then, from conservation of momentum, $(2M)v + (M)(-v) = (2M+M)v_f$, where v_f is the final velocity of both vehicles after the inelastic collision. Solving for v_f, we have $v_f = v/3$.

(a) The change in momentum of the truck is $\Delta p_t = (2M)v_f - (2M)v = -(4/3)Mv$.

(b) The change in momentum of the car is $\Delta p_c = (M)v_f - (M)(-v) = +(4/3)Mv$.

(d) $\text{KE}_\text{i} = (1/2)(2M+M)v^2 = (3/2)Mv^2$; $\text{KE}_\text{f} = (1/2)(2M+M)v_f^2 = (1/9)\text{KE}_\text{i}$. Therefore, 8/9 = 89% of the initial KE was lost as a result of the collision.

15-1 If the rider speeds up the back wheel, then it's gaining angular momentum. By conservation of angular momentum, this requires that the bike's body "nose up." Slowing down the back wheel makes it "nose down." This allows the rider to control the orientation of the bike so that it makes a two-point landing.

15-2

No. It is momentarily at rest, but not at equilibrium, because there is a force on it, the force of gravity, which is not canceled out by any other force. Another way of putting it is that it has $p = 0$ at that moment, but not $\mathrm{d}p/\,\mathrm{d}t = 0$, which is what equilibrium means.

15-3 No, it just means that any torques that do act are adding up to produce zero *total* torque. For instance, a merry-go-round rotating with constant angular velocity has torques acting on it, but the motor's torque and the torque due to friction are just canceling out to produce zero total torque. Of course it is also *possible* that no torques at all are acting (e.g., the earth has no torques on it at all), but it would be incorrect to infer that the merry-go-round's motion at constant angular velocity implied that there were no torques on it.

15-4 (a) $L = (4\pi/5)MR^2/T = 1.1\times10^{42}\ \text{kg}\cdot\text{m}^2/s$.

(b) Angular momentum is conserved because no outside torques are acting, so the angular momentum is the same.

(c) Solving the same equation for T gives $T = 150$ s.

15-5 (a) Alice could be in a frame of reference in which Cathy isn't moving. For instance, if Alice and Cathy were in an airplane together, it would be natural for both of them to adopt the frame of reference of the plane. Bob could be using the frame of reference of the dirt.

(b) Your opinions about angular momentum will depend on what point in space you arbitrarily chose as the axis. Let's say Dong is sitting on some grass. Bob might be using the center of the spinning Earth is his axis, while Alice uses the spot Dong is sitting on.

15-6 No. The angular momentum of a particle is $p_\perp r$, so if the particles differ in the distance, r, from the axis or in their direction of motion, they can have different angular momenta. Furthermore, they could even have the same $|\mathbf{r}|$ and $|\mathbf{p}|$, but if they were located in different directions relative to the axis, or their directions of motion were different, they could have different angular momenta. As an example of the effect of the direction, a particle has zero angular momentum if it is moving straight in towards the axis or directly away from it ($\theta = 0$ or $180°$).

15-7 You can get the maximum torque by setting everything up with θ=90°, giving

$$\begin{aligned}\tau &= rF\sin\theta \\ &= rmg \\ &= 270\ \text{N}\cdot\text{m} \qquad .\end{aligned}$$

15-10 The total torque required is the torque needed to lift his arm plus the torque needed to lift the weight. Since his arm is not changing, she needs to tell him to set things up so that the torque needed to lift the weight is also the same as before. Torque is $rF\sin\theta$, and since he is decreasing r by a factor of 17/33, she should tell him to increase the mass by a factor of 33/17 so that the increased weight force will compensate for the decreased r. The new mass should be 2.1 kg × 33/17 = 4.1 kg.

15-11 For the purpose of calculating gravitational torques, we can treat each branch as if its weight were concentrated at its center of mass. The longer branch's center of mass is twice as far from the trunk, so the r is doubled. The gravitational force is also twice as much, so $\tau = rF$ is four times greater.

15-12 Astrid is right and Michelle is wrong. The force on the ball acts along the line of the string. This line is perpendicular to the ball's motion, so the string does no work on the ball, and the ball must therefore keep the same kinetic energy. After all, if the ball's kinetic energy was going to change, conservation of energy says that some other form of energy would have to change in the opposite direction, but the pole and the string aren't storing or releasing any energy.

The flaw in Michelle's reasoning is that the string's force on the ball is not straight toward the center of the pole, since the string is wrapping around the pole. The string makes a nonzero torque on the ball, and the ball's angular momentum changes. The ball is transferring angular momentum to the planet Earth.

15-13 Basically we have a situation with two equations in two unknowns. There is an equation that says the total torque on the foot is zero, and another equation that says the total force on the foot is zero. The two unknowns are the tension in the calf muscle, T, and the force exerted by the shinbone, F_s. We can however simplify the solution by choosing the axis to be the place where the shinbone attaches to the foot. With this choice, the shinbone makes no torque on the foot, and the torque equation thus has only the tension in the calf muscle as an unknown. There are two torques on the foot, a clockwise torque exerted by the muscle and a counterclockwise torque from the floor. The floor's upward force equals the person's weight, so if we use plus signs for clockwise torques the torque equation is

$$Ta - Wb = 0 \qquad ,$$

which is easily solved to give

$$T = W\frac{b}{a} \qquad .$$

Letting positive forces be upward, the force equation is

$$T - F_s + W = 0 \qquad ,$$

so

$$\begin{aligned}F_s &= T + W \\ &= W\frac{b}{a} + W \\ &= \left(\frac{b}{a} + 1\right)W \qquad .\end{aligned}$$

15-14 (a) We have two unknowns, F and T, so we're going to need to find two equations to solve for them. Setting the total vertical force on the ruler equal to zero gives one equation (+=up),

$$F - mg + T = 0 \qquad ,$$

and setting the total torque equal to zero gives a second one (axis on the right end, counterclockwise=+),

$$-bF + \frac{L}{2}mg = 0 \qquad .$$

Solving these, we find

$$F = \frac{L}{2b}mg$$
$$T = \left(1 - \frac{L}{2b}\right)mg$$

(b) In the case of $b = L$, we have $F = T = mg/2$, which makes sense because the finger and the string are at the ends of the rod, each supporting half the rod's weight. For $b = L/2$, we get $T = 0$ and $F = mg$, because the rod is balanced on the finger, which means the string is slack.

(c) For $b < L/2$, the tension in the string becomes negative, which doesn't make sense — you can't push with a rope. In this situation, there is no possible equilibrium, and the rod just flips over in the counterclockwise direction.

15-15 (a) There are three forces on the ladder, F_1, F_2, and its own weight, which can be thought of as acting at its center for the purpose of calculating torques. If it's going to remain at rest, the total horizontal force, total vertical force, and total torque must all be zero. Taking the axis of rotation as the center of the ladder, the weight force makes no torque. If ϕ was less than θ, both torques would be of the same sign (clockwise), so they could not cancel.

(b) The three conditions for equilibrium are:
zero total horizontal force:

$$F_1 \cos\phi - F_2 = 0 \qquad (1)$$

zero total vertical force:

$$F_1 \sin\phi - mg = 0 \qquad (2)$$

zero total torque:

$$\frac{L}{2}F_2 \sin\theta - \frac{L}{2}F_1 \sin(\phi - \theta) = 0$$

or

$$F_2 \sin\theta - F_1 \sin(\phi - \theta) = 0 \qquad (3)$$

For our numerical check, let's arbitrarily choose $mg = 1.000$ N. L is irrelevant, since it canceled out in eq. (3). Eq. (2) gives $F_1 = 1.118$ N. Eq. (1) gives $F_2 = 0.500$ N. As advertised, eq. (3) checks out.

15-16 In the previous problem, we got three equations, which involved several variables we want to get rid of: F_1, F_2 and m. It should be possible to solve three equations in three unknowns. We can simplify things right off the bat by realizing that eq. (2) is useless; it introduces another variable, m, which we don't know and don't want to know. So we're better off just ignoring it and considering this as two equations, (1) and (3), in two unknowns. The general strategy in solving simultaneous equations like this is to solve one of the equations for one of the unknowns, then substitute into the other equation(s) to eliminate it. Let's solve eq. (1) for $F_2 = F_1 \cos\phi$. Eliminating F_2 from eq. (3), we have

$$F_1 \cos\phi \sin\theta - F_1 \sin(\phi - \theta) = 0 \qquad ,$$

which we can now simplify by dividing by F_1, resulting in

$$\cos\phi \sin\theta - \sin(\phi - \theta) = 0 \qquad .$$

We've eliminated all the unknowns, and we now have a relationship involving only θ and ϕ, the things we're interested in. Using the trig identity as suggested, $\sin(\phi - \theta) = \sin\phi\cos\theta - \sin\theta\cos\phi$, so our equation becomes

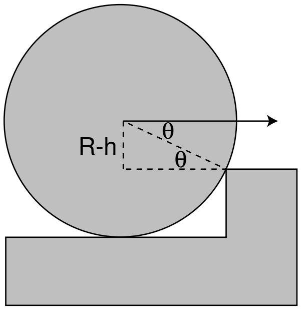
$$\cos\phi \sin\theta - \sin\phi\cos\theta + \sin\theta\cos\phi = 0 \qquad .$$

Simplifying, we get $\tan\phi = 2\tan\theta$, so $\phi = \tan^{-1}(2\tan\theta)$. As claimed, m and L are irrelevant.

15-17 (a) Let h be the height of the step, R the radius of the wheel, m its mass, and F the minimum force required. When the wheel is just about to lift off, the normal force of the ground under it goes to zero, so the only forces involved are gravity, F, and the force exerted by the corner of the step. If we're going to get it over the step, we have to make enough torque on it so that the total torque is clockwise. The minimum force corresponds to the case where the total torque is zero, so that any greater force would do the job. It's natural to consider the corner of the step as the axis, since that's the point around which the wheel will acutally rotate. With this choice, the only nonzero torques are the torque we're applying and the torque of gravity. Defining the angle θ shown in the figure, we have

$$\begin{aligned} F\sin\theta &= mg\sin(90^\circ-\theta) \\ F\sin\theta &= mg\cos\theta \\ F &= \frac{mg}{\tan\theta} \\ &= mg\frac{\sqrt{R^2-(R-h)^2}}{R-h} \\ &= mg\frac{\sqrt{1-(1-h/R)^2}}{1-h/R} \end{aligned}$$

(b) For $h=R$, the denominator equals zero, and the force required becomes infinite. This makes sense, because in this situation, you're just trying to pull the wheel straight into the wall of the step.



15-18 Conservation of energy gives

$$\frac{1}{2}mv^2-\frac{GMm}{r}=E \qquad .$$

Intuitively, this tells us that if r is going to get small, v has to get big. But notice that v is squared. That means that v doesn't really increase all that fast, because not much of a change in v is required in order to make v^2 get quite a lot bigger. At the same time, the bomb has to conserve angular momentum, which also requires that as r gets smaller, v has to get bigger to compensate. However, the angular momentum is $L=mvr\sin\theta$, with no squares or anything, and if v isn't getting bigger all that fast, then the only way to pull the angular momentum back up is if $\sin\theta$ increases. But $\sin\theta$ can only get as big as 1, when the bomb is flying horizontally, so that puts a limit on how small r can get:

$$\begin{aligned} L &= mvr\sin\theta \\ &< mvr \end{aligned}$$

From now on, let's just use r to mean the minimum value, rather than the clumsy sumbol r_{min}. Then solving the conservation of energy equation for v gives

$$v=\sqrt{\frac{2E}{m}+\frac{2GM}{r}} \qquad ,$$

so

$$\begin{aligned} L &= mvr \\ &= mr\sqrt{\frac{2E}{m}+\frac{2GM}{r}} \\ 0 &= \left(\frac{2E}{m}\right)+\frac{2GM}{r}-\frac{L^2}{m^2r^2} \\ 0 &= \left(\frac{E}{GMm}\right)r^2+r-\frac{L^2}{2GMm^2} \qquad , \end{aligned}$$

which we can solve using the quadratic formula to give the result originally claimed.

15-19 What makes this a little hard is that not only do we not know the weights w and W of the ship and bottle, we also can't tell the exact positions of their centers of mass. Let the bottle's center of mass lie at a distance H to the right of the fulcrum, and let the ship's center of mass be h to the right of the ship's stern. Suppose that for our first measurement we slide the ship around until its stern is a distance b_1 to the right of the fulcrum. Let the axis be at the fulcrum, so that the (unknown) force from the fulcrum doesn't make a torque. Then setting the total torque equal to zero gives

$$(b_1+h)w+HW-S_1\ell=0 \qquad ,$$

where S_1 is the scale's reading in this position, and ℓ is the scale's distance from the fulcrum. This is one equation in four unknowns (h, H, w, and W). It wouldn't do us any good to write down the equation that says that the total force is zero, because that would bring in another unknown: the force at the fulcrum.

It should now be fairly clear that a solution exists. Every time we slide the ship around to a new b, we generate a new equation, without introducing any new unknowns. It is therefore definitely possible to get a unique solution for all four unknowns simply by sliding the ship to locations b_1, b_2, b_3, and b_4.

Actually it turns out that only two locations are necessary. At a second location,

$$(b_2 + h)w + HW - S_2\ell = 0 \qquad .$$

Subtracting the two equations gives $(b_1 - b_2)w - (S_1 - S_2)\ell = 0$, which has only the unknown w in it.

15-20

(a) The equilibrium occurs where the interaction energy has a local minimum. Setting $a = 1$ as suggested, we have

$$\begin{aligned} \frac{\mathrm{d}U}{\mathrm{d}r} &= \frac{\mathrm{d}}{\mathrm{d}r} k\left(r^{-12} - 2r^{-6}\right) \\ &= k\left(-12r^{-13} + 12r^{-7}\right) \qquad . \end{aligned}$$

Setting this equal to zero,

$$\begin{aligned} 12r^{-13} &= 12r^{-7} \\ r^{-13} &= r^{-7} \\ r &= 1 \qquad . \end{aligned}$$

Since $a = 1$, this is the same as saying $r = a$.
(b) Making a graph shows that this is a minimum of U rather than a maximum. (You could also use the second derivative test.)
(c) At $r = \infty$, the equation gives $U = 0$. At $r = a$, we have $U = k(1 - 2) = -k$. The difference in energy is k.

15-21 The equal-area law is just a statement of conservation of angular momentum, which will still be true, because gravity is still directly toward the sun, and still makes zero torque on the planet.

The elliptical orbit law would have to be junked, and there's no easy way to figure out what the shape of the orbits would be instead. The arguments in section 4.1.6 depended completely on the $1/r^2$ nature of the gravitational force.

The law of periods would have to change, but it wouldn't be all that difficult to rework it. If the force law was $F = GMm/r^7$, we would have $a = F/m = GM/r^7 = v^2/r$, and $v = 2\pi r/T$, leading to $T \propto r^4$.

15-22 Let T be the time required for one rotation. Then the velocity of a ball rolling without slipping is $v = 2\pi R/T$, and we have $L/p = (4\pi mR^2/5T)/(2\pi mR/T) = (2/5)R$.

15-23 There is a kinetic frictional force on the ball, which reduces its momentum by Δp. Associated with this force there is also a torque. Just as force is the rate of change of momentum, torque is the rate of change of angular momentum, and since the ratio of torque to force is R, the ratio of ΔL to Δp is also R: $\Delta L = R\Delta p$. The ball has no angular momentum initially, so ΔL is the same as the final angular momentum L. We then have

$$\begin{aligned} \frac{p}{L} &= \frac{5}{2R} = \frac{p_\mathrm{o}}{R\Delta p} - \frac{1}{R} \\ \frac{5}{2} &= \frac{p_\mathrm{o}}{\Delta p} - 1 \\ \Delta p &= \frac{2}{7}p_\mathrm{o} \qquad , \end{aligned}$$

so the ball loses 2/7 of its initial momentum, and is left with 5/7.

15-25 (a) $\tau = mg(\ell/2 + b) - mg(\ell/2 - b) = 2mgb$ (b) This is the same as the torque we would get if the full weight $2mg$ acted at the center of mass, which is at a distance b from the fulcrum.

15-26

$$K = \frac{L^2}{2I}$$

$$\Delta\Theta = \omega_o\Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$W = \tau \Delta \Theta$$

15-27 Let the origin of our coordinate system be at the center of mass. By the assumption that the axes are parallel, we can ignore the existence of a dimension parallel to the axes, i.e., we can assume without loss of generality that the mass distribution is confined to a certain plane perpendicular to both axes. Offsetting the axis by a displacement $\mathbf{h}$ lying in this plane then amounts to simply making the change $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{h}$ in the integral for the moment of inertia.

$$\begin{aligned} I &= \int (\mathbf{r}+\mathbf{h})^2 \, \mathrm{d}m \\ &= \int r^2 \, \mathrm{d}m + 2\mathbf{h} \cdot \int \mathbf{r} \, \mathrm{d}m + h^2 \int \mathrm{d}m \\ &= I_\mathrm{o} + 0 + mh^2 \qquad , \end{aligned}$$

where the cross term vanishes because it's the same integral that occurs in the definition of the center of mass, and the center of mass is at the origin.

15-28 (a) Let's use subscripts r for the rope, g for gravity, and n for the floor's normal force. In the case where the block topples, it rotates and its center of mass rises as it tips up and over. In the case where it slides without tipping, neither of these is supposed to happen, so the vertical forces must cancel, and the torques must cancel. Cancellation of the vertical forces gives $F_n = mg$. The torques are most easily calculated using $\tau = r_\perp F$, with the axis taken to be at the center of the box. Note that in the condition where the box is not tipping, the normal force is actually being distributed over the whole bottom surface of the block, so it doesn't actually have a single, well-defined value of $r_\perp$; however, as soon as the tipping starts, the only point of contact is at $r_\perp = a$. The torques are therefore $\tau_r = bF_r$ and $\tau_n = amg$. If the box is just about to tip, i.e., the force from the rope has its maximum value, then these torques cancel, and $F_r = (a/b)mg$. The dependence on a/b makes sense, because a squat box with a large value of a would be more stable, allowing F_r to be greater. (What we've actually calculated here is the value of F_r for which the box would just barely *start* to tip. Once the box started to tip, we could ask ourselves whether it might then manage to right itself again. The answer is no, because once it starts to tip, both of the $r_\perp$ values start to decrease, but their ratio is still a/b.)

(b) The force of friction is $F_f = \mu F_n = \mu mg$. (In the case of static friction, this is the maximum force, which is what we're interested in because we want to see if we can unstick the box and get it to slide without tipping. In the case of kinetic friction, this is simply the value of the frictional force.) We now have a torque from friction, which is in the same clockwise direction as the torque from the rope. Cancellation of the torques gives $bF_r + b\mu mg = amg$, yielding $F_r = (a/b - \mu)mg$.

(c) The answer to part b, unlike the one from part a, could come out negative under certain circumstances. The negative solution would be unphysical, and the interpretation would be that it would be impossible to accelerate the box without making it tip over. This happens for $b > a/\mu$. That makes sense. A big value of b means a tall box, which would be less stable. In the limit $\mu \rightarrow 0$, we get $b \rightarrow \infty$, meaning that there is no maximum value of b, recovering the behavior of the frictionless case.

15-29 (a) Let the axis be the hinge, and let the length of the bar br ℓ. The gravitational torque can be calculated by pretending that it acts at the bar's center of mass, with $r = \ell/2$. Cancellation of the torques gives $(1/2)\ell mg = \ell T \sin\theta$, and the result is $T = mg/(2\sin\theta)$.

(b) To minimize T, we want to maximize $\sin\theta$, which means letting θ be 90°. In other words, the cable is vertical.

(c) As θ approaches zero, the tension approaches infinity. No matter how hard you pull, you can't support the weight of the bar with a horizontal cable.

15-30 (a) Let the hinge be the axis, and set the total torque on one bar equal to zero. With this choice of axis, the normal force at the hinge makes zero torque, so we have

$$\frac{1}{2}\ell mg\cos\alpha = \ell T\sin(\beta-\alpha) \qquad .$$

This is a single equation in two unknowns, α and T, so we're not ready to solve anything yet.

The brute-force technique would be to set the total horizontal and vertical force on this bar equal to zero, and this would give us two more equations, while introducing one more unknown (the horizontal normal force at the hinge). We would then have three equations in three unknowns.

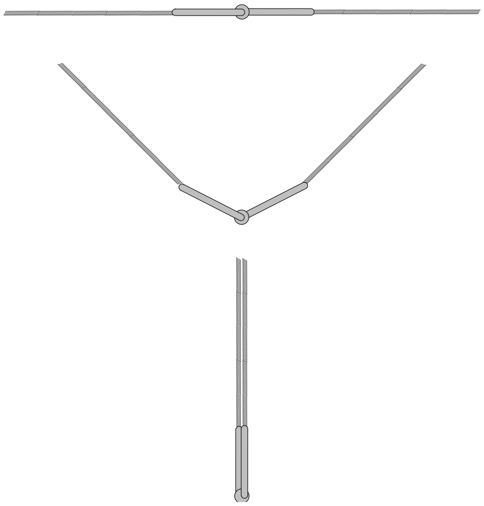
A trick that simplifies the calculation is to set the total vertical force on *both* bars equal to zero, in which case the normal force at the hinge becomes an internal force that we can ignore. We then have

$$2T\sin\beta = 2mg \qquad .$$

Solving the second equation for T and plugging in to the first equation, we get

$$\begin{aligned}\frac{1}{2}\cos\alpha &= \frac{\sin(\beta-\alpha)}{\sin\beta}\\ \frac{1}{2}\cos\alpha &= \frac{\sin\beta\cos\alpha-\cos\beta\sin\alpha}{\sin\beta}\\ \alpha &= \tan^{-1}\left(\frac{1}{2}\tan\beta\right) \qquad .\end{aligned}$$

(b) For $\beta = 0$, 45°, and 90°, we get $\alpha = 0$, 27°, and 90°, respectively. All three of these make sense physically. In particular, the intermediate case has $\alpha < \beta$, which looks intuitively right.



15-31 (a) For the Earth's rotation, $\omega = \Delta\theta/\Delta t = (2\pi)/(24\ \text{hr}) = 7.3\times10^{-5}\ \text{s}^{-1}$, and similarly for its yearly revolution around the sun, $\omega = 2.0\times10^{-7}\ \text{s}^{-1}$.
(b) The acceleration is $\omega^2 r$. The factor of ω^2 would tend to make the acceleration due to rotation bigger by a factor of $365^2 = 1.3\times10^5$, but the r factor would contribute a factor of (radius of orbit)/(radius of Earth) $= 2.3\times10^4$ in the opposite direction. The former effect is bigger, so the acceleration due to rotation is bigger. It's surprising, however, that the two numbers would even end up being near the same order of magnitude, given how different the ω and r values were.
15-32 Let the mass of each atom be m, and the center-to-center distance between neighboring atoms b.

$$\begin{aligned}\frac{I_{cross}}{I_{line}} &= \frac{m\cdot0^2+4mb^2}{m\cdot0^2+2mb^2+2m(2b)^2}\\ &= \frac{2}{5}\end{aligned}$$

15-33 Taking the vector cross product of these two vectors using the component form for the cross product, I got $-2\hat{\mathbf{x}}-5\hat{\mathbf{y}}+\hat{\mathbf{z}}$, with all the numbers in units of kg·m^2/s. As a check on my result, I decided to use $|\mathbf{r}\times\mathbf{p}| = |\mathbf{r}||\mathbf{p}|\sin\theta_{rp}$. The left-hand side is $\sqrt{30} = 5.48$. The angle between the two vectors can be found from $\mathbf{r}\cdot$

$\mathbf{p} = |\mathbf{r}||\mathbf{p}|\cos\theta_{rp}$, which gives $\theta = 2.40$ radians, resulting in $|\mathbf{r}||\mathbf{p}|\sin\theta = 5.49$, which checks out to within rounding errors.

15-34 The easiest way to find such a vector is to take the cross product.

$$\begin{aligned}(\mathbf{A}\times\mathbf{B})_x &= A_yB_z - B_yA_z = -3\\ (\mathbf{A}\times\mathbf{B})_y &= A_zB_x - B_zA_x = 6\\ (\mathbf{A}\times\mathbf{B})_z &= A_xB_y - B_xA_y = -3\end{aligned}$$

So our vector is $-3\hat{\mathbf{x}} + 6\hat{\mathbf{y}} - 3\hat{\mathbf{z}}$. We could also use any other vector lying along the same line, i.e., any scalar multiple of this one. I checked my result by computing its dot product with each of the two given vectors, verifying that it was zero in each case.

15-35 The definition says that the magnitude of the cross product is given by $|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}|\sin\theta_{AB}$, and its direction is perpendicular to $\mathbf{A}$ and $\mathbf{B}$, and right-handed. The magnitude part is easy to check, since the magnitude has the property $|k\mathbf{B}| = |k||\mathbf{B}|$. If k is positive, then the direction of $k\mathbf{B}$ is the same as that of $\mathbf{B}$, so the direction comes out as claimed. If k is zero, then $\mathbf{A}\times(k\mathbf{B}) = 0$, which is the same as $0\mathbf{A}\times\mathbf{B} = 0$. If k is negative, then $k(\mathbf{A}\times\mathbf{B})$ is in the opposite direction compared to $\mathbf{A}\times\mathbf{B}$, but this agrees with the direction of $\mathbf{A}\times(k\mathbf{B})$, since flipping the direction of $\mathbf{B}$ to $k\mathbf{B}$ causes the result of the right-hand rule to flip as well.

15-36 The definition of the vector cross product in terms of the components is $(\mathbf{A}\times\mathbf{B})_x = A_yB_z - B_yA_z$, and similarly for y and z. Reversing the roles of $\mathbf{A}$ and $\mathbf{B}$ interchanges the positive and negative terms on the right, so the sign flips.

15-37 Since it's not associative in general, we expect that almost any set of three vectors, chosen at random, will provide a suitable counterexample. I decided to use combinations of the unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$, since they're easy to calculate with. The first example I tried was $\hat{\mathbf{x}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}})$, which demonstrated Murphy's law by being one of those rare examples in which associativity *does* hold, because both $\hat{\mathbf{x}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}})$ and $(\hat{\mathbf{x}}\times\hat{\mathbf{y}})\times\hat{\mathbf{z}}$ are zero. My next example worked: $\hat{\mathbf{y}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}}) = \hat{\mathbf{y}}\times\hat{\mathbf{x}} = -\hat{\mathbf{z}}$, but $(\hat{\mathbf{y}}\times\hat{\mathbf{y}})\times\hat{\mathbf{z}} = 0\times\hat{\mathbf{z}}$ equals zero, which is different.

15-38 (a) The cross product inside the parentheses results in a vector, and the second cross product results in a vector. (b) The cross product gives a vector, and then dotting it with the other vector gives a scalar. (c) The dot product gives a scalar, but you can't take a cross product of a scalar with a vector. This one is nonsense.

15-39 (a) Since it's infinitesimally small, we can aproximate it as a rectangle. Its dimensions are $\mathrm{d}r$ and (using the definition of radian measure) $r\,\mathrm{d}\theta$, so the area is $\mathrm{d}A = r\,\mathrm{d}r\,\mathrm{d}\theta$.

(b) Now we get an (approximately) rectangular box with height $\mathrm{d}z$. Multiplying by the height of the box gives a volume $\mathrm{d}V = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$.

(c) The flavor of this problem is very similar to that of the calculation of the moment of inertia of a disk, $(1/2)Mr^2$, in section 4.2.5. In fact, we can save quite a bit of time by slicing the cone into disks and just integrating over all the disks. Each disk has infinitesimal height $\mathrm{d}z$ and mass $\mathrm{d}M$. It turns out to be convenient to pick a coordinate system in which $z = 0$ is the tip of the cone, and the cone extends to $z = h$ (i.e., if we want to visualize the positive z axis as being up, then the cone is upside-down). The advantage of this choice is that the surface of the cone is defined by the simple equation $r = (b/h)z$.

$$\begin{aligned}I &= \int \frac{1}{2}\,\mathrm{d}Mr^2\\ &= \int_0^h \frac{1}{2}(\rho A\,\mathrm{d}z)r^2\\ &= \int_0^h \frac{1}{2}\rho\pi r^2\,\mathrm{d}zr^2\\ &= \frac{1}{2}\rho\pi\left(\frac{b}{h}\right)^4\int_0^h z^4\,\mathrm{d}z\\ &= \frac{\pi}{10}\rho b^4h\end{aligned}$$

Our answer is in terms of ρ, but we're supposed to express it in terms of M. We can either look up the formula for the volume of a cone, $(1/3)\pi b^2h$, or evaluate it using a similar integral, omitting a factor of $(1/2)\rho r^2$ in the integrand.

The final result is

$$I = \frac{3}{10} M b^2 \qquad .$$

It makes sense that the result depends only on b, not h, since changing h without changing the mass would not change how far any of the mass was from the axis. It also makes sense that the numerical factor of 3/10 in front is smaller than the factor of 1/2 for a disk; a cone of radius b has more of its mass closer to the axis than does a disk of radius b.

15-40 This is exactly the same as the calculation of the moment of inertia of a cube in section 4.2.5, except for the limits of integration.

$$\begin{aligned}
I &= \rho \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} \left(y^2 + z^2\right) \mathrm{d}y \ \mathrm{d}z \ \mathrm{d}x \\
&= \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} \left(y^2 + z^2\right) \mathrm{d}y \ \mathrm{d}z \\
&= \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} y^2 \, \mathrm{d}y \ \mathrm{d}z + \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} z^2 \, \mathrm{d}y \ \mathrm{d}z \\
&= \rho a b \int_{-c/2}^{c/2} y^2 \, \mathrm{d}y \ + \rho a c \int_{-b/2}^{b/2} z^2 \, \mathrm{d}z \\
&= \frac{1}{12} \rho a b c^3 + \frac{1}{12} \rho a c b^3 \\
&= \frac{1}{12} \rho a b c (b^2 + c^2) \\
&= \frac{M}{12} (b^2 + c^2)
\end{aligned}$$

The result doesn't depend on a, and that makes sense, because changing a without changing the mass wouldn't change how far any of the mass was from the axis.

15-41 (a) The result from problem 19 was $E = L^2/2I$. This suggests plotting E as a function of L^2, which should give a line with a slope of $1/2I$. Since my spreadsheet software was rounding L^2 off to zero, I actually scaled both variables by powers of 10 before plotting them, as indicated on the labels on the axes.

The plot is indeed a pretty good straight line, although some small deviations from linearity are apparent. The fact that the y-intercept is nonzero is an indication that the form of the data is not quite $E \propto L^2$, which would give $E = 0$ for $L = 0$. The actual data do have $E = 0$ at $L = 0$, since all energies were measured relative to the lowest one. But since the graph has some curvature to it, the best-fit line doesn't go through the origin.

The slope is .1834, which has to be multiplied by 10^{-14+68}, giving $I = 2.7 \times 10^{-54}$ kg·m^2.

(b) The total volume of this nucleus is about 168 cubic femtometers, corresponding to a sphere with a radius of about 3 fm, and the moment of inertia of a sphere gives $I = (2/5) m r^2 \approx 1 \times 10^{-54}$ kg·m^2. The actual moment of inertia of the nucleus is about three times this value, which isn't bad agreement at all, considering that we're trying to describe a quantum-mechanical system using classical physics.

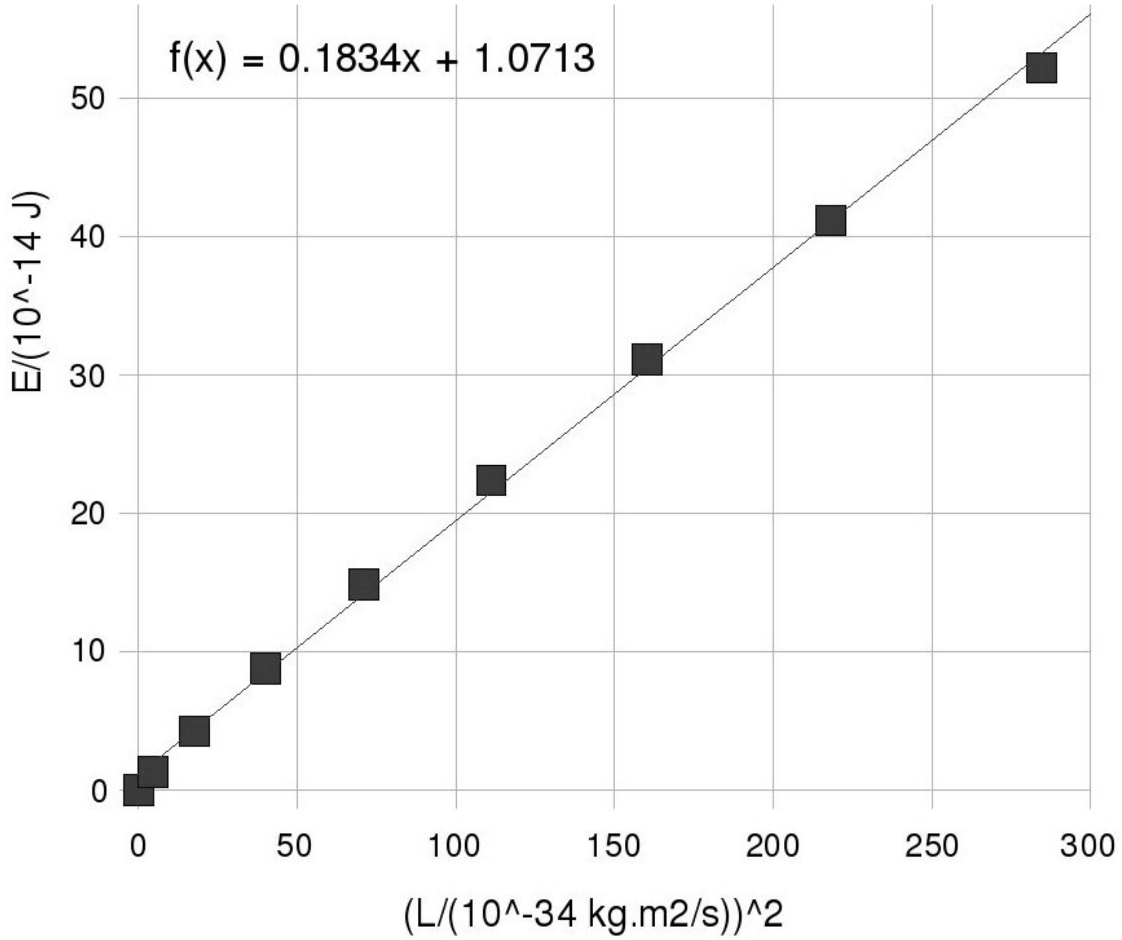
15-42 (a)

$$\begin{aligned}
(\mathbf{b} \times \mathbf{c})_y &= b_z c_x - c_z b_x \\
(\mathbf{b} \times \mathbf{c})_z &= b_x c_y - c_x b_y \\
(\mathbf{a} \times (\mathbf{b} \times \mathbf{c}))_x &= a_x (b_z c_x - c_z b_x) - a_z (b_x c_y - c_x b_y) \\
&= b_x (a_y c_y + a_z c_z) - c_x (a_y b_y + a_z b_z)
\end{aligned}$$

This can be put in the desired form by adding $a_x b_x c_x$ to the first term and subtracting it from the second.

(b) In the case where $\boldsymbol{\omega} = \omega \hat{\mathbf{x}}$, applying the rule found in part a to the integrand results in $\omega r^2 - x^2 \omega = \omega (y^z + z^2)$.

(c) Tackled by brute force, this is messy but straightforward. To get the result much more simply, let's start with a sphere of radius 1, whose moment of inertia is $(2/5) m$. Since this equals $\int (y^2 + z^2) \, \mathrm{d}m$, by symmetry we must have $\int y^2 \, \mathrm{d}m = \int z^2 \, \mathrm{d}m = (1/5) m$. What happens if we stretch it out along each axis, increasing its length along the x axis by a factor of a, and similarly for the other two axes? In the integral $\int y^2 \, \mathrm{d}m$, the stretching has had the effect of increasing the integrand by a factor of b^2, so it contributes $(1/5) m b^2$, and similarly for the z^2 integral. The results for the diagonal elements



Problem 15-41.

of the moment of inertia matrix are

$$I_{xx} = \frac{1}{5}m(b^2+c^2) \quad ,$$

$$I_{yy} = \frac{1}{5}m(a^2+c^2) \quad ,\text{and}$$

$$I_{zz} = \frac{1}{5}m(a^2+b^2) \quad .$$

15-43

$$\begin{aligned} I &= \int_0^\ell r^2\,\mathrm{d}m \\ &= \int_0^\ell r^2\frac{\mathrm{d}m}{\mathrm{d}r}\,\mathrm{d}r \\ &= \mu\int_0^\ell r^2 e^{r/\ell}\,\mathrm{d}r \end{aligned}$$

Although this looks like a definite integral because it has limits of integration, it's really more like an indefinite integral in the sense that the upper limit of integration is a variable. By switching to the unitless variable $u = r/\ell$, we can transform this into a definite integral that we'd be more likely to be able to find in a table of integrals, and this also has the advantage of making the calculus less messy. The result is

$$I = \mu\ell^3\int_0^1 u^2e^u\,\mathrm{d}u \quad .$$

The units are all in the factor in front: (kg/m)(m^3) = kg·m^2, which is correct for a moment of inertia. The integral can be done by integration by parts, by looking it up in a table, or using computer software.

$$\begin{aligned} I &= \mu\ell^3\; e^u(u^2-2u+2)\big|_0^1 \\ &= (e-2)\mu l^3 \end{aligned}$$

15-44 (a) Let N be the normal force of the cylinder on each bar. The total force on the bars in the vertical direction must be zero, so $2N\cos\theta = 2mg$.

The total torque on each bar must also be zero. Let the axis be at the hinge. The torque due to gravity is $mg(L/2)\sin(90°-\theta) = mg(L/2)\cos\theta$. Constructing the right triangle with vertices at the hinge, the point of contact, and the center of the cylinder, we find that the distance from the hinge to the point of contact is $r\tan\theta$, so the torque of the cylinder on each hinge is $Nr\tan\theta$.

We now have the two equations $2N\cos\theta = 2mg$ and $mg(L/2)\cos\theta = Nr\tan\theta$ in the two unknowns θ and N. Eliminating N, we find the result claimed in the problem.

(b) For large values of r/L, $\tan\theta$ must approach zero, so θ approaches zero. This makes sense, because the cylinder is so huge that the hinge is essentially lying on a flat surface.

For small values of r/L, $\tan\theta$ must approach infinity, so θ approaches 90 degrees. This is intuitively appealing if we imagine the cylinder as a hair-thin wire: the hinge folds flat.

15-45 (a) Let $\mathbf{r} = u\mathbf{A}+v\mathbf{B}$, where the interior of the triangle is defined by $u > 0$, $v > 0$, and $u + v < 1$. The parallelogram-shaped region defined by the infinitesimal intervals $\mathrm{d}u$ and $\mathrm{d}v$ then has area $\mathrm{d}A = |\mathbf{A}\times\mathbf{B}|\,\mathrm{d}u\,\mathrm{d}v$. The position of the center of mass is then given by

$$\begin{aligned} \mathbf{r}_{cm} &= \frac{\int\mathbf{r}\,\mathrm{d}A}{\int\mathrm{d}A} \\ &= \frac{\int_0^1\int_0^{1-v}(u\mathbf{A}+v\mathbf{B})\,\mathrm{d}u\,\mathrm{d}v}{\int_0^1\int_0^{1-v}\mathrm{d}u\,\mathrm{d}v} \quad , \end{aligned}$$

where we can treat the differential of area $\mathrm{d}A$ as if it were the differential of mass $\mathrm{d}m$ because the mass per unit area cancels out. The numerator and denominator can both be split up into two similar integrals, and straightforward calculus gives the result as claimed.

(b) Here we do need to explicitly introduce the mass per unit area $2m/|\mathbf{A}\times\mathbf{B}|$, but we get a cancellation between this factor of $|\mathbf{A}\times\mathbf{B}|$ and the one arising from the relation between $\mathrm{d}A$ and $\mathrm{d}u\,\mathrm{d}v$. The moment of inertia is

$$\begin{aligned} I &= 2m\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} r^2\,\mathrm{d}u\,\mathrm{d}v \\ &= 2m\left[\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} A^2u^2\,\mathrm{d}u\,\mathrm{d}v + \ldots + 0\right] \quad , \end{aligned}$$

where ... represents a similar term for vector $\mathbf{B}$, and 0 is a cross-term that vanishes. The integral

is separable, and the result is as claimed.
(c) Let I_o be the moment of inertia calculated in part b, and let $h = (1/6)|\mathbf{A}+\mathbf{B}|$ be the distance between the quadrilateral's center of mass and the center of mass of one of the triangles into which it can be split up, which follows from part a. Then $2(I_\text{o} + mh^2) = (1/6)m(A^2 + B^2)$, and solving for I_o gives the result claimed.

15-47 Newton's second law is

$$a = \frac{mg - T}{m} \qquad ,$$

where T is the tension in the string. The yo-yo's angular momentum is the same as the angular momentum of a single cylinder of mass m and radius R rotating about its axis:

$$L = \frac{\pi m R^2}{P} \qquad ,$$

where to avoid confusion I've used P rather than T for the period. The period is related to the yo-yo's downward velocity by $v = 2\pi r/P$, so we have

$$L = \frac{mR^2 v}{2r} \qquad .$$

The torque on the spindle is Tr, so

$$\begin{aligned} \tau &= \frac{\Delta L}{\Delta t} \\ Tr &= \frac{mR^2 a}{2r} \\ T &= \frac{mR^2 a}{2r^2} \end{aligned}$$

We now have two equations in the two unknowns T and a. One equation is Newton's second law, and the other is the equation just derived. Subsituting the latter into Newton's second law to eliminate T, we find

$$\begin{aligned} a &= g - \frac{R^2 a}{2r^2} \\ a &= \frac{g}{1 + R^2/2r^2} \qquad . \end{aligned}$$

Let's check this answer. The units work, because g is an acceleration, and the denominator is unitless. The denominator is greater than one, so $a < g$, which matches up with our real-world experience: a yo-yo doesn't drop like a rock. The acceleration gets smaller if R is bigger in relation to r, and that makes sense too, because increasing R makes it harder to spin (the same torque produces a slower change in the rate of rotation).

15-48 (a) We work by induction. Clearly $L_1 = w/2$. Given L_n, we can find L_{n+1} by thinking of the stack of $n + 1$ as being composed of a single book at the bottom with n books on top of it. We don't want the n books to tip, so they should protrude beyond the bottom book by L_n. In addition, we don't want the entire stack of $n + 1$ to tip over the edge of the table, i.e., its center of mass should be above the table's edge. Letting $\delta = L_{n+1} - L_n$, the horizontal position of this center of mass relative to the table's edge is $n\delta + (\delta - w/2)/(n + 1) = 0$. This results in $\delta = w/2(n + 1)$, so

$$L_n = \frac{w}{2} \sum_{m=1}^{n} \frac{1}{m} \qquad .$$

(b) If we draw rectangles on graph paper to represent the sum occurring in part a, then the curve $y = 1/(x + 1)$ always lies below the rectangles, so $L_n \ge (w/2)\int_0^n \mathrm{d}x/(x + 1) = (w/2)\ln(n + 1)$. Plugging in numbers proves the result claimed for $L > 7w$.

15-49 Since it's rotationally invariant, it must be built out of cross products, dot products, scalar products, and vector sums. Because its definition has to make sense regardless of the units of the vectors, the given information is enough to fix its behavior under scaling of the vectors. (I'll revisit this point below to make it more clear.) The given scaling information shows that if we write out an expression for $f(\mathbf{A}, \mathbf{B})$, each term must contain three factors of $\mathbf{A}$ and a single factor of $\mathbf{B}$. Using only cross products, we can write several expressions that fit these criteria:

$$\begin{aligned}&\mathbf{A}\times(\mathbf{A}\times(\mathbf{A}\times\mathbf{B}))\\&\mathbf{A}\times(\mathbf{A}\times(\mathbf{B}\times\mathbf{A}))\\&\mathbf{A}\times(\mathbf{B}\times(\mathbf{A}\times\mathbf{A}))\\&\mathbf{B}\times(\mathbf{A}\times(\mathbf{A}\times\mathbf{A}))\end{aligned}$$

Of these, the first two differ only in sign, while the third and fourth vanish identically. The first one fits all the desired criteria.

No other expression involving only cross products can fit the given criteria, since the only ones that have the right number of factors of $\mathbf{A}$ and $\mathbf{B}$ are the ones given, or trivial variations on them that can be put into one of these forms by using the anticommutativity of the cross product.

We also need to prove that no other expression exists that satisfies the given criteria and that is writen using some combination of dot products, cross products, scalar products, and vector sums. In fact, it is possible to write other expressions that satisfy the given criteria, but they are equivalent to the expression already found. Two examples of this type are $(\mathbf{A}\cdot\mathbf{A})(\mathbf{B}\times\mathbf{A})$ and $(1/2)\mathbf{A}\times(\mathbf{A}\times(\mathbf{A}\times\mathbf{B}))+(1/2)(\mathbf{A}\cdot\mathbf{A})(\mathbf{B}\times\mathbf{A})$.

In any term appearing in such an expression, there must be three multiplications. If none of these was a cross product, the result would be a scalar, and similarly if more than one was a dot product. Therefore the only possibility other than three cross products is a dot, a cross, and a scalar, as in the example already found. Rearranging the factors in every possible way fails to give anything that satisfies the given criteria.

It may be easier to understand now why it was necessary to assume that the function makes sense regardless of the units of the input vectors. We've shown that the function $A^2(\mathbf{B}\times\mathbf{A})$ satisfies the given criteria, where A^2 means $\mathbf{A}\cdot\mathbf{A}$. Suppose that we changed this to $(A^2+P(A^2))(\mathbf{B}\times\mathbf{A})$, where $P(x)$ is a polynomial having zeroes at $x=1$ and 4. This would also fit the given data. However, the condition $P(1)=0$ can only be meaningful if we define the units in which $x=1$.

15-50 (a)

$$\begin{aligned}I&=\int r^2\,\mathrm{d}m\\&=(m/b^2)\int r^2\,\mathrm{d}A\\&=(m/b^2)\int_0^b\int_0^b\left(x^2+y^2\right)\mathrm{d}x\ \mathrm{d}y\\&=(2/3)mb^2\end{aligned}$$

(b) Since there is no horizontal force, the center of mass drops along a vertical line. When the square has rotated through an angle θ, the height of the center of mass above the floor is $(b/\sqrt{2})\cos\theta$. Differentiating this, we find that at the final position, $\theta=45°$, the speed of the center of mass is $v=b\omega/2$. Conservation of energy gives

$$mg\frac{b}{\sqrt{2}}=mg\frac{b}{2}+\frac{1}{2}I\omega^2+\frac{1}{2}mv^2 \qquad .$$

The result is

$$\omega=\sqrt{\frac{12}{11}(\sqrt{2}-1)\frac{g}{b}}$$

15-51 (a) Changing the pitch by a given factor simply scales up the rate at which we move outward to larger circumferences, so the angular acceleration must be directly proportional to p.
(b) We expect the angular acceleration, which has units of s^{-2}, to depend on v, p, and r, and since the only one of these variables that contains seconds is v, the result must be proportional to v^2. Given that $\alpha\propto v^2p$, units constrain the dependence on r as well, so that $\alpha\propto v^2pr^{-3}$.
(c) The given information is that

$$\omega r=v=\text{constant}$$

and

$$\frac{\mathrm{d}r}{\mathrm{d}\theta}=\frac{p}{2\pi}=\text{constant},$$

To save writing we introduce the symbol $h=p/2\pi$. The spiral can be either left-handed or right-handed, and this can be accomodated by assigning a sign to p and h. By the chain rule, we have

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}\theta}{\mathrm{d}r} \cdot \frac{\mathrm{d}r}{\mathrm{d}t},$$

so that

$$\frac{1}{h} \cdot \frac{\mathrm{d}r}{\mathrm{d}t} \cdot r = v.$$

Separating variables gives $r\,\mathrm{d}r = vh\,\mathrm{d}t$, and integrating this, we find $\frac{1}{2}r^2 = vht$, where the constant of integration can be taken to be zero, since it won't matter in our final answer what time we define to be $t = 0$. We now find

$$\omega = (v/2ht)^{1/2},$$

and differentiating this gives

$$\begin{aligned}\alpha &= -\frac{1}{2}(v/2h)^{1/2}t^{-3/2}\\ &= -v^2hr^{-3}\\ &= -\frac{1}{2\pi}v^2pr^{-3},\end{aligned}$$

so the unknown unitless constant was $-1/2\pi$.

(d) I found this easiest to figure out by imagining the spiral to stand still, while a point moves along it like a bead on a wire. When we reverse the bead's direction of motion, nothing happens to the geometrical variables p and r, v reverses its sign, and α keeps the same sign (similar to the way that a rock thrown up in the air has the same acceleration on the way up and the way down). Because our equation contains v^2, it continues to make sense when we flip the direction of motion: neither side changes sign.

(e) To complete our check that the answer has the right symmetry properties, we need to account for the minus sign in the result. At first glance, this appears to have the *wrong* symmetry, since it seems to say that the motor's torque on the CD would always be in the clockwise direction — but anything that looks clockwise from one side is counterclockwise from the back. The solution to this puzzle is that p also has a sign. If we view the CD player from the back rather than the front, both p and α flip signs.

15-52 The constant acceleration equation $\theta = \omega_0 t + (1/2)\alpha t^2$ tells us that the number of revolutions is

$$\frac{\omega_0 t + (1/2)\alpha t^2}{2\pi \text{ radian/rev}} = 54811747219.1399 \text{ rev.}$$

The fraction of a revolution is real. The method used in the observations was capable of measuring small fractions of a rotation. The effect of the deceleration is much larger than this, however — about 100 revolutions over this period of time.

15-53 The relevant constant-acceleration equation is $\omega_f^2 = \omega_0^2 + 2\alpha\theta$. Setting $\omega_f = 0$ and solving for θ, we have $\theta = -\omega_0^2/2\alpha$. This is the number of radians. Ignoring the sign, the number of revolutions is $|\theta|/2\pi = \omega_0^2/4\pi\alpha = 290$ (2 sig figs).

15-54 One revolution is 2π radians, so $2\pi = (1/2)\alpha t^2$. The result is $t = 2\sqrt{\pi/\alpha} = 35$ s.

15-55 The rotational analog of the constant-acceleration equation $v_f^2 = v_0^2 + 2ax$ is $\omega_f^2 = \omega_0^2 + 2\alpha\theta$. The angular displacement is $\theta = 2\pi N$. Setting $\omega_f = 0$ and ignoring the sign, we have $\alpha = \omega^2/4\pi N$.

15-56 (a) $\omega = \mathrm{d}\theta/\mathrm{d}t = 3At^2 - B$.

(b) $\alpha = \mathrm{d}\omega/\mathrm{d}t = 6At$.

(c) Setting $\omega = 0$ and solving for t gives $t_+ = \sqrt{B/3A}$.

(d)

$$\begin{aligned}\omega_{\text{av}} &= \frac{\Delta\theta}{\Delta t}\\ &= At_+^2 - B\\ &= -2B/3\end{aligned}$$

15-57 (a) The easiest way to figure this out is to consider the case $r = 0$. Here the bug is just spinning in place, and clearly will not slip. (There is not even a preferred direction in which it would slip.) So the bug is more likely to slip at large values of r.

(b) The vertical forces on the bug cancel, so the normal force of the turntable acting on the bug equals mg, where m is the mass of the bug. The maximum force of static friction is $\mu_s mg$. Equating this to $m\omega^2 r$ gives $r = \mu_s g/\omega^2$.

15-58 The maximum force of static friction is $\mu_s mg$. The acceleration vector has radial component $\omega^2 r$ and tangential component αr, so its magnitude is $r\sqrt{\omega^4+\alpha^2}$. Applying Newton's second law, we get

$$\omega = \left[\left(\frac{\mu_s g}{r}\right)^2 - \alpha^2\right]^{1/4}$$

15-59 Newton's second law applied to the hanging mass gives

$$mg - T = ma,$$

where T is the tension in the string. The torque acting on the platform is TR, so

$$\alpha = TR/I.$$

The relationship between the linear acceleration of the string and the angular acceleration of the platform is

$$a = R\alpha.$$

These are three equations in the unknowns α, T, and I. Eliminating α and T gives

$$I = mR^2\left(\frac{g}{a} - 1\right).$$

15-60 Let positive x be to the left, positive y out of the page, and opsitive z up. Let the axis be at the universal joint. The total torque is

$$\begin{aligned}&\frac{1}{2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})\times(-\hat{\mathbf{z}})\\ &+\left(\frac{1}{2}\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}}\right)\times(a\hat{\mathbf{z}})\\ &+(\hat{\mathbf{x}}+\hat{\mathbf{z}})\times(-b\hat{\mathbf{y}})\\ &+\left(\frac{1}{2}\hat{\mathbf{y}}+\hat{\mathbf{z}}\right)\times(-c\hat{\mathbf{x}}).\end{aligned}$$

Computing the cross products and setting this equal to zero gives

$$\begin{aligned}0 =& \left(-\frac{1}{2}+a+b\right)\hat{\mathbf{x}}\\ &+\left(\frac{1}{2}-\frac{1}{2}a-c\right)\hat{\mathbf{y}}\\ &+\left(-b+\frac{1}{2}c\right)\hat{\mathbf{z}}.\end{aligned}$$

This is a system of three equations in the three unknowns. The solution is $a = 1/3$, $b = 1/6$, and $c = 1/3$.

15-61 (a) Suppose that $\mathbf{u}$ is a solution. Then if $\boldsymbol{\delta}$ is any vector for which $\mathbf{a}\cdot\boldsymbol{\delta} = 0$, then $\mathbf{u}+\boldsymbol{\delta}$ is also a solution. Thus if we fix the tail of $\mathbf{u}$ at the origin, its tip traces a plane perpendicular to $\mathbf{a}$. We also need to show that a solution exists. Since $\mathbf{a}$ is nonzero, one solution is $\mathbf{a}b/|\mathbf{a}|^2$.

(b) The demonstration of nonuniqueness is similar to the one for the dot product, except that now we want $\boldsymbol{\delta}$ *parallel* to $\mathbf{a}$. The solution set forms a line parallel to $\mathbf{a}$. Existence follows because given any line segment with nonzero length, we can form a (possibly degenerate) rectangle with a given area and having that segment as one of its sides.

(c) Because the plane and line described in parts a and b are perpendicular, they have a unique point of intersection.

15-62 First we need to relate the speed of the rollers to the speed of the slab. In the frame of reference of the rollers, the slab is traveling in one direction at a certain speed v, and the inclined plane is moving in the other direction at the same speed. Therefore back in the laboratory frame, the rollers have velocity v and the slab $2v$.

The problem can be solved either using Newton's laws or conservation of energy. Using Newton's laws is cumbersome, because it involves two normal forces and two static frictional forces, neither of which we know or want to know. Therefore let's use conservation of energy.

Although there are multiple rollers, the definitions of the total quantities m and I are such that it doesn't matter whether we sum over all the rollers or just pretend there is a single roller (along with a constraint that keeps the slab from tipping). Let the radius of the roller be r. It will clean things up if we define a unitless constant k so that $I = kmr^2$.

The kinetic energy of the roller has a part due to the roller's center-of-mass motion and another part due to its rotation. The former equals $(1/2)mv^2$. To find the latter, we use the fact that if the roller rotates through an angle $\mathrm{d}\theta$ in time

$\mathrm{d}t$, we have $\omega = \mathrm{d}\theta/\,\mathrm{d}t$, while the distance traveled by the roller is $r\,\mathrm{d}\theta$. The c.m.'s velocity is therefore $v = r\omega$, giving $\omega = v/r$. The energy of the rotation is $(1/2)I\omega^2 = (k/2)mv^2$. The total kinetic energy is

$$K = \frac{1}{2}mv^2(1+k) + \frac{1}{2}M(2v)^2$$

This is independent of r, which we could have anticipated since there would have been no way to incorporate r into a relation of this form while making the units make sense.

Let s be the distance that the rollers have traveled at a certain point in the motion, which means that the slab has gone $2s$. The change in potential energy since the beginning of the motion (which we take to have been at rest) is

$$\Delta U = -(2M+m)gs\sin\theta.$$

To simplify these expressions, let

$$\mu = 4M + m(1+k)$$

and

$$\gamma = (2M+m)g\sin\theta.$$

Conservation of energy then reads as

$$\frac{1}{2}\mu v^2 - \gamma s = 0.$$

This is identical to the statement of conservation of energy for a free-falling mass with mass μ, and $mg = \gamma$, so the acceleration of the rollers is γ/μ, while that of the slab is $2\gamma/\mu$, or

$$\frac{a}{g\sin\theta} = \frac{M + \frac{1}{2}m}{M + \frac{1}{4}m(1+k)}.$$

This expression has the right limiting behavior when $m \to 0$, since then we have $a = g\sin\theta$, as if the slab was sliding frictionlessly. We obtain the opposite limit by taking $M \to 0$ and $k \to 0$ (i.e., the mass of each roller is concentrated at its center). Then $a = 2g\sin\theta$. The interpretation of the factor of 2 is that the rollers roll down the slope with acceleration $g\sin\theta$, and the massless slab comes along at twice the acceleration.

So far we haven't done any check on the dependence on k. For a sphere, $k = 2/5$, and if we let $M = 0$, our result is that $a = (10/7)g\sin\theta$ for the slab, or $a = (5/7)g\sin\theta$ for the roller. This matches with the well known result.

15-63 (a) To find $2\mathbf{B}$ we double both the components of $\mathbf{B}$. Then to calculate $\mathbf{A} - 2\mathbf{B}$, we subtract those from the components of $\mathbf{A}$. This gives $\mathbf{A} - 2\mathbf{B} = (-7.0\hat{\mathbf{x}} - 28.0\hat{\mathbf{y}})$ meters. The pythagorean theorem results in a magnitude of 28.9 m.
(b) $\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y = -33.0\ \mathrm{m}^2$.
(c) Since $\mathbf{A}$ and $\mathbf{B}$ both lie in the x-y plane, the cross product will only have a z component: $(\mathbf{A}\times\mathbf{B})_z = A_xB_y - A_yB_x = 56.0\ \mathrm{m}^2$.
(d) This is a little less work to compute if we first simplify the expression to $\mathbf{A}^2 - \mathbf{B}^2$, where the square indicates taking the dot product of the vector with itself, i.e., finding the square of its magnitude. Subtracting the squares of the magnitudes, we get $-144\ \mathrm{m}^2$.
(e) $\theta = \cos^{-1}(\mathbf{A}\cdot\mathbf{B}/|\mathbf{A}||\mathbf{B}|) = 121°$

15-64 The rotational analog of $W = F\Delta x$ is $W = \tau\Delta\theta$. Both τ and $\Delta\theta$ are the same for the second revolution as for the first, so the work done in the second revolution is the same.

15-66 The moment of inertia of the rod for rotation about its center is $(1/12)m\ell^2$, where m is the mass of the rod. Using the parallel axis theorem, its moment of inertia about the fulcrum is $(1/12)m\ell^2 + mb^2$. The gravitational torque, with the fulcrum as the axis, is mgb. The resulting angular acceleration is $\alpha = \tau/I$. The tangential acceleration at the bug's position is $(\ell/2 + b)\alpha$, and setting this equal to g gives $b = \ell/6$.

15-67 The moment of inertia for a uniform rod about one end is $(1/3)m\ell^2$. The torque due to gravity is $(1/2)mg\ell$. The resulting angular acceleration is $(3/2)g/\ell$. The tangential acceleration at the bug's position is $a = b\alpha$. Subtracting this from g gives

$$g_a = \left(1 - \frac{3b}{2\ell}\right)g.$$

It is perhaps a little surprising that this can be negative for $b > (2/3)\ell$, meaning that the bug will lose contact with the door unless its feet are sticky enough.

16-2 0.07 to 0.10 s $\left(\frac{1}{15\text{ Hz}}\text{ to}\frac{1}{10\text{ Hz}}\right)$

16-3 Sixteen periods take about 8.8 ms, so one period is 0.55 ms. The frequency is 1/0.55 ms=1.82 kHz. The angular frequency is $(1.82\text{ kHz})(2\pi) = 1.1 \times 10^4$ rad/s.

16-4 (a) Pendulum 2 has then same total force on it when it is at the same angle, but when the two pendula are at the same angle, pendulum 2 is at twice the distance from equilibrium. Pendulum 2's $F - x$ graph will therefore look like pendulum 1's, but stretched out twice as wide. Since k is the slope of this graph, pendulum 2's k is half as much.

(b) $T = 2\pi\sqrt{m/k}$, so if pendulum 2's k is half as much, its period is greater by a factor of $\sqrt{2}$.

16-5 The spring constant is found by differentiating the force and evaluating the result at the equilibrium position x_o.

$$\begin{aligned} F &= ax^{-\beta} \\ k &= -\left.\frac{\mathrm{d}F}{\mathrm{d}x}\right|_{x_\text{o}} \\ &= a\beta x_\text{o}^{-\beta-1}\Big|_{x_\text{o}} \end{aligned}$$

We can find x_o from the condition that the forces cancel.

$$\begin{aligned} mg &= ax_\text{o}^{-\beta} \\ x_\text{o} &= (mg/a)^{-1/\beta} \\ k &= a\beta(mg/a)^{1+1/\beta} \\ \omega &= \sqrt{k/m} \\ &= \left[(a/m)\beta(mg/a)^{1+1/\beta}\right]^{1/2} \\ &= \beta^{1/2}g^{1/2+1/2\beta}\left(\frac{m}{a}\right)^{1/2\beta} \end{aligned}$$

16-6 We have

$$x = A\sin(\omega t + \delta),$$

and differentiating this gives

$$v = A\omega\cos(\omega t + \delta).$$

The total energy is

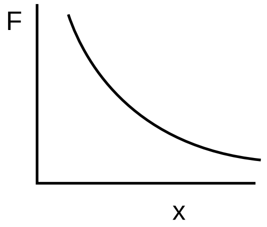
$$\begin{aligned} PE + KE &= \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \\ &= \frac{1}{2}A^2(k\sin^2\ldots + m\omega^2\cos^2\ldots), \end{aligned}$$

and, using $\omega = \sqrt{k/m}$, this becomes

$$\begin{aligned} PE + KE &= \frac{1}{2}A^2(k\sin^2\ldots + k\cos^2\ldots) \\ &= \frac{1}{2}A^2k, \end{aligned}$$

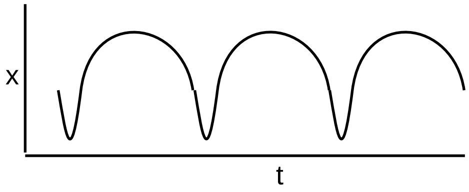
which is a constant, as required by conservation of energy.

16-7 The $F - x$ graph will look like this:



(Either graph a few points, or use your general math experience.) The graph in the previous problem only covered a small portion of this range, so it looked linear.

We only get simple harmonic motion (sinusoidal motion) when the $F - x$ graph is linear, so for large vibrations, where the $F - x$ graph can no longer be considered approximately linear, there will not be simple harmonic motion. Using the hint, we find that the motion must look something like this:



16-8 (a) The volume of the displaced water is $h_\text{o}b^2$, so its mass is $h_\text{o}b^2\rho$, and its weight is $h_\text{o}b^2\rho g$. Setting this equal to mg gives $h_\text{o} = m/b^2\rho$.
(b) A calculation similar to the one in part a gives $hb^2\rho g$ for the weight of the displaced water. Letting positive forces be upward, we have $F = hb^2\rho g - mg$.
(c) The equation $F = hb^2\rho g - mg$ is a line of slope $b^2\rho g$ when F is graphed against h, so the "spring constant" is $k = b^2\rho g$. The period is

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ &= 2\pi\sqrt{\frac{m}{b^2\rho g}} \\ &= 2\pi\sqrt{\frac{h_\text{o}}{g}} \end{aligned}$$

16-9 This is similar to the spring-and-lever example in section 16.2. Let x be the vertical position of one of the kids relative to equilibrium. The distortion of each spring is $(b/a)x$, so the total energy in the two springs is $PE = (2)(1/2)k[(b/a)x]^2$. The second derivative of this quantity gives the effective spring constant $k' = 2k(b/a)^2$. The moving mass is $m' = 2m$, so $\omega = \sqrt{k'/m'} = (b/a)\sqrt{k/m}$. (We have canceling factors of 2 due to the two kids and the two springs.) The resulting period is $T = 2\pi/\omega = 2\pi(a/b)\sqrt{m/k}$.

16-10

The 2π is unitless. The rest of the right-hand side has units of $[\text{kg}/(\text{N/m})]^{1/2} = [\text{kg}\cdot\text{m}/(\text{kg}\cdot\text{m/s}^2)]^{-1/2} = \text{s}$, which matches the units on the left.

16-11 $T \propto k^{-1/2} \propto g^{-1/2}$, so $g \propto T^{-2}$. Therefore a fractional change in T causes twice the fractional change in g. For example, a 1% increase in T will cause a 2% decrease in the inferred value of g. In symbols, $\Delta g/g = 2\Delta T/T$. The ΔT that would cause a $\Delta g = .024\ \text{m/s}^2$ is about 1 ms.

16-12 The result from problem 33 was $h = mg/k$. The constant-acceleration equation $h = (1/2)gT_{fall}^2$ results in

$$T_{fall} = \sqrt{\frac{2h}{g}} \qquad .$$

The total interaction energy is $(1/2)ky^2 + mgy$, and its second derivative is k, just as it would have been without the gravitational term. The interpretation is that hanging the mass vertically from the spring merely shifts the equilibrium position, without changing the period of the oscillations, which is still

$$T_{osc} = 2\pi\sqrt{\frac{m}{k}} \qquad .$$

The ratio of the two times is

$$\begin{aligned} \frac{T_{osc}}{T_{fall}} &= 2\pi\sqrt{\frac{m}{k}\cdot\frac{g}{2h}} \\ &= 2\pi\sqrt{\frac{m}{k}\cdot\frac{g}{2(mg/k)}} \\ &= \pi\sqrt{2} \\ &\approx 4.45 \qquad . \end{aligned}$$

Is this reasonable? In the case where the mass is falling, the distance h traveled is the same as the distance that it would have traveled in 1/4 of a cycle of oscillation. Roughly, then, we might expect the result to be about 4, and that is about what it is. The reason it's a little more than 4 is that in the free-fall case, the force acting on the mass is constant, whereas in the oscillating case, the force starts out being just as strong as gravity, but weakens and becomes zero at the end of a quarter cycle. Since the force in the oscillating case is always less than or equal to the force in the free-falling case, this lengthens the time for the oscillation, making the ratio slightly higher than the crude estimate of 4.

16-13 Let L be the amount by which the spring has been stretched or compressed with respect to equilibrium, so that its energy is $U_{spring} = (1/2)kL^2$. Let y be the height of the mass, and for convenience, let's define $y = 0$ to be the height at which the spring is relaxed, so that

$L = 0$ coincides with $y = 0$. If the mass goes down by a certain amount, the spring is stretched by half that amount, so $L = y/2$. (We don't care about positive or negative signs of L, since only L^2 occurs in the spring's energy.) The total interaction energy of the system is then

$$U = mgy + \frac{1}{8}ky^2 \qquad .$$

The effective spring constant experienced by the mass is not really k, but rather

$$\begin{aligned} k' &= \frac{\mathrm{d}^2}{\mathrm{d}y^2}\,U \\ &= \frac{\mathrm{d}^2}{\mathrm{d}y^2}\left(mgy + \frac{1}{8}ky^2\right) \\ &= \frac{1}{4}k \end{aligned}$$

In other words, this arrangement makes the spring appear to be four times less stiff than if the weight had just been hung directly from it. The period is $T = 2\pi\sqrt{m/k'} = 4\pi\sqrt{m/k}$.

16-14 (a)

$$\begin{aligned} U &= 2U_{one\ spring} \\ &= (2)(\frac{1}{2}k(L-b)^2) \\ &= k(L-b)^2 \\ &= k(\sqrt{b^2+y^2}-b)^2 \end{aligned}$$

(b) The effective spring constant is the second derivative of this quantity with respect to y, evaluated at $y = 0$.

$$\begin{aligned} \frac{\mathrm{d}^2 U}{\mathrm{d}y^2} &= \frac{\mathrm{d}^2}{\mathrm{d}y^2}k\left(\sqrt{b^2+y^2}-b\right)^2 \\ &= \frac{\mathrm{d}}{\mathrm{d}y}\left[2k\left(\sqrt{b^2+y^2}-b\right)\frac{1}{2}\left(b^2+y^2\right)^{-1/2}(2y)\right] \\ &= \frac{\mathrm{d}}{\mathrm{d}y}\,2ky\left[1-b(b^2+y^2)^{-1/2}\right] \\ &= 2k\left[1-b(b^2+y^2)^{-1/2}\right]+2ky\left[\ldots\right] \qquad , \end{aligned}$$

where the ... indicates stuff we don't need to evaluate, because it won't contribute to the derivative evaluated at $y = 0$. The result is zero, which implies that the period of small oscillations is infinite. This is a loophole in the text's statement that small oscillations are *always* simple harmonic. If you were to start this system doing large oscillations, and then let them die out slowly, their period would get longer and longer.

16-15 (a) The definition of k for motion in one dimension is $k = \mathrm{d}^2 U/\,\mathrm{d}x^2\big|_0$. Translating the linear quantities x and k into the angular ones θ and κ, this becomes $\kappa = \mathrm{d}^2 U/\,\mathrm{d}\theta^2\big|_0 = Bm$.

(b) Translating k and m to κ and I, $\omega = \sqrt{k/m}$ becomes $\omega = \sqrt{\kappa/I} = \sqrt{Bm/I}$.

16-16 I did this by modifying the program from section 4.6.

```
import math
def f(k,b,x):
  return -k*x-b*x*x*x
def period(m,k,b,amplitude,n):
  print("A=",amplitude,"  b=",b)
  x = amplitude
  v = 0
  w0 = math.sqrt(k/m) # frequency it would have if b=0
  t0 = 2.*3.1416/w0  # ...and the corresponding period
  t_max = t0*10.     # guess the longest time it could tak
  dt = t_max/n       # Divide t into n equal parts.
  t = 0.
  for i in range(n):
    t = t+dt
    dx = v*dt
    x = x+dx         # Change x.
    a = f(k,b,x)/m
    dv = a*dt
    v = v+dv
    if v>0.:
      return 2.*t
  return -999.       # If we get here, t_max was too short
```

Let's try it:

```
$ python -i simulate-anharmonic-force.py
>>> print(period(1.,1.,0.,1.,1000))
A= 1.0   b= 0.0
6.283200000000008
```

```
5    >>> print(period(1.,1.,0.01,1.,1000))
6    A= 1.0   b= 0.01
7    6.283200000000008
8    >>> print(period(1.,1.,0.01,10.,1000))
9    A= 10.0   b= 0.01
10   4.775232000000003
```

On lines 2-5, we run the program with $b=0$, and get the same result of exactly 2π that we had in section 2.5. On lines 6-9, we try a small value of b, and we observe that there is a slight decrease in the period, which makes sense, because the spring is stronger. On lines 10-12, a bigger amplitude produces a much smaller period. This is not the simple (and counterintuitive) behavior we observed when the spring's energy only had an x^2 term. In the pure x^2 case, the spring's greater energy at large amplitudes was exactly enough to compensate for the greater distance the mass needed to travel, causing it to cover that distance in the same time. The x^4 function, however, has a more severe dependence on x: it starts off flat, but then blows up like crazy when x gets big. The increased stiffness at large amplitudes does more than just compensate for the extra distance of travel, it actually makes the mass travel the *greater* distance in a *smaller* time.

17-1 Power is proportional to energy, and energy is proportional to A^2, so $A \propto \sqrt{P}$. The ratio of the amplitudes is $\sqrt{50/20}=1.6$.

17-2 $FWHM = f_{res}/Q = 100$ Hz, so since the curve is approximately symmetric, the range will be ±50 Hz, i.e., 250-350 Hz.

17-3 To find the minimum of the quantity inside the square root, we set its derivative equal to zero,

$$0 = \frac{\mathrm{d}}{\mathrm{d}f}\left(4\pi^2 m^2\left(f^2-f_\mathrm{o}^2\right)^2 + b^2 f^2\right)$$

$$0 = 8\pi^2 m^2\left(f^2-f_\mathrm{o}^2\right)(2f) + 2b^2 f$$

One solution is $f=0$, but that's normally a minimum. We're interested in solutions with $f \neq 0$, so we can divide by $2f$ to simplify:

$$0 = 8\pi^2 m^2\left(f^2-f_\mathrm{o}^2\right) + b^2$$

$$f = \sqrt{f_\mathrm{o}^2 - \frac{b^2}{8\pi^2 m^2}}$$

17-4 (a) If W/E is the fraction of energy lost to heat per cycle, then the fraction remaining after one is $(1-W/E)$. After two cycles, the fraction remaining is $(1-W/E)^2$, and so on. After n cycles, the fraction remaining is $(1-W/E)^n$.
(b) The definition of Q amounts to $(1-W/E)^Q = 1/535$, or

$$(1-W/E)^Q = e^{-2\pi} \qquad .$$

(c) Taking natural logs of both sides,

$$Q\ln(1-W/E) = -2\pi \qquad .$$

Since W/E is small, we can use the approximation $\ln(1+x) \approx x$, so $QW/E = 2\pi$, or $1/Q \approx (1/2\pi)W/E$.

17-5 (a) The definition of Q says that when $n=Q$, the energy falls off by a factor of $e^{2\pi}$. Since energy is proportional to the square of the amplitude, this corresponds to a factor of $Z=e^{\pi}$ in amplitude. Now we need to generalize this to the case where the number of oscillations we actually observe isn't actually Q. If the amplitude drop in n oscillations is Z, then the drop in one oscillation is $f^{1/n}$. The drop in Q oscillations is then $(Z^{1/n})^Q$. Setting this equal to e^{π} and solving for Q, we find $Q = n\pi/\ln Z$.
(b) The figure is pretty small, so this will be a fairly rough estimate. After $n=2$ cycles of oscillation, it looks like the amplitude falls off by a factor of $Z=2$. Applying the result from part a, we have $Q \approx 9$.

17-6 The second derivative test tells us whether an extremum is a maximum or a minimum. The second derivative of the quantity inside the square root is $12\omega^2 - 2(2-1/Q^2)\omega_\mathrm{o}^2$, and evaluating this at $\omega=0$ gives $-2(2-1/Q^2)\omega_\mathrm{o}^2$. For small values of Q, it's positive, indicating that the quantity inside the square root is at a minimum, and the amplitude is at a maximum. The value of Q at which the expression

$-2(2-1/Q^2)\omega_o^2$ becomes zero, on its way to becoming negative, is $Q=1/\sqrt{2}$.

17-7 (a) The force is not constant, so we have to do the integral

$$\begin{aligned}\frac{1}{2}W_{damp} &= \int_{-A}^{A} F\,\mathrm{d}x \\ &= \int_{-A}^{A} -bv\,\mathrm{d}x\end{aligned}$$

To find v as a function of x, we can use conservation of energy:

$$\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$$

Solving this for v gives

$$v=\sqrt{\frac{k}{m}\left(A^2-x^2\right)} \qquad ,$$

so we now have

$$\begin{aligned}\frac{1}{2}W_{damp} &= -\int_{-A}^{A} b\sqrt{\frac{k}{m}\left(A^2-x^2\right)}\;\mathrm{d}x \\ &= -b\sqrt{\frac{k}{m}}\int_{-A}^{A}\sqrt{\left(A^2-x^2\right)}\;\mathrm{d}x\end{aligned}$$

The definite integral can be done by a change of variable and a trig substitution, or by looking it up in a table of integrals, or, most simply of all, by recognizing the integrand as the equation of a semicircle of radius A, so that the area under the curve equals half the area of a circle, $\pi A^2/2$.

$$\frac{1}{2}W_{damp}=-\frac{\pi}{2}A^2b\sqrt{\frac{k}{m}}$$

We wish to get this in terms of f, which equals $(1/2\pi)\sqrt{k/m}$, so

$$W_{damp}=-2\pi^2A^2bf$$

(b,c,d) The energy is $kA^2/2$, so the rest follows immediately.

17-8 Let's not worry about units, since this is just a comparison with the approximate answers. Setting all those variables equal to 1 for convenience, the expression for the amplitude becomes

$$A=\left[(\omega^2-1)^2+\omega^2/36\right]^{-1/2}$$

By trial and error, I found that the maximum amplitude was 6.02, at $\omega=0.993$. The FWHM is going to be the difference between the two frequencies where the square of the amplitude equals $(6.02^2)/2$. By trial and error, I found that this occurred at frequencies of 0.906 and 1.074, giving a FWHM of 0.168, which is pretty close to 1/6.

4 Conceptual Physics

1-2 (a) 8 cm; (b) 1.33 m; (c) doesn't make sense; (d) 60 miles/hour

1-3 23.4 m + 2 × 0.294 m = 23.988 m, which should be rounded to 24.0 m (3 significant figures)

1-4

$$\begin{aligned} &3.0\times10^{8}\ \text{m/s}\\ &\quad\times\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)\left(\frac{1\ \text{inch}}{2.54\ \text{cm}}\right)\left(\frac{1\ \text{foot}}{12\ \text{inches}}\right)\\ &\quad\times\left(\frac{1\ \text{yard}}{3\ \text{feet}}\right)\left(\frac{1\ \text{furlong}}{220\ \text{yards}}\right)\\ &\times\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{hr}}\right)\left(\frac{24\ \text{hr}}{1\ \text{day}}\right)\left(\frac{14\ \text{days}}{1\ \text{forthnight}}\right)\\ &\quad= 1.8\times10^{12}\ \text{furlongs/fortnight}\end{aligned}$$

1-5 (a) 10^4 μg; (b) 10^{10} μg; (c) 10^{10} μg; (d) 10^{11} μg; (e) 1 μg

1-6 (a)

$$\begin{aligned}\frac{0.5\ \text{mg}}{0.2\ \text{ng/hamburger}} &= \frac{0.5\times10^{-3}\ \text{g}}{0.2\times10^{-9}\ \text{g}}\ \text{hamburgers}\\ &= 2\ \text{million hamburgers!}\end{aligned}$$

(Note how the fake units of "hamburgers" help to reassure us that dividing the first number by the second one gives the right result.)

(b) (0.5 mg)/(300 ng/serving)=2000 servings

1-7 Gravitational energy depends on how far apart the masses are. When you're at the top of your jump, your mass is as far as it's ever going to be from the mass of the planet, so that's when your gravitational energy is the greatest. Energy is conserved, so your greatest kinetic energy must be when your gravitational energy is the least: when you've just taken off and when you're about to hit the ground again. (You have *less* than your maximum KE when you're still in contact with the ground — during this time, you're converting chemical energy in your body into KE and GE. Likewise you have less than maximum KE after you hit the ground, because KE and GE are now being converted into heat and sound.)

1-8 Kinetic energy doesn't depend on the direction of the motion, so since both pennies start out at the same speed, they both start out with the same kinetic energy. They also start out at the same height, so they have the same gravitational energy and the same total energy. Energy is conserved, so if they start out with a certain amount of energy, that's the same amount of energy they'll always have; their total energies will always be equal to each other. Although the impacts with the ground will happen at different times, we don't care about that. On impact they have the same gravitational energy and the same total energy, so they must have the same kinetic energy on impact as well. They hit the ground with equal kinetic energies and equal speeds.

1-9 (a) The piece of string connected to B has three vertical parts: one connecting to the ceiling, one between the two pulleys, and one going down to B. If the part connecting to B lengthens by, say 2 cm, then this extra 2 cm has to come out of the other two segments. In other words, 1 cm of string is lost from the part connected to the ceiling, and 1 cm is lost from the part between the two pulleys. Therefore, if B goes down 2 cm, A rides up by only 1 cm.

(b) If they move, one will lose gravitational energy while the other gains gravitational energy. If A was a boulder and B was a feather, then A could release more gravitational energy by dropping than B would consume by rising. Likewise if B was ridiculously heavy compared to A, you'd release gravitational energy by letting B drop. We want to figure out how to arrange it so that there can't be any release of gravitational energy: even if one rises and the other falls, no gravitational energy will be transformed into kinetic energy. Since B travels twice as much as A, we want B to have half the mass. Then if B drops 2 cm, it will lose the same amount of gravitational energy that A gains by rising 1 cm.

1-10 (a) Magnetic energy plus kinetic energy stays the same, so if it's speeding up, the KE is increasing and the magnetic energy must be decreasing. (b) By similar reasoning, magnetic energy must be increasing as they approach each other.

1-11 To raise her body temperature by six degrees, the amount of energy required will be 6×200 kJ, or 1200 kJ. Converting to the basic units of joules, this is 1.2×10^6 J. A watt is a joule per second, so every second, the heat energy added to her body will be 200 J. That means that the time it will take for her to die will be $(1.2 \times 10^6 \text{ J})/(200 \text{ j}) = 6000$ s, or 100 minutes.

1-12 We're testing conservation of energy,

$$E_{total,i} \quad =? \quad E_{total,f}.$$

The only form of energy we can determine from the photo is kinetic energy, so this amounts to testing whether

$$KE_{1,i} \quad =? \quad KE_{1,f} + KE_{2,f},$$

where the initial time is before the collision and the final time is after the collision is over. Using $KE = (1/2)mv^2$, and taking advantage of the fact that the masses are equal, this becomes

$$v_{1,i}^2 \quad =? \quad v_{1,f}^2 + v_{2,f}^2.$$

Now $v = d/t$, where t is the time between flashes, and the common factor of t^{-2} can be divided out on both sides, so we end up with

$$d_{1,i}^2 \quad =? \quad d_{1,f}^2 + d_{2,f}^2,$$

where d is the distance between one position and the next. That is, the square of the distance is our measure of energy.

But rather than measuring directly from one position to the next, it's more accurate to measure the distance over several of the time intervals in succession. In four flashes, the incoming ball moves 6.0 cm. In the same amount of time after the collision, it moves 2.8 cm, and the target ball moves 5.0 cm.

Since we're leaving out lots of factors that have units, I'll write the numbers below without units.

$$\begin{aligned} 6.0^2 &=? \quad 2.8^2 + 5.0^2 \\ 36.0 &=? \quad 7.8 + 25.0 \\ 36.0 &=? \quad 32.8 \end{aligned}$$

There appears to have been a slight loss of energy, but it's really at the limit of our ability to measure things from the figure. We also know that there will be some extra energy after the collision in the form of heat and sound. The only thing that would really disprove conservation of energy in this collision would be if the final KE was *greater* than the initial KE, and if the increase was significant given the limited accuracy of the measurements.

1-13 First we take Newton's law of gravity, $F = Gm_1m_2/r^2$, and throw out the constant factor of Gm_1m_2, turning it into the proportionality $F \propto 1/r^2$, or $r \propto 1/\sqrt{F}$. Changing this into a statement about ratios, we have $r_1/r_2 = \sqrt{F_2/F_1} = 10$. At the earth's surface, r equals the earth's radius. To make it ten times bigger, we have to put the rocket nine earth radii above the surface.

1-14 One method for exploiting the changing mass of the electrons would be the same as the method given in the text for exploiting a change in G: raise a large weight on a day when the electrons' mass was small, then lower it and get back the energy on a day when it was high. Energy would not be conserved. The amount of energy released on the way back down would be greater than the amount stored on the way up. You could sell the excess energy. You could repeat the cycle as many times as you liked, and get rich.

A completely different method would be to store kinetic energy in a flywheel on days when the mass was low, and extract it on days when the mass was high. Note that this method has nothing to do with gravity.

1-15 The mass change in that example came out to be so small because we were dividing by c^2, which is big. To make it an easily measurable mass change, we'd have to make the decimal place move over by about 8 places on the final result, i.e., make c^2 about 10^8 times smaller. That means making c about 10^4 times smaller. The speed of light would have to be about 10^4 or 10^5 m/s.

1-16 (a) In units of 10^{-27} kg, the amount of mass lost is

$$1.67495 - 1.67265 - 0.00091 = 0.00139 \qquad ,$$

i.e., 0.00139×10^{-27} kg. This mass has been converted into energy:

$$\begin{aligned} E &= mc^2 \\ &= (0.00139 \times 10^{-27})(3 \times 10^8)^2 \\ &= 1.25 \times 10^{-13} \text{ joules} \qquad . \end{aligned}$$

(In the meter-kilogram-second system of units, kilograms multiplied velocity squared give units of joules.)

(b) For this process, the total mass changes from 1.67265 to 1.67586 (again in units of 10^{-27} kg). This is an *increase* in mass, which means that we would have to have a source of energy to make the reaction happen. A free proton has no such source of energy; it's all by itself, so there's nothing for it to get energy from.

2-1 The surface of the beer is slanted. This is a real physical effect, and the strong principle of inertia tells us that straight-line, constant-speed motion doesn't produce any real physical effect. The train must either be changing speed or changing its direction of motion. The train could be moving to the right and speeding up, moving to the left and slowing down, or going around a curve.

We can't tell the train's speed. It could even be at rest at this instant in time. For instance, it's possible that the train has been going to the left and slowing down, is at rest at this instant, and is now going to start speeding up again and moving to the right.

2-2 If the balloon is being carried along by the wind at constant velocity, then it's at rest with respect to the air. Only relative motion can cause a physical effect — absolute motion isn't even a well-defined concept. Since there is no relative motion between the balloon and the air, there can't be any physical effect. The flag is limp.

2-3 Suppose you're being observed by a cop in a car that was initially next to you and matching your speed. Let's say that as you take your foot off the gas, the cop car continues at the same speed, so you fall behind. In the cop's frame of reference, you were initially at rest, and then started moving backward, so you were speeding up.

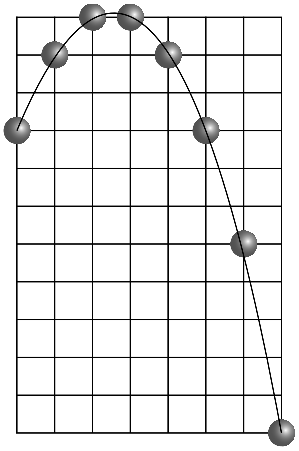
2-4 It would only violate conservation of energy. Momentum has a direction in space, and momenta in opposite directions can cancel out. Since all the air molecules are moving in different, random directions, their total momentum is initially zero. After settling down on the floor, their total momentum is still zero. There's nothing here that violates conservation of momentum. We started with zero, and ended up with zero.

Kinetic energy, however, is always positive, and can't cancel out. The molecules don't have zero kinetic energy to start with, and they do have zero kinetic energy after settling down on the floor. Conservation of energy is what keeps this from happening in real life.

2-5 Force is a transfer of momentum from one object to another. Equivalently, we can describe force as an interaction between two objects. Whichever way we say it, it takes two to tango — you can't have a force unless two objects are involved. There is no force when the bullet is in the air. There is a force when the bullet is passing through the book. This force will cause the bullet to lose momentum and the book to gain some.

Energy, on the other hand, exists at all times in this story. The bullet has kinetic energy before, during, and after the time when it passes through the book. (While it's passing through the book, some of this energy is converted to heat and sound.)

2-6 (a)



1 step over and 2 steps up
1 step over and 1 step up
1 step over and 0 steps up
1 step over and 1 steps down
etc.

(b) The very top of its arc is a little bit above the top line of the graph paper.

(c) The two figures represent the same motion seen in two different frames of reference. They have to be consistent, and we've seen that the top of the arc is a little higher than the highest position depicted in the figures.

2-7 The dark ball isn't moving at all. It's centered in the camera's field of view in the first frame, and in the other frames it's still centered in the camera's field of view.

As you move your eyes from the picture of the dark ball in the first frame to the picture of it in the second frame, the rightward motion of your eyeballs is following the passage of time, not a change in position. The film strip is like a graph whose y axis is position but whose x axis is time.

Similarly, the light-colored ball isn't moving to the right at all. In the first frame, it's directly below the dark ball. In the second frame, it's moved up a little, but it's still directly below the dark one. The light ball is moving straight up.

2-8 (a) It's speeding up. Between the first frame and the second, it only moves a tiny bit. Between the fourth and fifth frames, it moves a lot.

(b) (see figure)

(c) In the original frame of reference, shown in the book, there was a small decrease in gravitational energy due to the shortening distance between the ball and the earth, and there was an equal, small increase in the ball's kinetic energy. That made sense. Energy was conserved.

In this new frame, the ball is standing still, and the object that's changing its motion is the planet earth. The decrease in gravitational energy is the same as before, but in this frame of reference the increase in kinetic energy is huge, because the whole mass of the earth suddenly starts moving.

(d) No it doesn't violate the principle of inertia, and it's not true that all frames of reference are equally valid. This new frame of reference is a bogus, noninertial frame.

2-9 (a) The amount of KE to be converted into heat is the same in both cases, so the amount of work that has to be done is the same in both cases. Since work equals force times distance, and the work and force are the same, the stopping distance is the same.

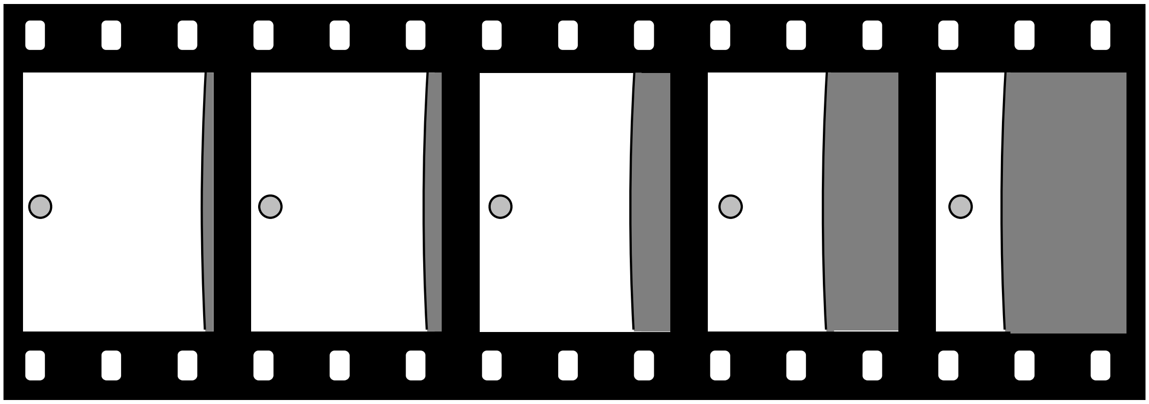
(b) Since the stopping distances are equal, if you watched videos of the two boats stopping, the videos would look exactly the same except for a rescaling of the time. That is, the slower boat's motion would look like the faster one's, but in slow motion. Therefore the time for the slower boat to stop will be greater.

2-10 (a) +, foot moves same direction as foot's force on ground
(b) +, force and motion are both forward
(c) −, force is backward, motion is forward
(d) −, force is up and motion is down

2-11 (a) As Weiping raises the rock, the sign of the force and the sign of d are the same (both are up), so she does 1 joule of work. On the way down, her force is up and the motion is down, so she does -1 joule of work. The total is zero.

Bubba's force was in the same direction as the motion during both halves of his motion, since, unlike gravity, the force of friction reversed directions on the way back. Bubba did 2 J of work.

(b) Work is defined as a transfer of energy. Weiping added gravitational PE to the rock on



Problem 2-8.

the way up, but on the way back down, she took it back out. The rock ends up with the same gravitational PE it started with, so by the definition of work, we verify that her work on the rock is zero.

As Bubba slides the table, he's adding heat energy to it. When he slides the table back in the opposite direction, he's not refrigerating it! Therefore, based on the definition of work, we know that the total work he does is not zero. This agrees with the result from part a.

3-1 You can get the maximum torque by setting everything up with θ=90°, giving

$$\begin{aligned}\tau &= rF\sin\theta\\ &= rmg\\ &= 270\ \text{N}\cdot\text{m}\qquad.\end{aligned}$$

3-2 The total torque required is the torque needed to lift his arm plus the torque needed to lift the weight. Since his arm is not changing, she needs to tell him to set things up so that the torque needed to lift the weight is also the same as before. Torque is $rF\sin\theta$, and since he is decreasing r by a factor of 17/33, she should tell him to increase the mass by a factor of 33/17 so that the increased weight force will compensate for the decreased r. The new mass should be 2.1 kg × 33/17 = 4.1 kg.

3-3 No, it just means that any torques that do act are adding up to produce zero *total* torque. For instance, a merry-go-round rotating with constant angular velocity has torques acting on it, but the motor's torque and the torque due to friction are just canceling out to produce zero total torque. Of course it is also *possible* that no torques at all are acting (e.g., the earth has no torques on it at all), but it would be incorrect to infer that the merry-go-round's motion at constant angular velocity implied that there were no torques on it.

3-5 For the purpose of calculating gravitational torques, we can treat each branch as if its weight were concentrated at its center of mass. The longer branch's center of mass is twice as far from the trunk, so the r is doubled. The gravitational force is also twice as much, so $\tau = rF$ is four times greater.

3-6 (a) Alice could be in a frame of reference in which Cathy isn't moving. For instance, if Alice and Cathy were in an airplane together, it would be natural for both of them to adopt the frame of reference of the plane. Bob could be using the frame of reference of the dirt.

(b) Your opinions about angular momentum will depend on what point in space you arbitrarily chose as the axis. Let's say Dong is sitting on some grass. Bob might be using the center of the spinning Earth is his axis, while Alice uses the spot Dong is sitting on.

3-7 Basically we have a situation with two equations in two unknowns. There is an equation that says the total torque on the foot is zero, and another equation that says the total force on the foot is zero. The two unknowns are the tension in the calf muscle, T, and the force exerted by the shinbone, F_s. We can however simplify the solution by choosing the axis to be the place where the shinbone attaches to the foot. With this choice, the shinbone makes no torque on the foot, and the torque equation thus has only the tension in the calf muscle as an unknown. There are two torques on the foot, a clockwise torque exerted by the muscle and a counterclockwise torque from the floor. The floor's upward force equals the person's weight, so if we use plus signs for clockwise torques the torque equation is

$$Ta - Wb = 0 \qquad ,$$

which is easily solved to give

$$T = W\frac{b}{a} \qquad .$$

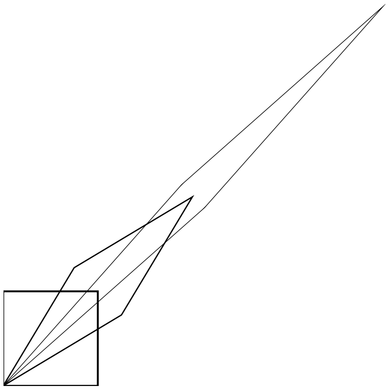
Letting positive forces be upward, the force equation is

$$T - F_s + W = 0 \qquad ,$$

so

$$\begin{aligned} F_s &= T + W \\ &= W\frac{b}{a} + W \\ &= \left(\frac{b}{a}+1\right)W \qquad . \end{aligned}$$

4-1 To get good precision, you need to do this on a full-size printout, as suggested by the problem. To keep from using up a whole page on my solutions handouts, I've shown the idea below at about 25% of the actual size.



First I drew the long diagonal twice as long again, meaning about 276 mm. To keep the area the same, I had to make its "cross-bar" half as long as on the preceding parallelogram, or about 17.5 mm. The speed represented by this new parallelogram is equal to the slope of its bottom edge. To get this slope, which is the rise divided by the run, we have to draw the rise and run accurately, i.e., lined up straight with the edges of the paper. I did this by using a second piece of paper as a right angle. The result is that the slope is (rise)/(run) = 91 mm/105 mm = .86, representing 86% of the speed of light.

4-2 (a) She says her own velocity is zero.

Time dilation happens because of relative motion. We can conclude that A and B are not in motion relative to one another, but that there is relative motion between them and C.

(b) Alice says B is at rest, but C is moving. (c) Betty says A is at rest, but C is moving. (d) Cathy says both A and B are moving, and she sees them moving in the same direction at the same speed.

4-6 Its kinetic energy is equal to its mass-energy minus its resting value,

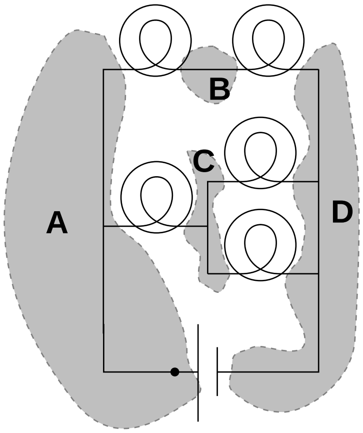
$$\begin{aligned} K &= \mathcal{E} - mc^2 \\ &= m(\gamma - 1)c^2 \\ &= (8.0\times10^7\ \text{kg})(\frac{1}{\sqrt{1-(1/2)^2}} - 1)c^2 \\ &= 1.1\times10^{24}\ \text{J} \end{aligned}$$

This is a thousand times greater than the total energy content of the world's nuclear arsenals.

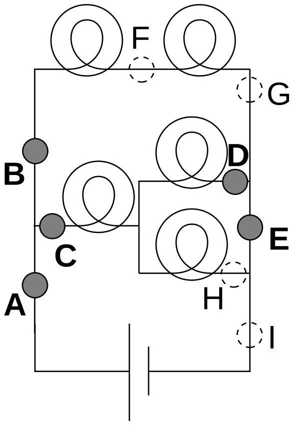
In other words, the Enterprise is the ultimate weapon of mass destruction. If it crashed into a planet (as it did, in one of the movies), it would destroy all life on the planet's surface.

5-1 Electricity is an inverse square force, so if r is smaller by a factor of 206.77, the force will be greater by a factor of $206.77^2 = 42754$.

5-2 There is constant voltage throughout a conductor. The wires are good conductors, but the bulbs aren't, so all we have to do is count how many parts there are in the circuit that consist of nothing but wire. As shown in the figure, there are four such parts. For instance, if we connect one probe of the voltmeter at the dot, and one in region D, we'll get a certain voltage reading, and we'll get the *same* voltage reading even if we move the second probe to some other place in region D. We can get a total of four unique voltage readings, but region A gives zero, so there are only three unique, nonzero readings we can get.



5-3 There are five unique currents you could measure, by inserting an ammeter at A, B, C, D, and E.



The key here is that charge is conserved. That means that, for example, you'll get exactly the same current reading at B, F, and G, because any charge that flows up through B will also have to go through F and G; it can't "drive off the road," and it can't get used up in the bulbs, because that would violate conservation of charge. Likewise, I and A give the same reading.

You also can't get two different results from D and H. This isn't because of conservation of charge, it's because the two bulbs are identical, and have the same voltage drop across them.

6-1 ————*rewrite*————

One way to get at this issue is to consider where the rules come from. They come from figure o. If you reverse the direction of time in figure o, then all the particles are going in the opposite direction. But the relativistic length contraction effect only depends on one thing's speed *relative* to another. Flipping everything over has no effect on the amount of relativistic length contraction.

By analogy, imagine that you're driving in heavy traffic on a one-way street. If you're going the same speed as everyone else, then none of you have any motion *relative* to each other. If you suddenly stop and put your car in reverse, you're going to die, because you've changed your motion relative to the other cars. But if everybody starts going backwards all at once, it's OK, because they aren't in motion *relative* to each other.

A whole different way of thinking about it is that each of the four diagrams shown in figure p can be time-reversed. If such a time-reversal resulted in a contradiction of one of the other figures, then the rules would violate time-reversal symmetry. However, there is no such contradiction. Time-reversing p/1 gives two dark-type charges moving to the left and attracting each other magnetically. This doesn't contradict the repulsion in figure p/4, because p/4 doesn't have two charges moving to the left, it has one going right and one going left. For similar reasons, p/2 doesn't contradict p/3 when you time-reverse it.

6-2

```
1                 2
  --->              --->
  --->              --->
  attraction        attraction

3                 4
  --->              --->
  <---              <---
  repulsion         repulsion
```

The simplified rule is that currents in the same direction attract magnetically, while currents in the opposite direction repel.

6-3 Replacing either charge in p/1 with a light-colored charge moving backward in time gives a diagram like p/2, which is still an attraction. Doing something similar to p/4 results in p/3, which is still a repulsion. It also works the other way around, e.g., you can do it to the bottom charge in p/2, which gives something like p/1.

The other two possibilities are ones that are not explicitly shown. You can replace the top charge in p/2 with a light charge moving to the left. This results in a diagram with two light-colored charges moving to the left, which isn't one of the diagrams we've already seen. However, p/1 implies that any like pair of charges moving in the same direction will experience a magnetic attraction.

Likewise, changing the top charge in p/3 gives a diagram with two light-colored charges moving in opposite directions, and repelling. this makes sense, because p/4 implies that like charges moving in opposite directions experience a magnetic repulsion.

6-4 To make something move in a circle, you have to make a force toward the center of the circle. As the electrons come over the top, there must be a downward magnetic force on them.

The electrons have the light-colored type of charge (negative, in standard terminology), so we have to use the left-hand rule in figure t. To get your fingers lined up the right way, you need to hold your flattened left hand out to the left with your thumb down, so you're looking at your knuckles. Bending your fingers, you find that they point back behind you. This means that the magnetic field points into the page in figure u.

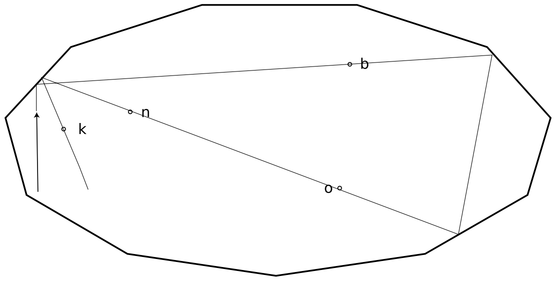
6-5 (a) In the visible part of the spectrum, the shortest wavelength is violet.

(b) No, because the wavelength of visible light is thousands of times bigger than the size of an atom.

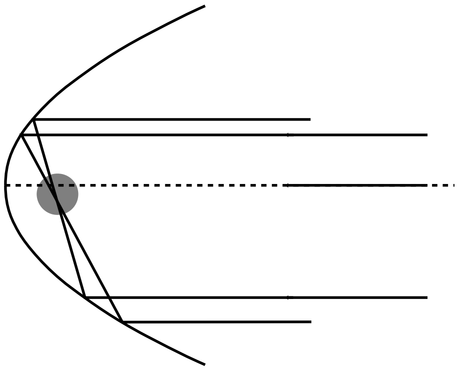
6-6 The distance between one hot spot and the next is on the order of 5 cm, i.e., 5×10^{-2} meters, which is in the microwave range, so this is indeed a reasonable explanation.

6-7 The correspondence principle says that new models have to be backward-compatible with old ones, i.e., the new model has to explain successfully all the same observations that the old model was able to deal with. The beginning of the book used the ray model, but now we've learned about the wave model. The ray model was able to explain all about mirrors and lenses, but the wave model can too. The difference comes when you get down to very short distance scales. For instance, the ray model doesn't give us any reason to believe it's impossible to make images of atoms using visible light, but that's wrong. Because an atom is smaller than a wavelength of light, we know that won't work. The models agree when light is interacting with things much bigger than the wavelength of light (lenses and mirrors), but with things that are microscopic, we need the wave model.

7-1 It spells "bonk."



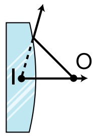
7-2 (a) The rays all cross at pretty much the same place, given the accuracy with which we can draw them.
(b) It could be used to cook food, for instance. All the sunlight is concentrated in a small area.
(c) Put the lightbulb at the point where the rays cross. The outgoing rays will then form a parallel beam going out to the right.



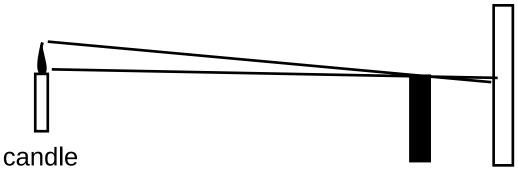
7-3 An image is where *all* the rays that started from a certain point come together for one big family reunion. Only 3 represents an image. Points like 1, 2, and 4 look special, but only because of the particular rays we chose to trace, out of the infinite number of rays fanning out from the point on the object. If we had chosen a different set of rays, they would all have crossed at 3 as before, but not at 1, 2, and 4.

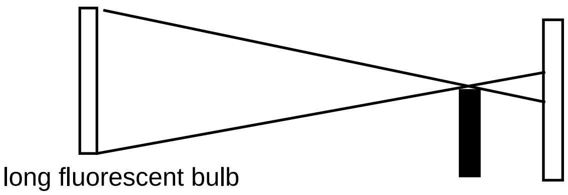
7-4 (a) The image is closer to the surface of the mirror, so the magnification is less than one.

(b) It's virtual, because the rays only appear to have passed through point I. There is no way it could make a real image, because in order to make a real image it would have to bring the rays back together.



7-5 The fuzzy part of the shadow is the part that is only illuminated by some of the rays from the light source. This is the area between the extreme rays shown in the figures. The fuzzy area is larger in the second figure.

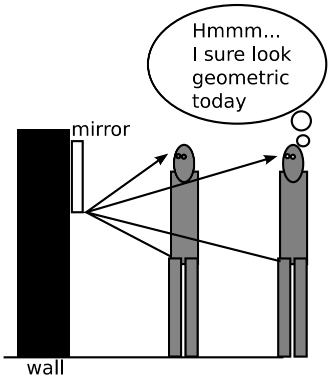




7-6 The image is at an equal distance behind the mirror, so the distance between the man and his image is twice his distance from the mirror, and his separation from his image is reducing at a rate of 2.0 m/s.

7-7 If you're going to see a particular part of your body, then rays from that part of your body have to be able to go to the mirror, be reflected, and go into your eye. If we're trying to find the lowest part you can see, then we're interested in the case where the ray hits the very bottom of the mirror. Backing up doesn't help, because the triangles are isoceles by the law of specular reflection, and the top of the base is fixed in height.

Another way of understanding the result is that if you go twice as far away, the mirror subtends half the angle, but your image also subtends half the angle, so the mirror still encompasses the same amount of your image.



7-8 See figures a-f.

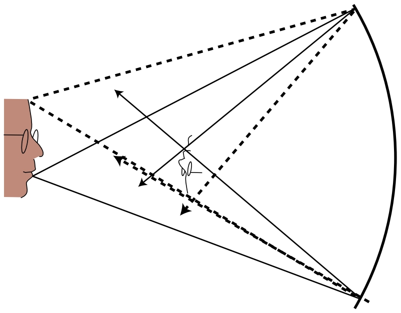
a and b: converging mirror, virtual image; increased object distance gives increased image distance

c and d: diverging mirror, virtual image: increased object distance gives increased image distance

e and f: converging mirror, real image: increased object distance gives decreased image distance

7-9 Usually you need to draw in a normal in order to get an accurate drawing of the reflected rays. Here, however, we already have one set of rays, and it's pretty easy to do the new rays so that they fan in or fan out by the correct angle compared to the old ones.

As claimed, the image of the forehead (where the dashed rays cross) is below the image of the lips.



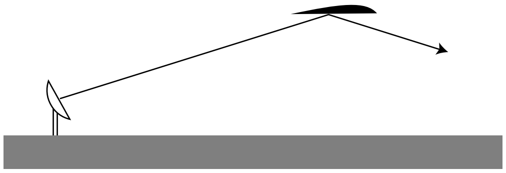
7-10 The magnification is the ratio of the image's size to the object's size. It has nothing to do with the person's location. The angular magnification, however, does depend on the person's location, because things farther away subtend smaller angles. The distance to the actual object is not changed significantly, since it's zillions of miles away in outer space, but the distance to the image does change if the observer's point of view changes. If you can get closer to the image, the angular magnification is greater.

7-11 The problem can be simplified conceptually by replacing the hypothetical square watch with a square hole in a piece of paper. This version is also easy to construct and test experimentally. The bundle of rays transmitted through the hole is identical to the bundle that would be reflected by a specular-reflecting square of the same size. Let A be the aperture size, i.e., a linear dimension of the hole, and let d be the distance from the aperture to the surface on which the spot is being projected. The results depend on how A and d compare. When d is small compared to A, the spot is square, but when d is large compared to A, we get a pinhole camera, which forms a circular spot shaped like the sun.

7-12 The time it takes light to travel a few meters, say 3 m, is $(3\ \text{m})/(3\times10^8\ \text{m/s}) = 10^{-8}$ s.

7-13 $f = c/\lambda = (3\times10^8\ \text{m/s})/(10^{-9}\ \text{m}) = 3\times10^{17}$ Hz, or call it 10^{17} Hz, since it's just an estimate. It would be UV or an x-ray.

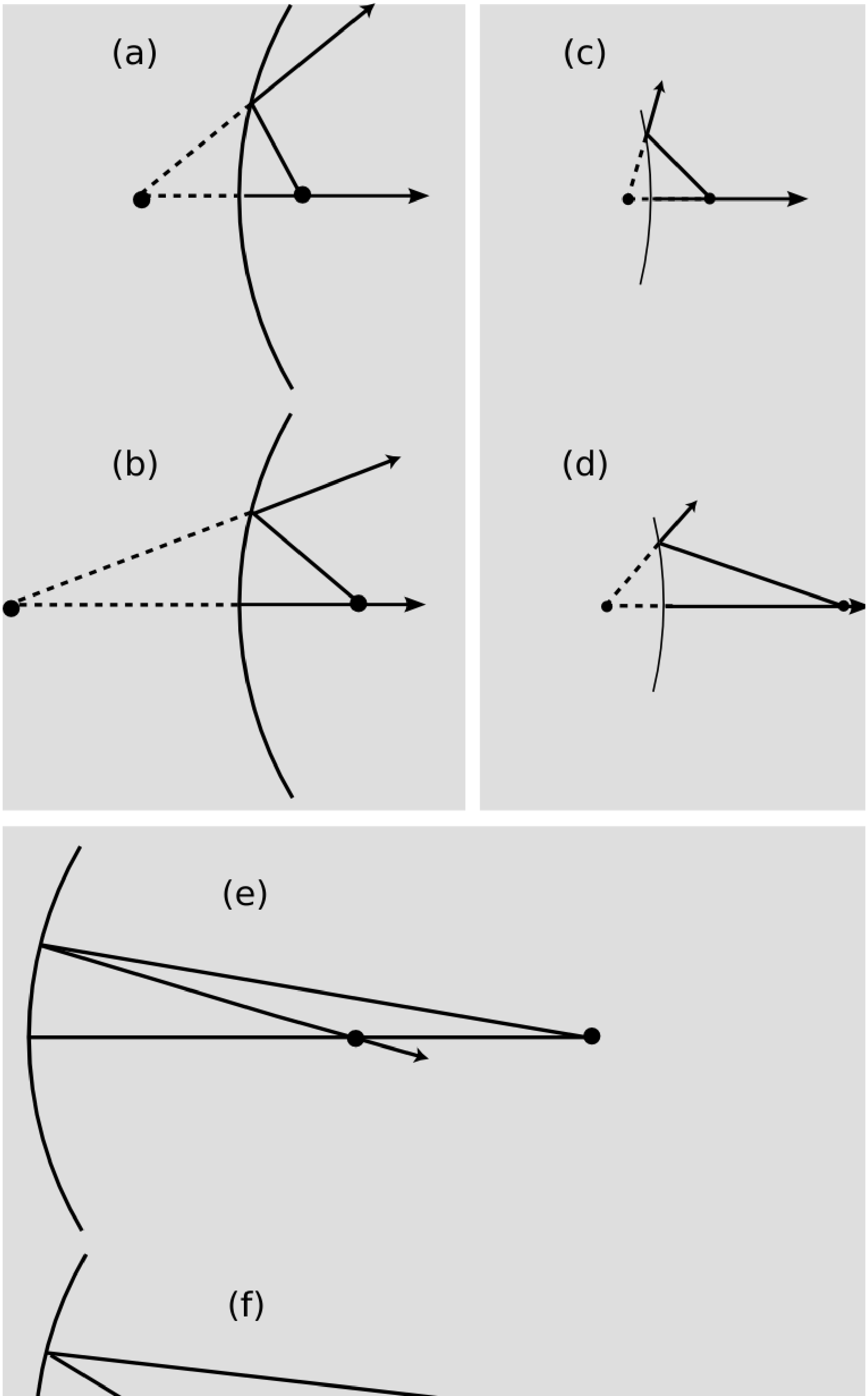
7-14 Because the surfaces are flat, you get specular reflection. In specular reflection, all the reflected rays go in one direction. Unless the plane is directly overhead, that direction won't be the right direction to make the rays come back to the radar station.



This is different from a normal plane, which has complicated, bumpy surfaces. These surfaces give diffuse reflection, which spreads the reflected rays randomly in more or less every possible direction.

8-1 0.07 to 0.10 s $\left(\frac{1}{15\ \text{Hz}}\ \text{to} \frac{1}{10\ \text{Hz}}\right)$

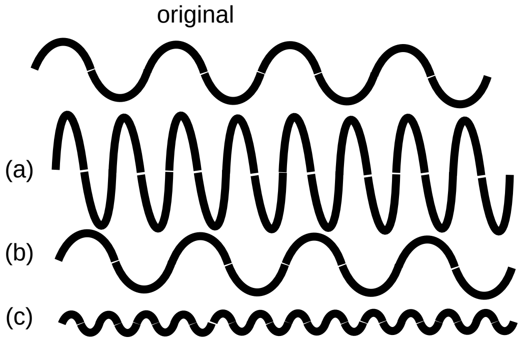
8-2 (a) Pendulum 2 has then same total force on it when it is at the same angle, but when the


(a)
(c)
(b)
(d)
(e)
(f)

two pendula are at the same angle, pendulum 2 is at twice the distance from equilibrium. Pendulum 2's $F-x$ graph will therefore look like pendulum 1's, but stretched out twice as wide. Since k is the slope of this graph, pendulum 2's k is half as much.

(b) $T=2\pi\sqrt{m/k}$, so if pendulum 2's k is half as much, its period is greater by a factor of $\sqrt{2}$.

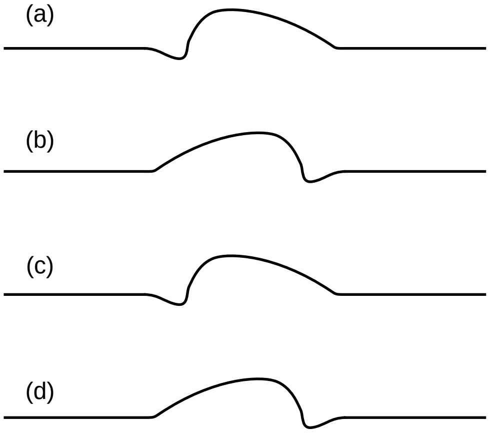
8-3 Since it's a graph of the wave as a function of position, what it shows is amplitude and wavelength. (a) The amplitude is doubled, and $\lambda=v/f$, so the wavelength is cut in half. (b) Since $v=f\lambda$, doubling both velocity and frequency leaves the wavelength unchanged. This is exactly the same as the orginal wave. (c) The information about the velocity is irrelevant. We're given the wavelength and amplitude directly, so that's all we need to show. (The change in velocity implies an additional change in frequency for the given wavelength, but the wavelength is 1/3 the original, not 1/6.)



8-4 (a) The scale on the time axis is not comparable to the scale of the distance axis, so there is no reason why your answer has to be the same width on the page. As the pulse passes through a certain point traveling to the right, the water will first go down a little, then up, so the graph should look similar, but reversed.

(b) As the pulse travels to the left, the water will first do the big, slow rise, then the small, short dip. The graph is not reversed compared to the original.

(c) and (d) are the same idea – the roles of question and answer are just reversed.



8-5

There are a couple of things that make it difficult to devise an accurate experiment: (1) it's hard to produce waves in a controlled way, and (2) it's hard to measure the height of the superposing waves in real time.

Possible solution: Find a machine that's capable of making spreading ripples of a consistent height, e.g., a wind-up toy car whose wheels go around. This takes care of problem 1. You could put two such cars in the water, and see if the peaks formed by constructive interference become twice as tall at a point equidistant from the two cars, compared to their height when only one car was turned on.

The toy cars also help to solve problem 2, because the waves are steady. You can put a ruler in the water, and watch the steady up-and-down vibration of the surface, measuring the height of the wave. This could be made easier by taking a video on a cell phone and then clicking through it frame by frame.

8-6 $T=1/f=3.82$ ms, and $\lambda=v/f=(340\text{ m/s})/f=1.3$ m.

5 Fields and Circuits

1-1 $B_y/B_x = \tan 59\,\text{deg} = 1.7$

1-2 (a) $B_m/B_a = \tan\theta$

(b) For $\theta = 0$, we have $B_m/B_a = 0$, which makes sense because the compass needle hasn't been deflected at all. For $\theta = 45°$, $B_m/B_a = 1$, which makes sense because the compass needle is making an even compromise between two fields of equal strength. For $\theta = 90°$, $B_m/B_a = \infty$, which makes sense because the only way B_m can win completely is if B_a is completely absent.

1-3 The maximum is 3 T, which is achieved when the fields are in the same direction. The minimum is 1 T, when they are in opposite direction. By varying the angle between the vectors from 0 to 180 degrees, all values in between can be achieved as well.

1-4 This problem has to be done analytically, not graphically, because it requires such high precision. The two given angles are both measured counterclockwise from the right, which is convenient; for angles defined this way, we can always find x components using a cosine, and y components using a sine, and the plus and minus signs will be come out automatically. The first vector's components are

$$\begin{aligned} E_{1,x} &= (35.24\ \text{N/C})\cos 217.3° \\ &= -28.03\ \text{N/C} \\ E_{1,y} &= (35.24\ \text{N/C})\sin 217.3° \\ &= -21.36\ \text{N/C} \qquad , \end{aligned}$$

and the second one's are

$$\begin{aligned} E_{2,x} &= (48.01\ \text{N/C})\cos 11.7° \\ &= 47.01\ \text{N/C} \\ E_{2,y} &= (48.01\ \text{N/C})\sin 11.7° \\ &= 9.74\ \text{N/C} \qquad . \end{aligned}$$

Adding components gives

$$E_x = 18.98\ \text{N/C}$$

and

$$E_y = -11.62\ \text{N/C} \qquad .$$

Again, the signs and sizes look about right. The magnitude of the total field is given by the Pythagorean theorem,

$$\begin{aligned} |\mathbf{E}| &= \sqrt{\Delta x_3^2 + \Delta y_3^2} \\ &= 22.25\ \text{N/C} \qquad . \end{aligned}$$

The direction is

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{\Delta y_3}{\Delta x_3}\right) \\ &= -31.4° \text{ or } 148.5° \end{aligned}$$

(measured counterclockwise from the right). We can tell from looking at a sketch that the first arctangent is the correct one. (I've given the final answers with four sig figs of accuracy, although you could argue that it should be three, since the least accurate piece of given data is the 11.7° angle. In reality, however, that angle isn't any less accurate than the 217.3° one, i.e., changing either one by a tenth of a degree will have about the same effect on the final result.)

1-5 (a) The cross product inside the parentheses results in a vector, and the second cross product results in a vector. (b) The cross product gives a vector, and then dotting it with the other vector gives a scalar. (c) The dot product gives a scalar, but you can't take a cross product of a scalar with a vector. This one is nonsense.

1-7

$$\begin{aligned} \mathbf{a}\cdot\mathbf{b} &= |\mathbf{a}||\mathbf{b}|\cos\theta_{ab} \\ \theta_{ab} &= \cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right) \\ &= 12.9° \end{aligned}$$

1-8 The easiest way to find such a vector is to take the cross product.

$$\begin{aligned} (\mathbf{A}\times\mathbf{B})_x &= A_yB_z - B_yA_z = -3 \\ (\mathbf{A}\times\mathbf{B})_y &= A_zB_x - B_zA_x = 6 \\ (\mathbf{A}\times\mathbf{B})_z &= A_xB_y - B_xA_y = -3 \end{aligned}$$

So our vector is $-3\hat{\mathbf{x}}+6\hat{\mathbf{y}}-3\hat{\mathbf{z}}$. We could also use any other vector lying along the same line, i.e., any scalar multiple of this one. I checked my result by computing its dot product with each of the two given vectors, verifying that it was zero in each case.

1-9 (a) Applying the Pythagorean theorem gives 30.0 μT.
(b) For a uniform field, the relationship $dU_B=(c^2/8\pi k)B^2dv$ is equivalent to $U_B=(c^2/8\pi k)B^2v$, which gives 1.43×10^{-2} J.

1-10 The energy density is proportional to the square of the field, so if the field patterns are identical except for a scaling up by a factor of 2, the energy in 3He is greater by a factor of 4.

1-11 The energy density of the magnetic field is proportional to the square of the field, so the ratio of the energy densities is $[(66\ \mu\text{T})/(23\ \mu\text{T})]^2=8.2$.

1-12 Setting $(c^2/8\pi k)B^2=(1/8\pi k)E^2$ gives $E/B=c$.

1-13 (a) The energy per unit volume is

$$\frac{E}{V}=\frac{1}{8\pi k}|\mathbf{E}|^2 \qquad .$$

(E is energy, $\mathbf{E}$ is electric field.) At the moment when the lightning strikes, the field equals E_c. The volume of the air in which the field exists is L^2h.

$$E=\frac{1}{8\pi k}E_c^2L^2h$$

(b) A greater h gives a greater energy, so it's more dangerous if it's higher.
(c) Let's use 1000 m for both L and h. Plugging in, the result is about 10^{10} J.

1-14 The energy density is proportional to the square of the field strength, so decreasing the field by a factor of 10 is equivalent to cutting the energy by a factor of 100.

1-15 (a) The energy depends on the square of the field, so we have

$$\begin{aligned}\alpha &= \frac{(\mathbf{E}_1+\mathbf{E}_2)^2}{E_1^2}\\ &= \frac{(\mathbf{E}_1+\mathbf{E}_2)\cdot\mathbf{E}_1+\mathbf{E}_2)}{E_1^2}\\ &= \frac{E_1^2+2\mathbf{E}_1\cdot\mathbf{E}_2+E_2^2}{E_1^2}\\ &= 2+2\cos\theta\\ \theta &= \cos^{-1}\left(\frac{\alpha}{2}-1\right).\end{aligned}$$

(b) For $\alpha=0$, 2, and 4, we get $\theta=180°$, $90°$, and 0. The first one makes sense because the vectors cancel. The second makes sense because putting the vectors tip to tail forms a square, and the vector sum has a magnitude greater by $\sqrt{2}$ compared to the original vectors. The last one makes sense because the fields are in the same direction, so the sum is double the individual vectors.

1-19 The definition of the vector cross product in terms of the components is $(\mathbf{A}\times\mathbf{B})_x=A_yB_z-B_yA_z$, and similarly for y and z. Reversing the roles of $\mathbf{A}$ and $\mathbf{B}$ interchanges the positive and negative terms on the right, so the sign flips.

1-20 Since it's not associative in general, we expect that almost any set of three vectors, chosen at random, will provide a suitable counterexample. I decided to use combinations of the unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$, since they're easy to calculate with. The first example I tried was $\hat{\mathbf{x}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}})$, which demonstrated Murphy's law by being one of those rare examples in which associativity *does* hold, because both $\hat{\mathbf{x}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}})$ and $(\hat{\mathbf{x}}\times\hat{\mathbf{y}})\times\hat{\mathbf{z}}$ are zero. My next example worked: $\hat{\mathbf{y}}\times(\hat{\mathbf{y}}\times\hat{\mathbf{z}})=\hat{\mathbf{y}}\times\hat{\mathbf{x}}=-\hat{\mathbf{z}}$, but $(\hat{\mathbf{y}}\times\hat{\mathbf{y}})\times\hat{\mathbf{z}}=0\times\hat{\mathbf{z}}$ equals zero, which is different.

1-21 Let $\mathbf{v}=c\hat{\mathbf{x}}+d\hat{\mathbf{y}}$, where c and d are to be found. We don't expect to be able to determine both c and d uniquely, because for any $\mathbf{v}$ that works, so will any nonzero scalar multiple of it. That is, we only expect to be able to find the ratio c/d, which expresses how big c is in proportion to d.

We want $\mathbf{v}$ to be perpendicular to $\mathbf{u}$, so $\mathbf{u}\cdot\mathbf{v}=$

0, or $ac + bd = 0$. Solving for the unknown ratio gives $c/d = -b/a$ (assuming $a \neq 0$). Concretely, a choice that works is $c = b$ and $d = -a$, so that

$$\mathbf{v} = b\hat{\mathbf{x}} - a\hat{\mathbf{y}}.$$

(Writing it this way also works in the special case where $a = 0$.) To check that this is a solution, we can compute $\mathbf{u} \cdot \mathbf{v} = ab - ba = 0$, as desired.

2-2 (a) The electric flux is $\Phi_E = \int \mathbf{E} \cdot \mathrm{d}\mathbf{A}$. The electric field is constant, so as with any integral, we can take the constant factor outside, giving $\Phi_E = \mathbf{E} \cdot \int \mathrm{d}\mathbf{A}$. Integrating all the little areas $\mathrm{d}\mathbf{A}$ gives the total area vector $\mathbf{A}$, and since this area is perpendicular to the top of the cube, it's parallel to the field, and we have $\Phi_E = |\mathbf{E}||\mathbf{A}| \cos 0 = Eh^2$.

(b) On the sides, the area vector is perpendicular to the field, so the dot product is zero.

(c) On the bottom, the area vector is in the opposite direction compared to the field, so the cosine factor is -1, and we have $\Phi_E = -Eh^2$.

(d) Adding up the results of the previous parts, we get a cancellation between the top and bottom, so the total is zero.

(e) By Gauss's law, the total charge inside the cube is zero.

2-3 (a) The expression $\operatorname{div} P$ doesn't make sense, because P is some scalar such as pressure, but the divergence operator takes a vector as an input.

(b) The expression $\mathbf{v} + \operatorname{div} \mathbf{w}$ doesn't make sense because the output of the divergence operator is a scalar, and we can't add a scalar to a vector.

2-4 The positive and negative charges attract each other.

2-5 (a) The total force acting on the particle is $mg + qE$, so by Newton's second law its acceleration is $a = g + (q/m)E$. This makes sense because when $q = 0$, we get $a = g$. Applying the constant-acceleration equation $v_f^2 = v_0^2 + 2a\Delta x$ gives $h = -v^2/2(g + qE/m)$. In the case where gravity is dominant, the negative value of g gives $h > 0$, which makes sense.

(b) Plugging in gives 83 m. This is comparable to the asteroid's "football-field" size, so the approximation of g as a constant may not be correct. Our result should probably be taken as a lower bound.

2-6 (a) Let the x axis be in the initial direction of motion and y the direction of the electric field. Newton's laws tell us that the x and y motion are independent, so the x velocity is constant, and the time taken to pass through the capacitor is $t = w/u$. The final velocity in the y diection is $v = at = (q/m)Et = qEw/mu$. The deflection angle is $\theta = \tan^{-1}(v/u) = \tan^{-1}(qEw/mu^2)$.

(b) Converting the given data to SI and plugging in, the result is $\theta = 0.98$ radians, or a little more than a 45 degree angle.

2-7 (a) $F = ma$ and $F = qE$, and since the field is uniform, the force and acceleration will be constant. For motion with constant acceleration, we have $v = at$, and eliminating the unknown F gives $v = qEt/m$.

(b) You can conclude that they have the same ratio of q/m, but you can't tell whether they have the same q and the same m. One could have twice the charge and twice the mass compared to the other, for instance.

2-8

$$\begin{aligned} q &= \int_{x=0}^{x=\infty} \mathrm{d}q \\ &= \int_0^\infty \frac{\mathrm{d}q}{\mathrm{d}x}\,\mathrm{d}x \\ &= \int_0^\infty ae^{-bx}\,\mathrm{d}x \\ &= \frac{a}{b} \end{aligned}$$

2-9 (a) If positive is the direction from the negative charge to the positive charge, and we pick a point on the positive side, then adding the fields of the two charges gives

$$E = kq\left[(r - \ell/2)^{-2} - (r + \ell/2)^{-2}\right].$$

This is an odd function of r, but in fact the field is physically an even function of r. If we had evaluated the field on the opposite side of the axis, we would have gotten an expression equal to minus this one.

(b) As suggested in the hint, we define a small, unitless parameter $u = \ell/2r$. With this change of variable, we have

$$E=\frac{kq}{r^2}\left[(1-u)^{-2}-(1+u)^{-2}\right].$$

The Taylor series of $(1+x)^p$ is $1+px+\ldots$, so this becomes $E\approx(kq/r^2)(1+2u-1+2u)=2kq\ell/r^3$. As claimed, this is proportional to r^{-3} and depends only on the dipole moment $q\ell$. It is double the field in the mid-plane at the same distance.

2-11 The result of problem 10 applies, so that each plate contributes a field $\pm2\pi k\sigma$. These fields cancel outside the capacitor, but reinforce on the inside, giving $E=4\pi k\sigma=4\pi kq/A$. The result is independent of h.

2-12 Let the square's sides be of length a. The field at the center is the vector sum of the fields that would have been produced individually by the three charges. Each of these individual fields is kq/r^2, where $r_1=a/\sqrt{2}$ for the two charges q_1, and $r_2=a/2$ for q_2. Vector addition can be done by adding components. Let x be horizontal and y vertical. The y components cancel by symmetry. The sum of the x components is

$$E_x=kq_1r_1^{-2}\cos45°+kq_1r_1^{-2}\cos45°-kq_2r_2^{-2}$$

Substituting $\cos45°=1/\sqrt{2}$ and setting this whole expression equal to zero, we find $q_2/q_1=1/\sqrt{2}$.

2-13 (a) The setup is similar to the preceding problem. We again choose as our Gaussian surface a finite cylinder of length ℓ and radius R. Gauss's law gives $2\pi R\ell E=4\pi kq_{in}$, where $q_{in}=\pi R^2\ell\rho$. Solving for the field, we have $E=2\pi kR\rho$.

(b) At $R=0$, we get $E=0$. This makes sense, because the field has to vanish on the axis by symmetry. If it were to be nonzero, there would be no way to decide which direction it should point.

(c) The units of $E=2\pi kR\rho$ are

$$\frac{\mathrm{N}}{\mathrm{C}}=?\left(\frac{\mathrm{N{\cdot}m^2}}{\mathrm{C^2}}\right)(\mathrm{m})\left(\frac{\mathrm{C}}{\mathrm{m^3}}\right),$$

which does check out.

2-14 (a) Let each fixed charge be Q, and let the charge of the ball be q. When the ball is at position x, its distance from one of the fixed charges is $\sqrt{b^2+x^2}$, so the magnitude of each force on it is $|\mathbf{F}|=kQq/(b^2+x^2)$. There are two such forces, acting at different angles, so we have to add them using vector addition. Their y components cancel, and their x components are the same, so for the total force on the ball, we have

$$\begin{aligned}F&=2F_x\\&=\frac{2kQq}{b^2+x^2}\cdot\cos\theta\\&=\frac{2kQq}{b^2+x^2}\cdot\frac{x}{\sqrt{b^2+x^2}}\\&=2kQq\left(b^2+x^2\right)^{-3/2}x\end{aligned}$$

(b) By $a=F/m$, the maximum acceleration occurs at the same point as the maximum force. To find the maximum force, we have to set the derivative of the force equal to zero and solve for x. The constants out in front don't affect the location of the maximum, so we can ignore them.

$$\begin{aligned}0&=\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(b^2+x^2\right)^{-3/2}x\right]\\&=-\frac{3}{2}\left(b^2+x^2\right)^{-5/2}(2x)x+\left(b^2+x^2\right)^{-3/2}\end{aligned}$$

We can simplify by multiplying both sides by $\left(b^2+x^2\right)^{5/2}$.

$$\begin{aligned}0&=-3x^2+b^2+x^2\\x&=\frac{b}{\sqrt{2}}\end{aligned}$$

2-15 (a) The two particles are already at the greatest distance from one another that is allowed by conservation of energy, so neither can escape.

(b) By bringing two of the particles close together, we can reduce the gravitational potential energy to an arbitrarily large negative value. This allows the third particle to escape without violating conservation of energy. By similar reasoning, we expect that systems such as our solar system may, over long time scales, eject some of their members.

(c) The particles repel one another, so they will fly away to an infinite distance.

(d) This is the same as a.

(e) This is similar to b. We can bring one of the positive charges and one of the negative ones close together in order to release as much energy as required to eject the other positive one.

2-16 There can be no stable equilibrium in which the string is slack, since one could always move the movable charge along a path at a constant distance from the fixed charge, bringing it to a position on the circumference at which its gravitational energy would be lower.

The result can only depend on the unitless quantity $\beta = kq/mg\ell^2$. To make the writing easier, we take $\ell = 1$. The distance between the charges is $r = 2\sin\theta/2$, and for this reason it will be convenient to define the variable $p = \sin\theta/2$, which equals 0 at the bottom of the circle and 1 at the top.

For $\beta < 4$ there is an equilibrium in which the movable charge is directly above the fixed one, with the string slack. These are unstable with respect to lateral motion, and not very interesting.

For large β there is an equilibrium at $\theta = \pi$, but we need to investigate when this is stable.

Using the identity $1 - \cos\theta = 2\sin^2\theta/2$, we find for the potential energy

$$U = \frac{1}{2}\beta p^{-1} + 2p^2,$$

where the first term is electrical and the second gravitational. Although it is θ that is physically relevant, it is easier to do as much of the calculus as possible with p as the variable, since the expressions are simpler. Expressed as a function of p, the energy is always concave up, so any point with $\mathrm{d}U/\,\mathrm{d}p = 0$ will be a stable equilibrium. In addition, we have $\mathrm{d}U/\,\mathrm{d}\theta = 0$ automatically at $\theta = \pi$, since $\mathrm{d}p/\,\mathrm{d}\theta = 0$ there.

We have $\mathrm{d}U/\,\mathrm{d}p = -(1/2)\beta p^{-2} + 4p$, which equals zero for $p = (\beta/8)^{1/3}$. As remarked earlier, this is always stable. However, it only exists for $\beta \le 8$, since we must have $p \le 1$.

To evaluate the stability of the upright equilibrium, we evaluate $\mathrm{d}^2U/\,\mathrm{d}\theta^2$ at $\theta = \pi$, which gives $\beta/8 - 1$. This is positive if $\beta > 8$. Thus although levitation straight up, with the cord under tension, is possible for $4 < \beta < 8$, this equilibrium is not stable against lateral motion for these values of β.

In summary, we find that for $\beta \le 8$ the stable equilibrium is at

$$\sin\frac{\theta}{2} = \left(\frac{\beta}{8}\right)^{1/3},$$

while for $\beta \ge 8$ the stable and only equilibrium is the one in which the cord is taut and upright.

3-2 (a) The total force on the carbon is zero by symmetry. Let positive be to the right, and consider the forces acting on the oxygen on the right. The total force is $kqQ/\ell^2 + kQ^2/(2\ell)^2$, where ℓ is the spacing between the atoms. Setting this to zero gives $qQ + Q^2/4 = 0$, or $q/Q = -1/4$.

(b) The total charge is $2Q + q = (7/4)Q$, which is nonzero.

(c) Counting all three pairs, the total electrical potential energy is $kQ^2/2\ell + kqQ/\ell + kqQ/\ell$, which comes out to be zero. This makes sense because if we moved the oxygen atoms outward symmetrically at the same time, the force on them would always be zero, and therefore we could bring them to infinity with zero work. But this is another feature of this model that fails when compared to reality, since in reality carbon dioxide molecules are stable, bound systems.

3-3 Electricity is an inverse square force, so if r is smaller by a factor of 206.77, the force will be greater by a factor of $206.77^2 = 42754$.

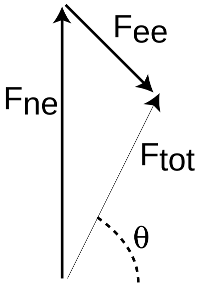
3-4 In the decay

$$\text{proton} + \text{electron} \rightarrow \text{neutron} + \text{neutrino},$$

the charges are $e + (-e) = 0 + 0$, so charge is conserved.

3-5 Let's use positive numbers for forces to the right, and negative numbers for forces to the left. There are two forces on the right-hand electron: a repulsive force from the other electron, and an attractive force from the nucleus. These will be represented as positive and negative numbers, respectively. The charges of the electrons are $-e$, and the charge of the nucleus is $+2e$, so the electron-electron force is $+ke^2/(0.2\text{ nm})^2$, and the nucleus-electron force is $-k(e)(2e)/(0.1\text{ nm})^2$, for a total force of -4×10^{-8} N.

3-6 You should always start this type of vector addition problem by drawing a rough diagram first so that you can check that your results make sense at every step.



The nucleus's force on the bottom electron, F_{ne}, is attractive, so it points up. The other electron's force on the bottom electron, F_{ee}, is repulsive, so it points down and to the right at a 45-degree angle. We'll state the final result using the angle θ defined in the figure. The magnitude of F_{ne} is given by Coulomb's law, $|F_{ne}| = k(e)(2e)/(0.1\text{ nm})^2 = 4.6\times10^{-8}$ N. The magnitude of F_{ee} is smaller by a factor of four: a factor of two because of the smaller charge, and another factor of two because its distance from the bottom electron is greater by a factor of $\sqrt{2}$. Thus, $|F_{ee}| = 1.1\times10^{-8}$ N. To do vector addition, we need to add components. Let positive x components be to the right, positive y components up. $\mathbf{F}_{ne}$ has components $F_{ne,x} = 0$ and $F_{ne,y} = 4.6\times10^{-8}$ N. $\mathbf{F}_{ee}$ has components

$$F_{ee,x} = +(1.1\times10^{-8}\text{ N})(\cos 45°) \qquad \text{and}$$
$$F_{ee,y} = -(1.1\times10^{-8}\text{ N})(\sin 45°) \qquad .$$

At this point, you can check the + and − signs of the components against the figure. The total force has components

$$F_{tot,x} = F_{ne,x} + F_{ee,x} = 7.8\times10^{-9}\text{ N} \qquad \text{and}$$
$$F_{tot,y} = F_{ne,y} + F_{ee,y} = 3.8\times10^{-8}\text{ N} \qquad .$$

Again, you can check the + and - signs against the figure. The magnitude of the total force is $\sqrt{F^2_{tot,x}+F^2_{tot,y}} = 4\times10^{-8}$ N (one sig fig). The angle θ equals $\tan^{-1}(F_{tot,y}/F_{tot,x}) = 80°$ (one sig fig).

3-7 The lithium is located nice and symmetrically in the lattice, so if there was no defect, the total force on it would be zero. Removing a Cl^- to make the hole removes a certain amount of force, so the total force without the Cl^- is equal to minus the force the Cl^- would have made, which we can find from Coulomb's law. The distance between the lithium and the hole is $\sqrt{(2.5\times0.3\text{ nm})^2+(0.5\times0.3\text{ nm})^2} = 0.77$ nm, so the magnitude of the force is $ke^2/(0.77\text{ nm})^2 = 3.9\times10^{-10}$ N, or call it 4×10^{-10} N. The Li^+ would have been attracted by the Cl^-, so removing that attractive force leaves a total force that points directly away from the hole.

4-2 (a) The gravitational potential energy is $U = mgh$, where h is the height relative to some arbitrary reference point. A difference in potential energy is $\Delta U = mg\Delta h$, and the difference per unit mass is $\Delta\phi = g\Delta h$. We then have $\phi_B - \phi_A = 3g\ell$ and $\phi_C - \phi_B = 8g\ell$.

(b) The "electric field" is $\Delta\phi/\Delta x$, which gives $(3/4)g$ for the first segment and $4g$ for the second. The ratio is 16/3.

(c) The difficulty of the climbing doesn't depend on altitude, only on differences in height.

4-4 In each case, the electric field is minus the derivative of the potential.

Potentials 1 and 2 give the same field, $E = -a$. These are uniform fields. The constant a is the strength of the field, and b is just an arbitrary constant with no physical significance.

Potentials 3 and 4 both give $E = a/x^2$. This looks like the field of a point charge. The constant a is basically the charge (or the charge multiplied by k), and as before b has no physical significance.

4-6 (a) The field is uniform, so it's conservative, and therefore going from A to D by either path should take the same amount of work per unit charge, 1 J/C. To check this, we can break the square down into parts. The work from A to C or B to D is zero, since the motion is perpendicular to the force. Therefore the work per unit charge from A to B must be 1 J/C. Since the field is uniform, this is the same as the work done from C to D, which makes sense: the total along ACD is 1 J/C.

(b) Visualizing a curl-meter, we can see that this field has a curl. It's not conservative, so there is no guarantee that the work along EGH is the same as along EFH. In fact, the work along EGH is zero. First we go from E to G, and the work is zero because the field is zero. Then we go from G to H, and the work is again zero, because the motion is perpendicular to the field.

(c) A potential can be defined if and only if the field is conservative. Pattern a is conservative and b is not.

4-7 When a function has a constant derivative, we don't really need calculus and can find $\Delta\ldots/\Delta\ldots$, which is the same as $d\ldots/\ldots$ Ignoring signs, the field is

$$\begin{aligned} E &= (\text{ignored } -)\frac{\Delta\phi}{\Delta x} \\ &= \frac{1.5\ \text{V}}{4.2\times10^{-2}\ \text{m}} \\ &= 36\ \text{V/m}. \end{aligned}$$

4-8 The relationship between field and potential is $E=-dV/dx$. We don't have any information about the field pattern, so for the sake of this estimate we can only assume it's constant, so that $E=-\Delta V/\Delta x$. Ignoring signs, which would only be meaningful if we had established a coordinate system,

$$\begin{aligned} \Delta V &= (\text{ignored } -)E\Delta x \\ &= (1.5\times10^{5}\ \text{V/m})(1\times10^{-4}\ \text{m}) \\ &= 15\ \text{V}. \end{aligned}$$

This is a fairly reasonable voltage to have inside a piece of consumer electronics.

4-10 $\phi=-\int E dr=(1/2)br^{-2}$

4-11 This is the opposite of the previous problem. To find the field given the voltage, we take the derivative:

$$\begin{aligned} E &= -\frac{\mathrm{d}}{\mathrm{d}r}\frac{k}{r^3} \\ &= \frac{3k}{r^4} \end{aligned}$$

4-12 $E=-d\phi/dx=-(4/3)cx^{1/3}$.

4-13 (a) Its volume is (length) × (circumference) × (thickness) = $(L)(2\pi r)(t)$. The electric field is $E=V/t$. The energy stored is

$$\begin{aligned} \frac{1}{8\pi k}|\mathbf{E}|^2\times(\text{volume}) &= \frac{1}{8\pi k}\left(\frac{V}{t}\right)^2(2\pi rLt) \\ &= \frac{V^2rL}{4kt} \end{aligned}$$

(b) Making r small means that the neuron needs less energy stored in it to have voltage V across its membrane, which may explain why they evolved to be so skinny. There is a limit on how small r can be, however, because the cell can't be so skinny that it lacks room inside for its machinery. Having a thicker membrane would also reduce the energy needed (dividing by a larger number gives a smaller result), but a cell membrane can't be arbitrarily thick, because it needs to take in nutrients and expel wastes through it.

4-14

$$\begin{aligned} V &= -\int E\,\mathrm{d}r \\ &= -2k\lambda\ln r+\text{const.} \end{aligned}$$

This is a little bit of a strange-looking expression, because normally we can't evaluate a transcendental function on an input that has units. However, we can get away with this in the case of a log, because $\ln(a/b)=\ln a-\ln b$, so if we wanted to make the units not look so wrong, we could write this as $-2k\lambda(r/r_0)$, where r_0 is related to the original constant by $r_0=\exp(-\text{const.})$.

4-15 (a) $\mathbf{E}=-\nabla\phi$. In terms of components, we have

$$E_x=-2ax,$$

and similar equations for y and z.

(b) The Pythagorean theorem gives $|E|=2ar$.

(c) The result from the example was that for a sphere of uniformly distributed charge, $E = (4\pi k\rho/3)r$, which is the same form as our field, with $a = 2\pi k\rho/3$.
(d) It would have had no effect. Mathematically, the gradient operator ∇ is a kind of derivative, and derivatives are linear and have the property that the derivative of a constant is zero. Therefore adding a constant onto an expression never has any effect on its derivative. Physically, the potential ϕ is a measure of potential energy (per unit charge), and a potential energy can always have a constant added onto it without affecting the physics.

4-16 (a) Pattern 1 has a nonzero divergence at the two points where the field lines begin and end. Pattern 2 has a zero divergence everywhere, because the field lines don't begin or end. If pattern 1 is really the physically correct electrostatic field of a two-charge dipole, then it has zero curl. This looks plausible if we imagine inserting a curl-meter. Pattern 2 clearly has a nonzero curl, as can be seen, for example, by visualizing a curl-meter inside one of the D-shaped loops.
(b) Pattern 1 can't be a magnetic field pattern, because it has a nonzero divergence.
(c) Pattern 2 can't be a static electric field, since it has a nonzero curl.
(c) The nonzero curl of pattern 2 also implies that it can't be expressed as the gradient of a potential.

4-17 (a) The voltage at the surface of the large sphere is kq_a/a, and on the small one kq_b/b. Since they're connected by a wire, they must be at equal voltage. Equating the two expressions gives $q_a/q_b = a/b$.
(b) The area of a sphere is proportional to r^2, so the charge per unit area is proportional to qr^{-2}. Therefore

$$\frac{\text{density of charge on sphere } b}{\text{density of charge on sphere } a} = \frac{q_b b^{-2}}{q_a a^{-2}} = \frac{a}{b}$$

4-18 The basic idea is to find the voltage V at the center of the triangle (defining the voltage infinitely far away to be zero, as usual). We then use conservation of energy to find the minimum initial kinetic energy, which corresponds to some minimum initial speed. A little trig shows that the distance r from the center to one of the vertices is given by $r = (b/2)/\cos 30° = b/\sqrt{3}$, so

$$\begin{aligned} V &= \frac{3kQ}{r} \\ &= \frac{3\sqrt{3}kQ}{b} \\ qV &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2qV}{m}} \\ &= \sqrt{\frac{6\sqrt{3}kQq}{mb}} \end{aligned}$$

4-19 (a) Conservation of energy gives

$$\begin{aligned} U_A &= U_B + K_B \\ K_B &= U_A - U_B \\ \frac{1}{2}mv^2 &= e\Delta V \\ v &= \sqrt{\frac{2e\Delta V}{m}} \end{aligned}$$

(b) Plugging in numbers, we get 5.9×10^7 m/s. This is about 20% of the speed of light, so the nonrelativistic assumption was good to at least a rough approximation.

4-20

4-21 (a) This is similar to the example of the spherical capacitor in section 3.6. The electric field in this geometry was found in section 2.8 to be proportional to $1/r$, by applying Gauss's law to a cylindrical Gaussian surface: as the cylinder's radius grows, its area grows like r to the first power, so the field must fall off by the same factor in order to keep the flux constant.

So you could say that we already know the electric field, but we only know it up to a constant factor. As in the spherical example, the problem boils down to finding that constant. The potential difference between the inner and outer conductors is (ignoring signs)

$$\Delta\phi = \text{(ignored sign)} \int_a^b \frac{c}{r}\,\mathrm{d}r,$$

where the constant c is to be determined. This gives $\Delta\phi = c\ln(b/a)$, or $c = \Delta\phi/\ln(b/a)$, so the result for the field is

$$E = \frac{\Delta\phi}{\ln(b/a)} \cdot \frac{1}{r}.$$

To make the sign meaningful, we would have to pick a coordinate system, and would also have to know whether $\Delta\phi$ was defined as $\phi_b - \phi_a$ or $\phi_a - \phi_b$, which wasn't specified in the statement of the problem.

(b) When $b = a$, we have $\ln(b/a) = 0$, and $E = \infty$. This makes sense because the only way to have a finite voltage change over an infinitesimal distance is if the field is infinite. When $b < a$, the log is negative, which flips the sign of the field, and that makes sense, because it's like we've switched the electrodes around.

4-22 If there was no hole in the lattice, then by symmetry, the potential at any of the available gaps would be the same. Removing the Cl^- to make a hole will then change the potentials by an amount equal to minus its original contribution. Its original contribution was $kq/r = -ke/r$, so the potential at each point in space is changed by $+ke/r$. Now the lithium is going to jump from the gap it occupies in the figure, where the potential is V_1, to some neighboring gap with potential V_2, losing potential energy of $e(V_1 - V_2)$. Without the hole, we would have had $V_1 - V_2 = 0$, but removing the Cl^- to make the hole changes both V_1 and V_2. V_1 changes by an amount ke/r_1, where r_1 is the distance from the original gap to the hole, and similarly for V_2. The potential energy lost by the lithium is therefore $ke^2(1/r_1 - 1/r_2)$, so that is also the amount of KE it gains. To get its speed, solve $KE = (1/2)mv^2$ for $v = \sqrt{2KE/m} = e\sqrt{2k(1/r_1 - 1/r_2)/m}$. It is going to jump to the gap to the left, since that is the direction closest to the direction of the force on it as found in the previous homework problem. The Pythagorean theorem gives $r_1 = 0.77$ nm and $r_2 = 1.06$ nm, the mass of a lithium atom is about 7 times the mass of a proton, and plugging in to the equation for the velocity gives 3.7×10^3 m/s, or call it 4000 m/s.

4-23 (a) The answer must be of the form $\alpha q\sqrt{k/mg}$, where α is a unitless constant.

(b) We can place an image charge $-q$ above the conducting plane, at a distance 2ℓ from the real charge. The force is $(1/4)kq^2/\ell^2$. Balancing this against the gravitational force mg gives $\ell = (1/2)q\sqrt{k/mg}$.

(c) If the charge moves upward from the equilibrium position, the upward electrical force gets stronger, so there is a net upward force. Similarly, a downward displacement gives a net downward force.

4-24 (a) Because θ is unitless, it can only depend on the unitless ratio y/x.

(b) The trick here is to use three image charges: charges $-q$ at $(x, -y)$ and $(-x, y)$, and a $+q$ at $(-x, -y)$. This is exactly analogous to the formation of images in a pair of mirrors placed at right angles to one another. If we take this arrangement of charges, flip it across the y axis, and then negate all the charges, it becomes the same arrangement again. It follows that E_y vanishes, as required, on the y axis. A similar argument holds for the x axis.

By the result of part a, we can ignore factors of q and the Coulomb constant. Vector addition gives $4F_x = -1/x^2 + x/r^3$ and $4F_y = -1/y^2 + y/r^3$. The result is

$$\tan\theta = \frac{y - r^3/y^2}{x - r^3/x^2},$$

which can also be expressed in terms of y/x, but without much apparent simplification.

5-1 The momentum density needs to be in the $+x$ direction. Since it is perpendicular to both **E** and **B**, the magnetic field can't be along the x axis. It also can't be along the y axis, because then it would be parallel or antiparallel to **E**, and the the cross product $\mathbf{E} \times \mathbf{B}$ would be zero. The remaining possibilities are the $\pm z$ directions, and of these, the one that gives momentum in the $+x$ direction is if **B** is in the $-z$ direction.

5-3 We're discusssing the force acting *on* the *right*-hand sphere. There is tension parallel to the direction of the field lines in the center, which produces a force up and to the left. There is also pressure perpendicular to the field lines, which makes a force up and to the right. We would expect the vector sum to be approximately upward. By symmetry, the force acting on the left-hand sphere should be downward.

This is somewhat counterintuitive because we're used to forces that act from center to center. One thing that's unusual about this type of force is its effect on angular momentum. If the spheres are released from rest in this position, they will start to spin counterclockwise. To be consistent with conservation of angular momentum, there needs to be some clockwise angular momentum somewhere, and presumably this would be in the form of electromagnetic waves radiated by the spheres.

5-4 The electric force vector is

$$\begin{aligned}\mathbf{F}_E &= q\mathbf{E} \\ &= (1\ \text{N})\hat{\mathbf{y}} \qquad .\end{aligned}$$

The magnitude of the magnetic force is

$$\begin{aligned}|\mathbf{F}_B| &= |q\mathbf{v}\times\mathbf{B}| \\ &= q|\mathbf{v}\times\mathbf{B}| \\ &= q|\mathbf{v}||\mathbf{B}|\sin\theta_{vB} \\ &= 2\ \text{N} \qquad ,\end{aligned}$$

and its direction is determined by the right hand rule to be along the z axis, so

$$\mathbf{F}_B = (2\ \text{N})\hat{\mathbf{z}} \qquad .$$

The total force on the particle is

$$\mathbf{F} = (1\ \text{N})\hat{\mathbf{y}} + (2\ \text{N})\hat{\mathbf{z}} \qquad .$$

5-5 The electric force is either in the same direction as the electric field or in the opposite direction, depending on whether the particles are positively or negatively charged. Since the magnetic force is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ and the question states that the electric field is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$, the magnetic force must be either in the same direction as the electric force or in the opposite direction. Since the goal is to make the forces cancel, we'll assume the directions of the fields are chosen so the forces are in opposite directions. That means the forces cancel if they're chosen to have equal magnitudes. The electric force equals $q\mathbf{E}$. The magnitude of the magnetic force equals $qvB\sin\theta$, and since the $\mathbf{v}$ and $\mathbf{B}$ vectors are perpendicular, $\sin\theta = 1$. Therefore we need $qE = qvB$. Canceling q from both sides and solving for v, we have $v = E/B$.

5-6 There must be either $\mathbf{E}$ or $\mathbf{B}$ or both, since some force caused the particle to accelerate. It can't be a pure magnetic field, because $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$ is zero for $\mathbf{v} = 0$, so the particle would not have accelerated from rest. It can't be a pure electric field, because a uniform electric field would just have caused the particle to accelerate in a straight line. There must be both $\mathbf{E}$ and $\mathbf{B}$ present.

5-7 (a) $\mathbf{F} = q\mathbf{v}\times\mathbf{B} = 0$, so the particle remains at rest.
(b) $\mathbf{F} = q\mathbf{v}\times\mathbf{B} = q|\mathbf{v}||\mathbf{B}|\sin 0° = 0$, so there is no force, and the particle travels in a straight line with constant speed.
(c) The force will always be perpendicular to the z axis, so since the particle initially has no velocity in the z direction, it will remain in the same plane, parallel to the x-y plane. The particle orbits in a circle in this plane, at constant speed.

5-8 The second data-point isn't very surprising, because multiplying charge by -2 and the velocity by -1 multiplies the magnetic force by $(-2)(-1) = 2$. The only thing the second data-point really tells us is that the field is purely magnetic. If the $\mathbf{F}$, $\mathbf{v}$, and $\mathbf{B}$ vectors had to be perpendicular, there would only be one possibility for this field, $\mathbf{B} = (1\ \text{kT})\hat{\mathbf{z}}$. But a magnetic field along the same direction as the motion exerts no force, so we can throw in an arbitrary x component, giving a solution such as $\mathbf{B} = (1\ \text{kT})\hat{\mathbf{z}} + (37\ \text{kT})\hat{\mathbf{x}}$.

5-9 It can't be a purely electric field, because then the velocity would have no effect on the force. It can't be purely magnetic, either, because tripling the velocity didn't simply triple the force, as it would if the only force was $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$.

5-10 Particle 1 isn't moving, so it experiences only an electric force. The electric field causing this force must be

$$\frac{(1\ \text{N})\hat{\mathbf{y}}}{1\ \text{C}} = (1\ \text{N/C})\hat{\mathbf{y}} \qquad .$$

Particle 2 experiences the same force as particle 1, even though it is moving along the x axis. Therefore, the magnetic field, if any, must lie along the x axis.

The magnetic and electric forces on particle 3 cancel, so the magnetic force on it must be $(-1\ \text{N})\hat{\mathbf{y}}$. A magnetic field along the x axis that produces this force must be $(-1\ \text{T})\hat{\mathbf{x}}$.

5-11 At first it might seem possible, since we're trying to determine six numbers (E_x, E_y, E_z, B_x, B_y, and B_z), and two experiments would provide six pieces of raw data (F_{1x}, F_{1y}, F_{1z}, F_{2x}, F_{2y}, and F_{2z}). Ordinarily, we expect that six equations in six unknowns can be solved uniquely. However, this isn't true if the equations are redundant. For example, the system of equations

$$\begin{aligned} a &= 2b \\ 2a &= 4b \end{aligned}$$

can't be solved for a and b.

In fact, we can never solve these six equations for the six unknowns. Suppose we fix the design of your experiment by specifying $\mathbf{v}_1$ and $\mathbf{v}_2$. We may as well use unit charge in both cases, so these two velocities give a complete specification of the design of the experiment. But given this design, there will always be different fields that give the same results. To see this, let's find $\mathbf{E}$ and $\mathbf{B}$ that produce zero force in both measurements. We want fields such that

$$\begin{aligned} \mathbf{E} + \mathbf{v}_1\times\mathbf{B} &= 0 \\ \mathbf{E} + \mathbf{v}_2\times\mathbf{B} &= 0 \qquad . \end{aligned}$$

Subtracting,

$$(\mathbf{v}_1 - \mathbf{v}_2)\times\mathbf{B} = 0 \qquad .$$

It's easy to show that the procedure won't work if the two velocities are the same, so let's just assume the vector $\mathbf{v}_1 - \mathbf{v}_2$ is nonzero. We can take $\mathbf{B}$ to lie along $\mathbf{v}_1 - \mathbf{v}_2$, and let $\mathbf{E} = -\mathbf{v}_1\times\mathbf{B}$.

5-12 (a) See the answer to the self-check.
(b)

$$\begin{aligned} a &= \frac{F}{m} \\ \frac{v^2}{r} &= \frac{qvB\sin 90°}{m} \\ \frac{v}{r} &= \frac{q}{m}B \\ r &= \frac{m}{q}\cdot\frac{v}{B} \end{aligned}$$

(c) From the definition $B = F/(qv\sin\theta)$, we see that the Tesla unit breaks down into N·s/(C·m).

$$\begin{aligned} \frac{\text{kg}}{\text{C}}\cdot\frac{\text{m/s}}{\text{T}} &= \frac{\text{kg}\cdot\text{m}}{\text{C}\cdot\text{s}}\cdot\frac{\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}} \\ &= \frac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot\frac{1}{\text{N}}\cdot\text{m} \\ &= \text{m} \end{aligned}$$

(d) The equation derived in part b has m and v on the top, so it states that increasing either of these variables will increase the radius of the circle. The opposite is true for q and B.
mass: Increasing the mass of the particles makes it harder for a given force to turn the beam away from its initial path, so the turn will be wider.
charge: Increasing the charge of the particles makes it easier for a given field to act on them, causing them to execute a tighter curve.
velocity: This is like driving a car. Freeway off-ramps have gentle curves, because a high-velocity car is harder to turn.
field: Increasing the magnetic field results in more force on each particle, making the beam curve more tightly.
(e)

$$\begin{aligned} v &= \omega r \\ \omega &= \frac{v}{r} \\ &= \frac{qB}{m} \end{aligned}$$

5-13 (a) In time t, the length that passes by a fixed point is $\ell = vt$, and the charge in this portion of the line is $q = \lambda\ell = \lambda vt$. The current is $I = q/t = \lambda v$.

(b) The charge on this portion of the wire is $q = \lambda\ell$, so the Lorentz force law gives the force acting on it as $F = qvB = \lambda\ell vB = I\ell B$, which leads to the claimed result.

5-14 They're the same. When we have a charge distribution with zero total charge, its dipole moment is unchanged when we move it around without rotating it, since the dipole moment is defined by the potential energy, and the potential energy is unchanged when we do this. These two distributions therefore have the same dipole moment, since we can make one into the other by sliding one of the +− pairs.

5-16 (a) The maximum kinetic energy will occur when the electrical potential energy of the dipole is at a minimum, which occurs when the dipole is pointed in the same direction as the field, i.e., at 90 degrees compared to its initial orientation. Conservation of energy gives

$$\begin{aligned} U_i + K_i &= U_f + K_f \\ 0 &= -q\ell E + \frac{1}{2}I\omega^2. \end{aligned}$$

The moment of inertia is (2) $\left[(1/2)m(\ell/2)^2\right]$, so we end up with

$$\omega = \pm 2\sqrt{\frac{qE}{m\ell}}$$

(b) The square root can have either sign. This represents the physical fact that the dipole will oscillate indefinitely back and forth, achieving the maximum $|\omega|$ over and over again, but with opposite signs.

(c)

$$\begin{aligned} \text{s}^{-1} &=? \sqrt{\frac{\text{C}\cdot\text{N/C}}{\text{kg}\cdot\text{m}}} \\ &=? \sqrt{\frac{\text{N}}{\text{kg}\cdot\text{m}}} \\ &=? \sqrt{\frac{\text{kg}\cdot\text{m/s}^2}{\text{kg}\cdot\text{m}}} \\ &=? \text{s}^{-1} \end{aligned}$$

Yes, the units do check out.

5-17 The distant field of a dipole goes like r^{-3}. Since $B \propto r^{-3}$, we have

$$\frac{B(10r)}{B(r)} = \left(\frac{10r}{r}\right)^{-3} = 10^{-3}.$$

Since we're notating $B(r)$ as B, the field at the more distant point is $10^{-3}B$.

5-19 (a) In units of 10^{-27} kg, the amount of mass lost is

$$1.67495 - 1.67265 - 0.00091 = 0.00139 \qquad ,$$

i.e., 0.00139×10^{-27} kg. This mass has been converted into energy:

$$\begin{aligned} E &= mc^2 \\ &= (0.00139 \times 10^{-27})(3 \times 10^8)^2 \\ &= 1.25 \times 10^{-13} \text{ joules} \qquad . \end{aligned}$$

(In the meter-kilogram-second system of units, kilograms multiplied velocity squared give units of joules.)

(b) For this process, the total mass changes from 1.67265 to 1.67586 (again in units of 10^{-27} kg). This is an *increase* in mass, which means that we would have to have a source of energy to make the reaction happen. A free proton has no such source of energy; it's all by itself, so there's nothing for it to get energy from.

5-20 (a) Newton's second law gives $evB = mv^2/r$, and as analyzed in a previous problem, the condition for particles of velocity v not to be deflected in the first section is $E/B = v$. Eliminating v gives $h = 2r = 2Em/B^2e$, or

$$\frac{\Delta h}{\Delta m} = \frac{2E}{eB^2}.$$

(b) The units are

$$\begin{aligned}\frac{\mathrm{V/m}}{\mathrm{C{\cdot}T^2}} &= \frac{\mathrm{V{\cdot}C^2{\cdot}m^2}}{\mathrm{m{\cdot}C{\cdot}N^2{\cdot}s^2}} \\ &= \frac{\mathrm{N{\cdot}m^2}}{\mathrm{N^2{\cdot}s^2}} \\ &= \frac{\mathrm{m}}{\mathrm{kg}}.\end{aligned}$$

(c) It makes sense that the sensitivity increases with E, because a higher E allows the use of faster particles, which make wider circles. It also makes sense that the sensitivity decreases with B, because if B is small, there is little force causing the particles to curve, and the circles get big. Making B very small will help with sensititivity, but it comes at a price, which is that one has to build a very large vacuum chamber.

5-21 (a) In SI units, we need to show that $\mathrm{J/m^3} = \mathrm{N/m^2}$. But this is clearly true, since work equals force times distance, so 1 J = 1 N·m.
(b) We already know that the energy density is equal to E^2/k times a unitless constant. Therefore E^2/k also has the right units to be a pressure. We could worry about whether there is some other proportionality we could form from these quantities, with different exponents, that would also have units of pressure. But this can't be done, because the base SI units of k contain kilograms to the first power, and so do the base units of the field and the pressure.

6-1 The wave propagates in the direction of the Poynting vector, which is in the direction of $\mathbf{E} \times \mathbf{B}$. Using the right-hand rule, this is in the $-x$ direction.

6-2 Since it's a plane wave, the direction of the magnetic field is perpendicular to the electric field and the direction of propagation, so it must be in the $\pm y$ direction. What gives $\mathbf{E} \times \mathbf{B}$ in the positive z direction is if the magnetic field is in the $+y$ direction.

The Poynting vector is $\mathbf{S} = (c^2/4\pi k)\mathbf{E} \times \mathbf{B}$. Because $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other, the magnitude of their cross product is $|\mathbf{E}||\mathbf{B}|$. We also have $|\mathbf{E}| = c|\mathbf{B}|$ for a plane wave, so $|\mathbf{E} \times \mathbf{B}| = cB^2$. Putting this together,

$$a = \frac{c^2}{4\pi k},$$

so

$$B = \sqrt{\frac{4\pi k a}{c^3}}.$$

6-4 The given information is the Poynting vector, and we need to find the momentum. Comparing the expressions for the Poynting vector and the momentum density, we find

$$\frac{p}{v} = c^{-2}S.$$

The result is $p = 15$ kg·m/s.

6-7 The Poynting vector of the superimposed waves is $(\mathbf{E}_1+\mathbf{E}_2)\times(\mathbf{B}_1+\mathbf{B}_2)$. If we multiply this expression out, we get four terms. Two of these are just $\mathbf{S}_1$ and $\mathbf{S}_2$, the Poynting vectors of the separate waves. Now suppose the polarization of the two waves differs by an angle θ. Because they are propagating in the same direction, this angle θ is both the angle between the electric field vectors and the angle between the magnetic field vectors. The remaining terms in the cross product equal $(E_1B_2 + E_2B_1)\sin(\theta + \pi/2)$. As a function of the randomly chosen θ, the sine factor oscillates symmetrically through positive and negative values, and therefore averages to zero.

7-1 In the figure, a field that was purely electrical in the original frame would lie on the bottom edge of the square. A field that was purely magnetic in the new frame would lie on the slanted left side of the parallelogram. The answer to the question is no, because a Lorentz transformation will never bring one of these lines on top of the other; Lorentz transformations squish and stretch the coordinate grid, but they never flip it over.

7-2 (a) She says her own velocity is zero.

Time dilation happens because of relative motion. We can conclude that A and B are not in motion relative to one another, but that there is relative motion between them and C.

(b) Alice says B is at rest, but C is moving. (c) Betty says A is at rest, but C is moving. (d) Cathy says both A and B are moving, and she sees them moving in the same direction at the same speed.

7-3 The equation for gamma is $\gamma = 1/\sqrt{1-v^2}$. If v is negative, v^2 still comes out positive. Negative and positive values of v give the same results for gamma.

This makes sense, because positive and negative values of v indicate motion in opposite directions. Relativistic effects like time dilation and length contraction shouldn't come out different if you travel west rather than east, or north rather than south. Gamma indicates how strong these relativistic effects are, so gamma shouldn't depend on the direction of motion. This relates directly to the second postulate stated in section 7.2, that all directions in space have the same properties.

Note that it *is* OK for a directional (vector) quantity like velocity to be connected to some other direction quantity, such as acceleration, by a relation such as $a = dv/dt$. Then an observer who chooses the opposite orientation for their coordinate system will simply flip all the signs of the velocities and accelerations, and there is no way to tell in experiments which choice is "special." That's different from the situation with γ, which is not directional and can be measured without even choosing a coordinate system.

7-4 (a)

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-(v/c)^2}}\\ &= \frac{1}{\sqrt{1-\left(\frac{17000\ \text{m/s}}{3\times10^8\ \text{m/s}}\right)^2}}\\ &= 1.0000000016\end{aligned}$$

(b) Gamma gives us a comparison of the rate at which time flows on earth and on the probe. A gamma of one would indicate equal rates. Since the gamma is a tiny bit greater than one, the ratio is a tiny bit different from a one-to-one ratio. Gamma differs from one by 1.6×10^{-9}, i.e., 1.6 parts per billion. The disagreement between clocks on the probe and on earth that accumulates over each year is therefore

$$(31\times10^6\ \text{s})(1.6\times10^{-9}) = 0.050\ \text{s} \qquad .$$

This is a fairly big discrepancy. You might even be able to detect it with a consumer-grade clock. (The way NASA really notices the effect is that when the probe signals back to earth by radio, the radio waves vibrate a little more slowly than they should, i.e., the signal's frequency is shifted a tiny bit.)

7-5 Let $r = 6.4\times10^6$ m be the radius of the earth, $R = 1.5\times10^{11}$ m the radius of its orbit, and $T = 1$ year $= 3.2\times10^7$ s the period of its orbit. Then the velocity is $v = (2\pi R)/T$, and we have $\gamma = 1/\sqrt{1-v^2/c^2} = 1 + 4.8\times10^{-9}$. The resulting contraction of the diameter is about $(4.8\times10^{-9})(2r) = 6$ cm.

7-6 (a)

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-v^2/c^2}}\\ &= \frac{1}{\sqrt{1-9/25}}\\ &= \frac{1}{\sqrt{16/25}}\\ &= \frac{5}{4}\end{aligned}$$

(b) The $(3/5)c$ example came out simple because 3, 4, and 5 are integers that have the relationship $3^2+4^2=5^2$. This is just like a 3-4-5 right triangle, but there's no triangle involved here! We need to find another set of numbers like this. The simplest way is to reverse the role of the 3 and 4, in which case we get

$$v = \frac{4}{5} \qquad , \qquad \gamma = \frac{5}{3}$$

If that seems like a cheap shot, then you need to find another set of integers that form a right triangle; I was lazy, and just did a Google search for "Pythagorean triple," which turned up $11^2 + 60^2 = 61^2$ as an example. This provides the examples

$$v = \frac{11}{61} \quad , \quad \gamma = \frac{61}{60}$$

and

$$v = \frac{60}{61} \quad , \quad \gamma = \frac{61}{11} \quad .$$

8-2

$$\begin{aligned} I &= \mathrm{d}q/\,\mathrm{d}t \\ &= 2ct \\ P &= I\Delta V \\ &= I\cdot(IR) \\ &= I^2R \\ &= 4c^2t^2 \end{aligned}$$

8-3 $q = \int I\,\mathrm{d}t = \int bt\,\mathrm{d}t = \frac{1}{2}bt^2 + q_\mathrm{o}$
Here q_o is a constant of integration, representing the cloud's charge at time zero.

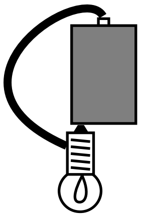
8-4 $I = \mathrm{d}q/\,\mathrm{d}t = b\sin\omega t + \omega(a+bt)\cos\omega t$

8-5 $R = \Delta V/I = (110\text{ V})/(0.9\text{ A}) = 120\ \Omega$

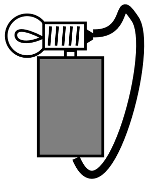
8-7 (a) $P = I\Delta V = I(IR) = I^2R$
(b) Energy dissipated in those wires is wasted in heating the neighborhood, so we want R to be small.

8-8 (a) $P = I\Delta V = (\Delta V/R)\Delta V = \Delta V^2/R$
(b) The power is proportional to ΔV^2, so cutting ΔV in half reduces the power by a factor of 4, to 50 W.

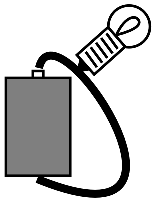
8-9



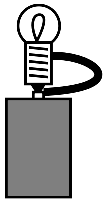
(a) good



(b) good



(c) no good



(d) no good

The bulb has two terminals. One is the bump on the end, and the other is the screw-threaded metal jacket. The battery also has two terminals: the bump on one end and the flat surface on the opposite end. In the two good methods, a and b, current can flow out of one terminal of the battery, into one terminal of the bulb, through the filament, out the other terminal of the bulb, and back in to the opposite terminal of the battery. In method c, there is a dead end in the bulb: the current could get in through the bump-shaped terminal, but there would be no way for it to get out of the screw-threaded terminal once it had gone through the filament. The other problem with *c* is that there is an easy way for current to go through the low-resistance wire, so the wire will get very hot and the battery will burn out. In method d, there is a complete loop of wire consisting of the bump-shaped terminal of the bulb, the filament, the screw-threaded terminal, and the wire that comes back to the bump-shaped terminal. This loop is all made out of conductors, and the voltage is equal at all points within a piece of conducting material. Therefore the voltage difference between the two ends of the filament is zero, and by Ohm's law, $V = IR$, no current will flow through the filament. Also, method d gives no way for current to get to the other terminal of the battery.)

8-10 (a) In one revolution, the amount of charge passing by a certain angular position is 1 nC. The time required for one revolution is $1/(33.3\ \text{min}^{-1})$, so $I = \Delta q/\Delta t = 3.3 \times 10^{-8}$ C/min = 600 pA.

(b) The idea is the same. The time for one orbit is $\Delta t = 2\pi r/v$, so $I = e/\Delta t = ve/2\pi r = 1$ mA.

8-11 To get negative values of P, you'd have to have opposite signs for I and ΔV. But you can't control the signs of I and ΔV independently — switching the wires reverses both. The current flows in response to the electric force, whose direction is related to the direction of the voltage drop. Another way of looking at it is that the electrons collide with the atoms, causing them to vibrate, and the vibration is what heat is. There is no way that randomly vibrating atoms will all hit the electrons in the same direction, producing a current.

8-12 (a) If the power is the same, then $P = I\Delta V$ tells us that tripling the voltage has to go along with cutting down the current to one third of what it was.

(b) By $P = I\Delta V$, the only way to get more power with the same voltage would be to increase the current. This would require fatter wires, which would be more expensive.

9-1 (a) The loop rule says that when you add up the voltage drops across the batteries plus the (opposite in sign) voltage drop across the bulb, you get zero. The batteries work the same as a single battery with a voltage equal to their sum.

(b) It apparently has thousands of cells in series. The "resistor" would be its prey.

9-2 (a) This has to be true based on the junction rule, so it doesn't tell you anything about the resistance values.

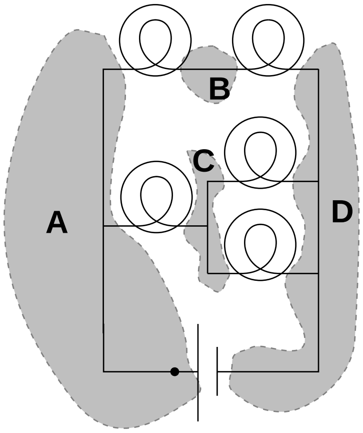
(b) We already knew without even touching the circuit that the currents were going to be equal, and if we find that the voltages are also equal then $V = IR$ says that the resistances must be equal.

(c) The loop rule tells us that the voltage drops are equal in the parallel circuit, and if we also find out the currents are equal, then $V = IR$ says the resistances must be equal.

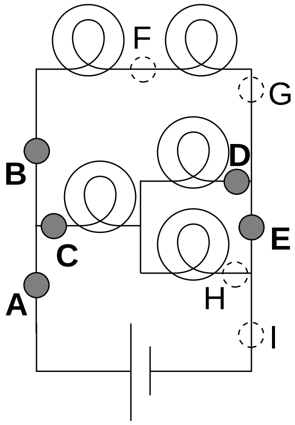
(d) This is not useful information for finding out about the resistors, since it has to be true based on the loop rule.

9-3 Using the junction rule at the junction A feeds into, we find that the current through A is 300 mA. Using the junction rule at the junction on the left, we then have 600 mA for the total current drawn from the battery, which is also the current through D. Applying the junction rule at the junction in the middle, we find a current of 100 mA for both B and C. All our unknowns are now determined, so we can use the junction on the right side to check that our solution is correct: 500 mA plus 100 mA gives 600 mA, which is indeed what we had for D.

9-4 There is constant voltage throughout a conductor. The wires are good conductors, but the bulbs aren't, so all we have to do is count how many parts there are in the circuit that consist of nothing but wire. As shown in the figure, there are four such parts. For instance, if we connect one probe of the voltmeter at the dot, and one in region D, we'll get a certain voltage reading, and we'll get the *same* voltage reading even if we move the second probe to some other place in region D. We can get a total of four unique voltage readings, but region A gives zero, so there are only three unique, nonzero readings we can get.



9-5 There are five unique currents you could measure, by inserting an ammeter at A, B, C, D, and E.



The key here is that charge is conserved. That means that, for example, you'll get exactly the same current reading at B, F, and G, because any charge that flows up through B will also have to go through F and G; it can't "drive off the road," and it can't get used up in the bulbs, because that would violate conservation of charge. Likewise, I and A give the same reading.

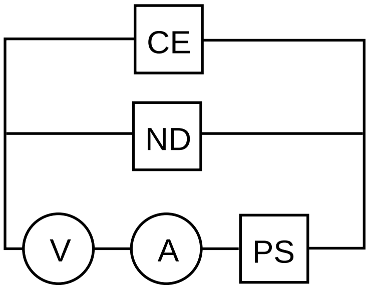
You also can't get two different results from D and H. This isn't because of conservation of charge, it's because the two bulbs are identical, and have the same voltage drop across them.

9-6 An open switch in (b) is the same circuit as a closed switch in (a): the light bulb is lit, and there's no problem. When you close the switch in (b), it does cause the bulb to go out (basically because the two sides of the bulb are connected with wire and therefore have no voltage difference, but see the more detailed note below).

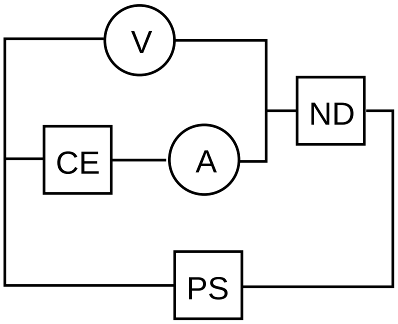
But huge amounts of current will now flow through the middle branch of the circuit. This is a short, which will certainly kill the battery, and may also cause enough heating to burn up the flashlight.

One subtle issue about this is that you might think closing the switch in (b) would have no effect on the bulb, since you're just adding a new parallel branch to a circuit. This would lead you to believe that although the short would kill the battery and burn things up, the bulb would stay on, contrary to the claim in the problem that it would go out. In fact, the bulb will go out in (b), because it's not a pure parallel circuit. You have to readjust your thinking a little, because ordinarily we assume that wires have negligible resistance, but in a short circuit their resistances aren't negligible anymore, because the equivalent resistance of the whole circuit is so low. That means you can't neglect the resistance of the wires going from the battery to the junctions. You basically have a series circuit now, consisting of the wire from the battery to the junction, the wire through the switch, and the wire from the switch back to the battery. This series circuit shares the voltage drop of the battery three ways, which means that the voltage across the middle piece is greatly reduced.

9-7

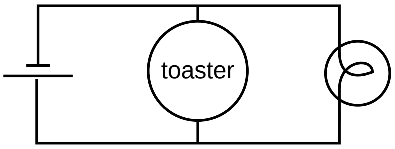


(a) They have correctly set up the CE and ND in parallel. The ammeter is measuring the total current out of the power supply, however, rather than just the current that is going to go through the ND. The voltmeter is hooked up in series, but it should be in parallel. A voltmeter has a very high resistance, so the circuit will not function when the voltmeter is hooked up this way.

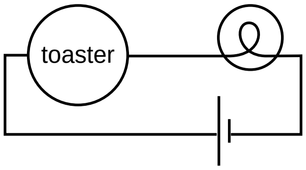


(b) The meters are hooked up correctly, but the CE and ND are in series rather than in parallel. (If they had hooked up the CE and ND in parallel, then they would have had to get the ammeter hooked up correctly to measure the current through the ND rather than the CE, but in this series circuit, both the CE and the ND have the same current through them.)

9-8



(a)



(c)

(a) See figure.

(b) The E-shaped piece of wire across the top is one big piece of conducting material, so it must have the same voltage throughout it. Similarly, the whole E-shaped piece on the bottom is at the same voltage. Therefore, the voltage difference across each of the three components is the same. Since we know the supply is 110 V, the voltage drops across the toaster and lamp must also be 110 V. Ohm's law gives $I_{toaster} = (110\ \mathrm{V})/R_{toaster} \approx 100$ A, and $I_{lamp} = (110\ \mathrm{V})/R_{lamp} \approx 50$ A.

(c) In this setup, which is not what's really used in your house, it's not the voltage drops that are equal but the currents — whatever current goes through one component also has to go through the others. The two resistances in series add together to make a total resistance of 3 Ohms, so $I_{toaster} = I_{lamp} = V/3\ \Omega \approx 40$ A. Ohm's law can now be used to find the voltage drop across each appliance, $V_{lamp} = I_{lamp}R_{lamp} = 73$ V, and $V_{toaster} = I_{toaster}R_{toaster} = 37$ V. (I didn't round the voltages off to one sig fig because I wanted to make it clear that they add up to 110 V.)

9-9 (a) Call the voltages V_5 and V_{10}. All three currents are the same, since it's a single closed loop, and there is no place to store charge. We therefore only need one symbol for current — just call it I. Kirchoff's loop rule says $V_5 + V_{10} =$ 9 volts. Ohm's law tells us $V_5 = (I)(5\ \Omega)$ and $V_{10} = (I)(10\ \Omega)$. This means that V_{10} must be twice as big as V_5, and since they have to add up to 9 volts, they must be $V_5 = 3$ V and $V_{10} = 6$ V.

(b) As explained above, all three currents are the same. One way to find the current is by solving the equation $V_5 = (I)(5\ \Omega)$ for I, giving $I = 0.6$ A. Another way would be to divide 9 V by the series resistance of 15 ohms.

(c) The 10-ohm bulb will go out, because its two sides are now connected with wire, and therefore there is no voltage difference between the two sides. The 5-ohm bulb now has the full voltage drop of the battery across it, i.e. 9 volts, and its current will be $(9\ \mathrm{V})/(5\ \Omega) = 1.8$ A. This means it will be brighter than it was before.

(d) You'll see exactly the same thing as in the original circuit. None of the reasoning used in solving part (a) made any assumption about what order the bulbs were in. The current does not "get tired" or "get used up" by having to go through one bulb first before it gets to the other. Conservation of charge says that charge can't "get used up."

9-11 All three in series gives one possibility. All three in parallel: one possibility. Two in series, in parallel with the third: three possibilities. Two in parallel, in series with the third: three possibilities. The total number of resistance values you can make is eight.

9-12 Resistance is inversely proportional to the cross-sectional area. The area depends on the square of the diameter, so the ratio of resistances is $(1.2)^2 = 1.4$.

9-13 (a) Let the voltage of the battery be V. In the original circuit, we have $P = IV = V^2/R$,

while in the new one $P' = V^2/(R+r)$. By forming the ratio of these two quantities, we can eliminate the voltage, and $P/P' = 1 + r/R$. The result is $r = (P/P' - 1)R$.
(b) The ratio P/P' is unitless, which makes sense because we're subtracting a unitless 1 from it. The result is that ohms equal ohms, which makes sense.
(c) For $P' = P$, we have $r = 0$, which makes sense: no additional resistance was introduced, no no change occurred. For $P' = 0$, we have $r = \infty$, which also makes sense. For example if the dirt is a perfect insulator, then we have an open circuit.
(d) The current in the modified circuit is $I = V/(R + r)$, which gives $P' = I^2R = V^2R/(R + r)^2$. After a little algebra, the result is

$$r = \left(\sqrt{\frac{P}{P'}} - 1\right) R,$$

which, remarkably, differs from the result of part a only by the presence of the square root.

9-14 The total resistance seen by the voltage source is $x + y$, so the current that flows is $I = \Delta V/(x + y)$. Using the result of problem 1 from ch. 3, the power dissipated in the lamp is $P = I^2y = \Delta V^2 y(x + y)^{-2}$. Thus we want to find the value of y that maximizes the quantity $y(x + y)^{-2}$. To find a maximum, we set the derivative equal to zero:

$$\begin{aligned}0 &= \frac{\mathrm{d}}{\mathrm{d}y} y(x+y)^{-2} \\ &= (x+y)^{-2} - 2y(x+y)^{-3}\end{aligned}$$

Multiplying both sides by $(x + y)^3$ simplifies this to

$$0 = (x + y) - 2y \qquad ,$$

which gives $x = y$.

Physical explanation: Very large values of y are like an open circuit, e.g. like opening a switch. Very little current flows, so no power is dissipated in the lamp. Very small values of y result in more current, but the current is still limited by the resistance of the cord, and this current, passing through a small resistance y, dissipates very little power. This is a specific example of something known as impedance matching: the maximum power is transmitted when the resistance of the load is matched to the resistance of the transmission line.

9-16 The resistor on the left looks different from the others, but actually it's connected to exactly the same things they are. The whole circuit is just four resistors in parallel, but drawn in a funny way. By the loop rule, the voltage across each of the resistors is the same as the battery's voltage V, and therefore each resistor has a current $I = V/R$. The total current is $4V/R$.

9-17 This is a problem that has a trick to it. If you don't think of the trick, it's very difficult, because the circuit can't be broken down into parallel and series parts. The trick is to realize that although the circuit has been drawn in an asymmetric way, it's actually very symmetric. The two rightmost bulbs play identical roles in the circuit, as do the two leftmost one. You can see this if you redraw the circuit so that the top left bulb and the top right bulb are on one horizontal line, and similarly for the bottom left and bottom right. The bulb in the middle has zero current through it by symmetry.

9-18 This setup could be made to work just fine at the extreme settings. Let R_k be the (variable) resistance of the knob, and R_h the resistance of the heater.

The "off" position would have to have $R_k = \infty$, i.e., it would be like an open circuit. The total resistance $R = R_k + R_h$ would be infinite as well, zero current would flow, and no power would be dissipated anywhere.

The full-power position would have $R_k = 0$, i.e., it would be like a piece of wire. Current would flow, but there would be no voltage drop across R_k, so the only heating would be in R_h.

The problem occurs at intermediate settings, where R_k is neither zero nor infinity. Here R_k would get hot, wasting power by heating up the knob. At $R_k = R_h$, for example, the knob would be dissipating the same amount of heat as the heating element itself! This would presumably melt the knob, or burn your hand.

9-19 Like the preceding problem, this one can't be solved by breaking the circuit down into parallel and series parts. Unlike that one, there is no easy way to solve it by symmetry. We simply have to use the loop rule and the junction rule to write down enough equations to solve for all the unknowns. Let's label the bulbs A, B, C, ... going from top to bottom and left to right, like reading a page of text. For simplicity, we think of the bulbs at the bottom as a single bulb, E, with a resistance of two ohms. (This is the only part of the circuit that can be simplified in this way.) Let A stand for the current through bulb A, and so on. The junction rule then gives

$$\begin{aligned} D &= B+E \qquad \text{and} \\ A+B &= C \qquad . \end{aligned}$$

Since every bulb except for E has a resistance of one ohm, the voltage drop across each bulb equals the current through it. That is, we can use the symbol A interchangeable for both the current through A (in amperes) and the voltage drop across A (in volts). The only exception is the double bulb E, which has a voltage $2E$ across it. We use the loop rule three times:

$$\begin{aligned} A+C &= 1 \\ D+2E &= 1 \\ A &= D+B \qquad . \end{aligned}$$

The result is five equations in five unknowns, which we solve by eliminating one variable after another. The final results are $A = 6/13$, $B = 1/13$, $C = 7/13$, $D = 5/13$, and $E = 4/13$. The total current is $A + D = 11/13$ amps, so the power drawn from the one-volt battery is (11/13 A)(1 V) = 11/13 W.

10-1

$$\begin{aligned} I &= \int_0^\ell r^2 \, \mathrm{d}m \\ &= \int_0^\ell r^2 \frac{\mathrm{d}m}{\mathrm{d}r} \, \mathrm{d}r \\ &= \mu \int_0^\ell r^2 e^{r/\ell} \, \mathrm{d}r \end{aligned}$$

Although this looks like a definite integral because it has limits of integration, it's really more like an indefinite integral in the sense that the upper limit of integration is a variable. By switching to the unitless variable $u = r/\ell$, we can transform this into a definite integral that we'd be more likely to be able to find in a table of integrals, and this also has the advantage of making the calculus less messy. The result is

$$I = \mu\ell^3 \int_0^1 u^2 e^u \, \mathrm{d}u \qquad .$$

The units are all in the factor in front: (kg/m)(m^3) = kg·m^2, which is correct for a moment of inertia. The integral can be done by integration by parts, by looking it up in a table, or using computer software.

$$\begin{aligned} I &= \mu\ell^3 \; e^u(u^2-2u+2)\Big|_0^1 \\ &= (e-2)\mu l^3 \end{aligned}$$

10-3 This is simplest if we take the center of the circle to be at the origin, and the center of the arc to be on the x axis. Then by symmetry the field has only an x component. The infinitesimal field contributed by the charge dq is $dE_x = -(kdq/b^2)\cos\theta$. We have $dq = \lambda d\ell = b\lambda d\theta$, and the integral from $-\alpha/2$ to α/s comes out to be $2k\lambda/b \sin\alpha/2$. To check our answer, we note that the units make sense and that the result vanishes, as it should, when $\alpha = 0$ or 2π.

10-4 This is exactly the same as the calculation of the moment of inertia of a cube in section

4.2.5, except for the limits of integration.

$$\begin{aligned}
I &= \rho \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} \left(y^2 + z^2\right) \mathrm{d}y \ \mathrm{d}z \ \mathrm{d}x \\
&= \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} \left(y^2 + z^2\right) \mathrm{d}y \ \mathrm{d}z \\
&= \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} y^2 \, \mathrm{d}y \ \mathrm{d}z + \rho a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} z^2 \, \mathrm{d}y \ \mathrm{d}z \\
&= \rho ab \int_{-c/2}^{c/2} y^2 \, \mathrm{d}y \ + \rho ac \int_{-b/2}^{b/2} z^2 \, \mathrm{d}z \\
&= \frac{1}{12} \rho abc^3 + \frac{1}{12} \rho acb^3 \\
&= \frac{1}{12} \rho abc(b^2 + c^2) \\
&= \frac{M}{12}(b^2 + c^2)
\end{aligned}$$

The result doesn't depend on a, and that makes sense, because changing a without changing the mass wouldn't change how far any of the mass was from the axis.

10-5 (a) Since it's infinitesimally small, we can aproximate it as a rectangle. Its dimensions are $\mathrm{d}r$ and (using the definition of radian measure) $r\,\mathrm{d}\theta$, so the area is $\mathrm{d}A = r\,\mathrm{d}r\,\mathrm{d}\theta$.
(b) Now we get an (approximately) rectangular box with height $\mathrm{d}z$. Multiplying by the height of the box gives a volume $\mathrm{d}V = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$.
(c) The flavor of this problem is very similar to that of the calculation of the moment of inertia of a disk, $(1/2)Mr^2$, in section 4.2.5. In fact, we can save quite a bit of time by slicing the cone into disks and just integrating over all the disks. Each disk has infinitesimal height $\mathrm{d}z$ and mass $\mathrm{d}M$. It turns out to be convenient to pick a coordinate system in which $z = 0$ is the tip of the cone, and the cone extends to $z = h$ (i.e., if we want to visualize the positive z axis as being up, then the cone is upside-down). The advantage of this choice is that the surface of the cone is defined by the simple equation $r = (b/h)z$.

$$\begin{aligned}
I &= \int \frac{1}{2} \mathrm{d}Mr^2 \\
&= \int_0^h \frac{1}{2}(\rho A \, \mathrm{d}z) r^2 \\
&= \int_0^h \frac{1}{2} \rho \pi r^2 \, \mathrm{d}z r^2 \\
&= \frac{1}{2} \rho \pi \left(\frac{b}{h}\right)^4 \int_0^h z^4 \, \mathrm{d}z \\
&= \frac{\pi}{10} \rho b^4 h
\end{aligned}$$

Our answer is in terms of ρ, but we're supposed to express it in terms of M. We can either look up the formula for the volume of a cone, $(1/3)\pi b^2 h$, or evaluate it using a similar integral, omitting a factor of $(1/2)\rho r^2$ in the integrand. The final result is

$$I = \frac{3}{10} M b^2 \qquad .$$

It makes sense that the result depends only on b, not h, since changing h without changing the mass would not change how far any of the mass was from the axis. It also makes sense that the numerical factor of 3/10 in front is smaller than the factor of 1/2 for a disk; a cone of radius b has more of its mass closer to the axis than does a disk of radius b.

10-6

$$\begin{aligned}
m &= \int \mathrm{d}m \\
&= \int \frac{\mathrm{d}m}{\mathrm{d}v} \, \mathrm{d}v \\
&= \int_0^\infty a e^{-br} \, \mathrm{d}v \\
&= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} a e^{-br} r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\phi \\
&= 2\pi \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} a e^{-br} r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \\
&= 4\pi \int_{r=0}^{\infty} a e^{-br} r^2 \, \mathrm{d}r
\end{aligned}$$

The r integral can be done by integration by parts, or with computer software. The result is

$$m = \frac{8\pi a}{b^3}$$

10-7 (a) Let $\mathbf{r} = u\mathbf{A} + v\mathbf{B}$, where the interior of the triangle is defined by $u > 0$, $v > 0$, and $u + v < 1$. The parallelogram-shaped region defined by the infinitesimal intervals $\mathrm{d}u$ and $\mathrm{d}v$ then has area $\mathrm{d}A = |\mathbf{A} \times \mathbf{B}|\,\mathrm{d}u\,\mathrm{d}v$. The position of the center of mass is then given by

$$\begin{aligned}\mathbf{r}_{cm} &= \frac{\int \mathbf{r}\,\mathrm{d}A}{\int \mathrm{d}A} \\ &= \frac{\int_0^1 \int_0^{1-v} (u\mathbf{A} + v\mathbf{B})\,\mathrm{d}u\,\mathrm{d}v}{\int_0^1 \int_0^{1-v} \mathrm{d}u\,\mathrm{d}v} \quad ,\end{aligned}$$

where we can treat the differential of area $\mathrm{d}A$ as if it were the differential of mass $\mathrm{d}m$ because the mass per unit area cancels out. The numerator and denominator can both be split up into two similar integrals, and straightforward calculus gives the result as claimed.

(b) Here we do need to explicitly introduce the mass per unit area $2m/|\mathbf{A} \times \mathbf{B}|$, but we get a cancellation between this factor of $|\mathbf{A} \times \mathbf{B}|$ and the one arising from the relation between $\mathrm{d}A$ and $\mathrm{d}u\,\mathrm{d}v$. The moment of inertia is

$$\begin{aligned}I &= 2m \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} r^2\,\mathrm{d}u\,\mathrm{d}v \\ &= 2m \left[\int_{-1/2}^{1/2} \int_{-1/2}^{1/2} A^2 u^2\,\mathrm{d}u\,\mathrm{d}v + \ldots + 0 \right] \quad ,\end{aligned}$$

where $\ldots$ represents a similar term for vector $\mathbf{B}$, and 0 is a cross-term that vanishes. The integral is separable, and the result is as claimed.

(c) Let I_o be the moment of inertia calculated in part b, and let $h = (1/6)|\mathbf{A} + \mathbf{B}|$ be the distance between the quadrilateral's center of mass and the center of mass of one of the triangles into which it can be split up, which follows from part a. Then $2(I_\mathrm{o} + mh^2) = (1/6)m(A^2 + B^2)$, and solving for I_o gives the result claimed.

10-8 (a) We know that the field of a ring, on its axis, is $kQz\left(b^2 + z^2\right)^{-3/2}$, so we can get the field of the cylinder by slicing it like a salami and integrating the contributions of the rings to the total field. Each ring has a width $\mathrm{d}z$, so its charge is $\mathrm{d}Q = \text{(circumference)(width)}\sigma = 2\pi b\sigma\,\mathrm{d}z$.

$$\begin{aligned}E &= \int k\,\mathrm{d}Qz\left(b^2 + z^2\right)^{-3/2} \\ &= \int_0^\infty 2k\pi b\sigma z\,\mathrm{d}z\left(b^2 + z^2\right)^{-3/2} \\ &= 2k\pi b\sigma \int_0^\infty z\,\mathrm{d}z\left(b^2 + z^2\right)^{-3/2}\end{aligned}$$

This is a good candidate for integrals.com, but if you want to do it by hand, you can substitute $u = z^2 + b^2$, which gives

$$\begin{aligned}E &= k\pi b\sigma \int_{b^2}^\infty u^{-3/2}\,\mathrm{d}u \\ &= -2k\pi b\sigma\, u^{-1/2}\Big]_{b^2}^\infty \\ &= 2k\pi\sigma\end{aligned}$$

(b) If you doubled the cylinder's radius, while keeping the charge density constant, you'd have a cylinder with four times the area, and therefore four times the charge. All the angles would be the same, but all the distances would be doubled. Doubling the distance makes the field four times smaller, and this cancels out the effect of the increase in charge.

10-9 (a) The surface area of a sphere is $4\pi r^2$, so the volume of the kind of spherical shell suggested in the hint is $\mathrm{d}V = 4\pi r^2\ \mathrm{d}r$. The energy is

$$\begin{aligned}U &= \int_0^\infty \frac{\mathrm{d}V}{8\pi k} E^2 \\ &= \int_0^\infty \frac{\mathrm{d}V}{8\pi k} \left(\frac{kq}{r^2}\right)^2 \\ &= \frac{kq^2}{2} \int_0^\infty r^{-2}\,\mathrm{d}r \\ &= \frac{kq^2}{2} \left(\frac{1}{0} - \frac{1}{\infty}\right)\end{aligned}$$

It's perfectly legal to take $1/\infty$, and the result is 0. However, $1/0$ gives infinity. In other words, the integral diverges near $r = 0$, but not at $r = \infty$. The interpretation is that the field has infinite energy close to the charge, but finite energy in the space farther away.

(b) The energy in the exterior volume equals the result from part a, with c set to infinity, $U_{ext} = kq^2/2b$. The energy in the interior volume is

$$\begin{aligned}U_{int} &= \int_0^b \frac{\mathrm{d}V}{8\pi k}\, E^2 \\ &= \int_0^b \frac{\mathrm{d}V}{8\pi k}\, \left(\frac{kqr}{b^3}\right)^2 \\ &= \frac{kq^2}{2b^6}\int_0^b r^4\,\mathrm{d}r \\ &= \frac{kq^2}{10b}\end{aligned}$$

The total energy is $U_{int} + U_{ext} = 3kq^2/5b$.

10-11 We can slice the cylinder like a salami into thin slices of thickness $\mathrm{d}z$, and then integrate, because we already know that the on-axis field of a disk is $2\pi\sigma k\left(1 - z/\sqrt{b^2+z^2}\right)$. Because the slices are so thin, the surface charge density of each slice is an infinitesimal number, $\mathrm{d}\sigma$. The field is

$$\begin{aligned}E &= \int \mathrm{d}E \\ &= \int 2\pi\,\mathrm{d}\sigma k\left(1 - \frac{z}{\sqrt{b^2+z^2}}\right)\end{aligned}$$

The relationship between $\mathrm{d}\sigma$ and $\mathrm{d}z$ is $\mathrm{d}\sigma = \rho\,\mathrm{d}z$. (One way to see this is to consider a tiny cube of charge inside the cylinder, with dimensions $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$. The cube's charge is $\rho\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$, and dividing by the area $\mathrm{d}x\,\mathrm{d}y$ gives $\rho\,\mathrm{d}z$.) We then have

$$\begin{aligned}E &= \int_0^\ell 2\pi\rho\,\mathrm{d}zk\left(1 - \frac{z}{\sqrt{b^2+z^2}}\right) \\ &= 2\pi\rho k\int_0^\ell \left(1 - \frac{z}{\sqrt{b^2+z^2}}\right)\,\mathrm{d}z\end{aligned}$$

The integral can be done with computer software, or by looking it up in a table, or by substituting $u = z^2$. The result is

$$E = 2\pi k\rho\left(\ell + b - \sqrt{b^2+\ell^2}\right)$$

10-12 We use the type of coordinate system suggested in the hint, and let the disk lie directly above the origin, in the first and second quadrants. In cylindrical coordinates, a little trig shows that the equation of the disk is $r \le 2b\sin\theta$. The voltage is

$$\begin{aligned}V &= \int \frac{k\,\mathrm{d}q}{r} \\ &= k\sigma\int\frac{\mathrm{d}A}{r} \\ &= k\sigma\int\frac{r\,\mathrm{d}r\,\mathrm{d}\theta}{r} \\ &= k\sigma\int_{\theta=0}^{\pi}\int_{r=0}^{2b\sin\theta}\mathrm{d}r\ \mathrm{d}\theta \\ &= k\sigma\int_{\theta=0}^{\pi} 2b\sin\theta\ \mathrm{d}\theta \\ &= 4bk\sigma \qquad .\end{aligned}$$

10-13 The result of problem 21 for the field of an infinite strip of width b was $E = 4k\sigma\tan^{-1}(b/2z)$. Slicing the box up into strips, with each strip having an infinitesimal surface charge density $\mathrm{d}\sigma = \rho\,\mathrm{d}z$, we have

$$\begin{aligned}E &= \int\mathrm{d}E \\ &= \int 4k\tan^{-1}\frac{b}{2z}\ \mathrm{d}\sigma \\ &= \int_0^b 4k\tan^{-1}\frac{b}{2z}\,\rho\,\mathrm{d}z \\ &= 4k\rho\int_0^b \tan^{-1}\frac{b}{2z}\ \mathrm{d}z\end{aligned}$$

Substituting $u = 2z/b$, we have

$$E = 2k\rho b\int_0^2 \tan^{-1}\frac{1}{u}\ \mathrm{d}u$$

The integral can be looked up in a table or done with computer software, and the result is

$$E = 2k\rho b\left(2\tan^{-1}\frac{1}{2}+\frac{1}{2}\ln 5\right)$$

10-14 Newton's laws won't give a solution in closed form, since the force and acceleration wouldn't be constant, and wouldn't be known as a function of time. Instead, we need to use conservation of energy. The loss in electrical energy $q\Delta V$ equals the gain in kinetic energy $(1/2)mv^2$. The voltage can be found by integrating over the surface of the pipe,

$$V = k\int\frac{\sigma\,\mathrm{d}A}{r} \qquad ,$$

where r is the distance between the point at which we're calculating the voltage and a point on the cylinder. The two points at which we need to calculate the voltage are the center of the cylinder and a point at infinity. In the latter case, r is infinite, so $V = 0$. In other words, ignoring signs, we can just calculate this integral for the center of the pipe, and use the result as ΔV.

$$\begin{aligned} V &= k\int_{-\ell/2}^{\ell/2}\frac{\sigma 2\pi b\,\mathrm{d}z}{r} \\ &= k\int_{-\ell/2}^{\ell/2}\frac{\sigma 2\pi b\,\mathrm{d}z}{\sqrt{b^2+z^2}} \end{aligned}$$

The integral can be done with computer software, or looked up in a table. The result is

$$V = 4\pi k\sigma b\,\operatorname{asinh}\left(\frac{\ell}{2b}\right) \qquad ,$$

where asinh is the inverse hyperbolic sine function, which can also be written as $\operatorname{asinh} x = \ln\left(x+\sqrt{x^2+1}\right)$. For the velocity when the charge is infinitely far away from the "gun," we have:

$$\begin{aligned} \frac{1}{2}mv^2 &= q|\Delta V| \\ &= q|0-V| \\ &= qV \\ v &= \sqrt{2\frac{q}{m}V} \\ &= \sqrt{8\pi k\sigma b\frac{q}{m}\operatorname{asinh}\left(\frac{\ell}{2b}\right)} \end{aligned}$$

11-1 (a) The only nonzero current is in the aluminum, and the area that matters is the cross-sectional area $\pi b^2 - \pi a^2$ of the aluminum, i.e., the area perpendicular to the direction of flow. Assuming the current density to be uniform throughout the aluminum, we have

$$j = \frac{I}{\pi(b^2-a^2)}.$$

(b) Converting to SI and plugging in numbers gives

$$\begin{aligned} j &= \frac{1.00\times10^3\ \mathrm{A}}{\pi((1.407\times10^{-2}\ \mathrm{m})^2-(4.76\times10^{-3}\ \mathrm{m})^2)} \\ &= 1.82\times10^6\ \mathrm{A/m^2}. \end{aligned}$$

11-2 The field of dipole 1, experienced by dipole 2, is along the z axis with component $B_z = -(km_1/c^2)r^{-3}$. This gives dipole 2 an energy $U = -B_z m_2 = (k/c^2)m_1m_2r^{-3}$. They are stable in the orientation that minimizes U, which would be the antiparallel one in which the signs of m_1 and m_2 are opposite. If they were parallel rather than antiparallel, then their magnetic energy would be maximized, and some of this energy could, for example, be converted into kinetic energy to rotate them into the stable orientation.

11-3 The g factor is $(e/m)(|\mathbf{L}|/|\mathbf{m}|) = 1.17$. This makes sense. If the nucleus had been made of two pointlike particles, each with charge e, then we should have gotten $g = 2$. Since one particle is neutral, we would expect to get get half the magnetic moment, cutting the g factor in half, so $g \approx 1$, which is roughly what we see. The remaining discrepancy is presumably because the neutron and proton are not point-like particles.

11-4 If N is even, then half the gears have dipole moments out of the page, and half of them into the page. The sum is zero.

If N is odd, then all but the odd one cancel, so the problem reduces to that of finding the dipole moment of a single gear. The period is $T = 1/f$. During this time, charge q flows past a fixed point on one gear, so the current is $I = q/T = qf$. The dipole moment is $m = IA = \pi qfr^2$.

11-5 (a) The dipole moment depends on the area *vector*, not just the scalar area. The area vectors cancel pairwise, so the total dipole moment is zero. This makes sense, because by symmetry there is no preferred direction for the dipole moment to point, and in fact every current in the cube cancels out.
(b) At each place where two squares are joined together edge to edge, their currents cancel along that edge. Therefore the only nonvanishing current is around the perimeter, and the presence of the little house doesn't matter. The dipole moment is therefore the same as for a square with sides of length $5b$, or $m = 25Ib^2$. We can also see this by considering the area vectors. The area vectors of the house's north and south walls cancel, as do the ones on the east and west.

11-6 The field at any point on the axis can be found by adding the contributions to the field from the two loops. The center of the entire apparatus does not, however, coincide with the center of either loop: in each case it is out of that plane by a distance $h/2$. By symmetry, both these fields are along the axis, so we can add them without worrying about vector addition. The field of a single circular loop would be

$$B = \frac{k}{c^2} \cdot 2\pi I b^2 \left(b^2 + z^2\right)^{-3/2}.$$

Substituting $z = h/2$ and $b = h$, and multiplying by $2N$, we get a total field

$$B = \frac{k}{c^2} \cdot 4\pi \left(\frac{4}{5}\right)^{3/2} \frac{IN}{h}.$$

11-7 (a) Using the equation for the magnetic field at the center of a current loop, we get 0.063 mT.
(b) Along the positive z axis.
(c) It would make a field with the same magnitude along the negative x axis.
(d) The total field is the vector sum of the two. Since they're perpendicular, the magnitude of the sum is given by the Pythagorean theorem, coming out to be 0.09 mT. It points at a 45° angle between the two, in the $x - z$ plane.

11-8 The hard part is visualizing the directions of the four fields that are adding together. Looking along the length of the apparatus, two of the fields point at a 45-degree angle down and to the left, while the other two point at 45 degrees down and to the right. By vector addition, this means that the total field is $2\sqrt{2}$ times the individual ones. Each individual field is $2kI/c^2R = 2\sqrt{2}kI/c^2b$ so the total field is $8kI/c^2b$.

11-9 (a) The Biot-Savart law gives

$$\mathbf{B} = \frac{kI}{c^2} \int \frac{\mathrm{d}\boldsymbol{\ell} \times \mathbf{r}}{r^3}$$

Here $\mathbf{r}$ is the vector from a point on the wire to the point at which we're calculating the field. Let the wire lie on the x axis, and let the point of interest be on the y axis at $y = R$. Let θ be the angle between the $\mathbf{r}$ vector and the wire, measured on the right side so that at large negative x, $\theta \to 0$. The magnitude of the cross product occurring in the integrand is $|\,\mathrm{d}\boldsymbol{\ell}||\mathbf{r}|\sin\theta$, or $\mathrm{d}x \cdot r \cdot \sin\theta$. We then have for the magnitude of the field

$$B = \frac{kI}{c^2} \int_{-\infty}^{\infty} \frac{\sin\theta\,\mathrm{d}x}{r^2}.$$

(b) The variables occurring in the integrand are x, r, and θ. We need to pick one of these variables to work with and reexpress the others in terms of it. Rewriting the variables x and r in terms of θ gives $x = -R\cot\theta$, $\mathrm{d}x = R\sin^{-2}\theta\,\mathrm{d}\theta$, and $r = R/\sin\theta$. These substitutions change the integral into

$$B = \frac{kI}{c^2R}\int_0^\pi \sin\theta\,\mathrm{d}\theta.$$

As promised, the integral now reduces to a trivial form that equals 2.

11-10 As suggested in the hint, we recycle the setup from the long, straight wire, which resulted in

$$B = \frac{kI}{c^2R}\int_0^\pi \sin\theta\,\mathrm{d}\theta.$$

This can be applied to one side of the square simply by changing the limits of integration. We then have

$$\begin{aligned} B_{\text{square}} &= 4\cdot\frac{kI}{c^2R}\int_{\pi/4}^{3\pi/4}\sin\theta\,\mathrm{d}\theta \\ &= 4\sqrt{2}\frac{kI}{c^2R}. \end{aligned}$$

The final result for the comparison with the circular loop is

$$\frac{B_{\text{square}}}{B_{\text{circle}}} = \frac{2\sqrt{2}}{\pi}.$$

This makes sense. It equals about 0.90, so the square's field is a little small, as we would expect because the corners are farther from the center.

11-11 This is a simple generalization of the preceding problem involving a square. The contribution from each side is proportional to $(1/h)\cos\theta$, where θ is the angle between a side and a line from the center to an adjacent vertex. Let the perimeter be ℓ, so that $\cos\theta = (\ell/2n)/R$. Then the field is proportional to $\ell/hR = \ell/\rho^2$.

In the case of $n = 2$, we have $h = 0$, so the field is infinite, which makes sense. For $n \to \infty$, we have $B = (Ik/c^2)\ell/\rho^2 = (Ik/c^2)2\pi r/r^2$, which is correct.

11-13 First we find the field of an infinite solenoid and confirm that we can get the same result as the one obtained more easily using Ampere's law. Let the solenoid have radius b, and let the z axis be the axis of symmetry. Then integrating the field of a loop gives

$$B = \int \frac{2\pi k}{c^2}\,\mathrm{d}I\cdot b^2(b^2+z^2)^{-3/2}.$$

It's nicer to express this in terms of the current density $\eta = \mathrm{d}I/\,\mathrm{d}z$.

$$B = \frac{2\pi k\eta b^2}{c^2}\int_{-\infty}^{\infty}\mathrm{d}z\cdot(b^2+z^2)^{-3/2}$$

We next do the obvious clean-up step of changing to the unitless variable $u = z/b$.

$$B = \frac{2\pi k\eta}{c^2}\int_{-\infty}^{\infty}\mathrm{d}u\cdot(1+u^2)^{-3/2}$$

If all we cared about was this definite integral, then the sensible thing to do would be to evaluate it using software, which gives the result 2. But we're going to want to generalize this to other limits of integration. Playing around with trig substitutions leads to the fact that the nice substitution is $\tan\theta = 1/u$, where θ has the interpretation stated in the problem. We then have

$$B = \frac{2\pi k\eta}{c^2}\int_{-\infty}^{\infty} -\sin\theta\,\mathrm{d}\theta.$$

Putting in the finite limits of integration produces the desired result.

12-1 (a) As suggested in section 10.5.9, let's identify sinusoidal functions with points in the complex plane. The function $4\sin\omega t$ corresponds to the real number 4, and $3\cos\omega t$ to $3i$. Their sum is the function corresponding to $4+3i$. Now we have to go backwards and find what sinusoidal function $4+3i$ represents. The magnitude of this complex number represents the amplitude, A, of the result, so $A = \sqrt{4^2+3^2} = 5$. The argument represents the phase angle $\delta = \tan^{-1}(3/4) = 37°$.

(b)

$$\begin{aligned} 5\sin(\omega t + 37°) &= 5[\sin\omega t\cos 37° + sin37°\cos\omega t] \\ &= 3.9\sin\omega t + 3.0\cos\omega t \end{aligned}$$

To within rounding errors, this verifies that the result was correct.

12-4 (a) Multiplication adds the arguments, so raising something to the third power triples its argument. The result is 3θ (or $3\theta + 2\pi n$, where n is any integer).

(b) If $a^3 = z$, then the argument of a should be 1/3 that of z. (There would be additional possibilities such that $\arg 3a = \arg z + 2\pi n$, but we're only talking about the first quadrant, so that can't happen.) The only one that looks like it has 1/3 the argument of z is x.

(c) The argument has to be 1/3 that of $a + bi$, i.e., $(1/3)\tan^{-1}(b/a)$.

(d) The easy way to do the cube root is to use the fact that if $y = x^3$, then $|y| = |x|^3$, and $\arg y = 3 \arg x$. Therefore if y is the quantity inside the exponent in the denominator, we have $|x| = (\sqrt{8})^{1/3} = \sqrt{2}$, and the easiest possibility for the argument is $\arg x = (1/3) \arg y = \pi/4$. (There are other possibilities such as $11\pi/12$, because arguments that differ by a multiple of 2π represent the same thing.) Division divides the magnitudes, giving a magnitude of 1. Division subtracts the arguments, giving 0. Therefore the result of the calculation is 1.

12-6 This is impractical to carry out by the methods of freshman calculus, without using complex numbers. The trick is to use Euler's formula to rewrite $e^x \cos x$ as $(1/2)e^x(e^{ix} + e^{-ix})$. The result is -2^{50}.

12-8 When you have an exponent with a funky base, the standard method for reexpressing it in terms of base e is to apply the identity $\ln(a^b) = b \ln a$, which gives $a^b = \exp(b \ln a)$. This gives $i^i = e^{i \ln i}$. This reduces the problem to one of finding the natural log of i. Euler's formula tells us that raising e to the power $(\pi/2)i$ gives i. (Actually there is more than one solution, but we'll come back to that.) Taking $\ln i = (\pi/2)i$, we find

$$i^i = e^{-\pi/2} \qquad .$$

Euler's formula tells us that e^x is the same as $e^{x+2\pi i}$, $e^{x+4\pi i}$, etc. That means that $\ln i$ has more than one possible value. It can be $(\pi/2 + 2\pi n)i$, where n is any integer. This means that i^i could be any real number of the form $e^{-\pi/2-2\pi n}$. It shouldn't be too surprising that an exponent such as i^i can have more than one value. For example, $4^{1/2}$ can equal either 2 or -2.

12-9 $FWHM = f_{res}/Q = 100$ Hz, so since the curve is approximately symmetric, the range will be ±50 Hz, i.e., 250-350 Hz.

12-10 (a) The definition of Q says that when $n = Q$, the energy falls off by a factor of $e^{2\pi}$. Since energy is proportional to the square of the amplitude, this corresponds to a factor of $Z = e^{\pi}$ in amplitude. Now we need to generalize this to the case where the number of oscillations we actually observe isn't actually Q. If the amplitude drop in n oscillations is Z, then the drop in one oscillation is $f^{1/n}$. The drop in Q oscillations is then $(Z^{1/n})^Q$. Setting this equal to e^{π} and solving for Q, we find $Q = n\pi/\ln Z$.

(b) The figure is pretty small, so this will be a fairly rough estimate. After $n = 2$ cycles of oscillation, it looks like the amplitude falls off by a factor of $Z = 2$. Applying the result from part a, we have $Q \approx 9$.

12-11 (a) Since $\arg(z \cdot 1/z) = 0$, and arguments add when we multiply, $\arg(1/z) = -\theta$.

(b) $|z| = \sqrt{3+1} = 2$.

(c) It forms a 30-60-90 right triangle, so the arg is 30 degrees.

(d) $|1/z| = 1/|z| = 1/2$.

(e) $\arg(1/z) = -\arg z = -30\ ^\circ$.

(f) $\text{Im}(1/z) = |z| \sin(-30^\circ) = -1/4$.

12-12 $\arg(1/\bar{z}) = -\arg \bar{z} = \arg z$.

13-1 Solving $\omega \approx 1/\sqrt{LC}$ for C, we get $C = 1/L\omega^2 = 1/4\pi^2 L^2 f^2$. Looking at the nearest handy FM radio dial, the range of frequencies is about 88-108 MHz, so the capacitor needs to cover the range from about 2.2 to 3.3 pF.

13-3 (a) We have an RC time constant $\tau = RC$, so

$$\begin{aligned} C &= \frac{\tau}{R} \\ &= \frac{5 \times 10^{-3}\ \text{s}}{1.0\ \Omega} \\ &= 5 \times 10^{-3}\ \text{F}. \end{aligned}$$

5 mF is kind of a large capacitance, so it may be that the explanation for this particular example is not stray capacitance in series, or maybe the

resistance is actually a lot bigger than an ohm.

(b) We need to check that units of s/Ω are the same as farads, where the farad is defined by $U = (1/2C)q^2$ to be 1 F = 1 C^2/J. We have

$$\begin{aligned} \frac{\text{s}}{\Omega} &= \frac{\text{s}}{\text{V/A}} \\ &= \frac{\text{s}\cdot\text{A}}{\text{V}} \\ &= \frac{\text{C}^2}{\text{J}}. \end{aligned}$$

13-4 Combining $U = q^2/2C$ with $q = CV$ gives $U = (1/2)CV^2$.

13-5 The capacitance is proportional to $1/h$. (a) At fixed voltage, we have $U = (1/2)CV^2$, so $U \propto 1/h$. (b) At fixed charge, $U = (1/2C)q^2$, so $U \propto h$.

13-6 (a) Doing this using the relation $U = q^2/2C$ would be fairly complicated, since the energy density is not constant. We would have to find the energy stored in a spherical shell between r and $r + \mathrm{d}r$, and then integrate to find U. It's much easier to do this using $C = q/V$, since we already know how to relate q and V from the analysis in the example: $q = V/k(1/a - 1/b)$. The result for the capacitance is $C = 1/[k(1/a - 1/b)]$.

(b) In the notation of the parallel-plate capacitor example, we have $A = 4\pi a^2$ (or $5\pi b^2$, it doesn't matter because a and b are nearly the same) and $b = a + h$. Our expression for the capacitance involves the quantity $1/a - 1/b$, which can be rewritten as $1/a - 1/(a+h) = h/a(a+h) \approx h/a^2$. We then have $C \approx a^2/kh = A/4\pi kh$, as for the parallel-plate case.

(c) Plugging in the radius of the earth yields 710 μF. This is a surprisingly small value, considering how big the earth is!

13-7 We need the trace on the left to show us where "zero" is on this exponential curve. The experimenter has adjusted the scope so that this is on the line above the bottom line. We need the trace on the right to show us enough time resolution to get decent precision. In the trace on the right, the voltage starts at 8.75 divisions. After a time equal to RC, this should drop off by a factor of e, to 3.22 divisions. This looks like it happens after about 8.65 divisions, which is $t = 4.32$ μs. We have $t = RC$, so $C = t/R = 920$ pF.

13-10 (a) We have

$$\begin{aligned} B &= 4\pi k c^{-2} IN/\ell \qquad &(5.1)\\ U &= (1/2)LI^2 \qquad &(5.2)\\ U &= (c^2/8\pi k)B^2 v \qquad \text{and} &(5.3)\\ v &= A\ell. &(5.4) \end{aligned}$$

Combining equations 1, 3, and 4, we have

$$U = \frac{c^2}{8\pi k}\left(\frac{4\pi k}{c^2}\cdot\frac{IN}{\ell}\right)^2 A\ell \quad = \frac{2\pi k}{c^2} I^2 N^2 A/\ell$$

Solving equation 2 for L then gives

$$L = (4\pi k/c^2)N^2 A/\ell.$$

(b) Plugging in gives an inductance of 0.9 H. Given the non-ideal nature of this solenoid, we expect this to be only a rough estimate. The actual measured inductance is about 20% lower.

13-11 The highest point is at an energy of about 1.9 units, and this really does seem to be about the top of the underlying bell curve, since the points on the sides are very nearly symmetrically placed on either side of it. The resonant frequency is then approximately the frequency of this point, or about 837 Hz.

To find the FWHM, we draw a pencil curve through the points and interpolate to estimate where the pencil curve cuts through the half-maximum height of 1.9/2 = 0.95. The distance between these points is about 1.6 divisions, or 40 Hz.

$Q = f_0/\text{FWHM} = 21$.

14-1 (a) The impedance of an inductor is $i\omega L$, so the argument is 90°.

(b) For a capacitor, $Z = -i/\omega C$, so $\arg Z = -90°$.

(c) The impedances are in series, so they add. The inductor's impedance has a bigger magnitude, so it wins, and the total impedance is on the positive imaginary axis.

(d) The magnitude of each impedance is 1 Ω, with one being positive real and one positive imaginary. The phase angle is 45°.

(e) The simple way to figure this out is that when the two imaginary impedances combine in parallel, the smaller one "wins," so $\arg Z = -90°$. One can also just plug in and find $Z = (Z_C^{-1} + Z_L^{-1})^{-1} = -2i\ \Omega$. It may seem a little weird, but is true, that the parallel impedance has a magnitude *bigger* than either of the individual magnitudes, which is the opposite of what we would expect from our experience with resistances. This is because the signs are opposite.

14-2 (a)

$$\begin{aligned} Z &= (1/Z_R + 1/Z_C)^{-1} \\ &= [1/R + 1/(-i/\omega C)]^{-1} \\ &= (2.70 + 6.28i)^{-1} \times 10^5\ \Omega \end{aligned}$$

The magnitude of a complex number's inverse is the inverse of its magnitude, so

$$\begin{aligned} |Z| &= \frac{1}{\sqrt{2.70^2 + 6.28^2}} \times 10^5\ \Omega \\ &= 1.5 \times 10^4\ \Omega \end{aligned}$$

(rounding the final result to two sig figs). The argument of a complex number's inverse is minus the argument of the original number, so $\arg Z = -\tan^{-1}(6.28/2.70) = -69°$.

(b) The amplitude of the current is

$$\begin{aligned} \tilde{I} &= \Delta\tilde{V}/|Z| \\ &= 68\ \text{nA} \end{aligned} \quad .$$

Suppose the current has the phase of a sine wave. Then we associate the current with a point on the real axis, and multiplying it by the impedance will give a voltage that has an argument of −69°, i.e., closer to the negative imaginary axis. The negative imaginary axis represents something with the phase of − cos, and if you sketch the graphs of sin and − cos, you'll see that − cos is behind by 90 degrees in phase. Therefore the voltage is behind the current, by 69 degrees.

Another method for doing this, which avoids having to memorize what represents what in the complex plane, is to note that an impedance with an argument of −90° (on the negative imaginary axis) is capacitive, so our parallel RC circuit, with its impedance having an argument of −69°, is dominated by its capacitor. For a purely capacitive circuit, the current leads the voltage, so we know that will also be true in our circuit. The mnemonic "eVIL" may come in handy here. It tells you that the voltage (V) leads the current (I) in an inductive (L) circuit, whereas it's the other way around in a capacitive one. (Some people know this as "ELI the ICE man," with E as a notation for emf, which is a concept, closely related to voltage, that we'll encounter later.)

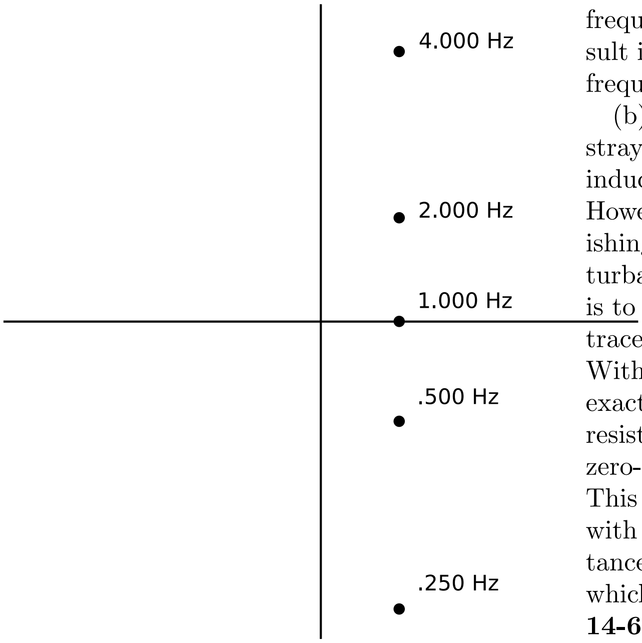
14-3 (a) Impedances add in series, so

$$\begin{aligned} Z &= R + i\omega L - \frac{i}{\omega C} \\ &= 1 + i(\omega - 1/\omega) \end{aligned} \quad ,$$

where, for ease of writing, I've omitted the units and sig figs. The results are:

ω (Hz)	Z (Ω)
0.250	$1.000 - 3.750i$
.500	$1.000 - 1.500i$
1.000	1.000
2.000	$1.000 + 1.500i$
4.000	$1.000 + 3.750i$

Here's what it looks like in the complex plane:



(b) The resonant frequency is $\omega \approx 1/\sqrt{LC} =$ 1.000 Hz. This is the frequency at which the magnitude of the impedance is the smallest, so the largest current will flow for a given voltage.

(c) $f = \omega/2\pi = 0.1592$ Hz

14-4 In parallel, it will be the inverse impedances that add. The total inverse impedance is $1/Z = (0.4 - 0.3i)\ \mathrm{k}\Omega^{-1}$. The real part of this can be contributed by the inverse resistance of a resistor with a resistance of 2.5 kΩ. For the imaginary part, we need a component whose inverse impedance $1/Z$ will be a negative imaginary value. That means that its impedance Z has to be a positive imaginary value, so it has to be an inductor, just as in the original series setup. We then have $L = Z/(i\omega) = Z/(2\pi i f) = 10$ mH.

14-5 (a) We have $Z^{-1} = Z_L^{-1} + Z_C^{-1}$, and if we want no current to flow, then Z should be infinite, so the right-hand side of this equation needs to vanish. One term is positive imaginary and the other negative imaginary, so this cancellation will happen whenever $|Z_L| = |Z_C|$. This results in $1/\omega C = \omega L$, so $\omega = 1/\sqrt{LC}$. This relation looks identical to the one for the resonant frequency of a *series* LC circuit, but here the result is opposite: we get *no* response at all at the frequency ω.

(b) The details will depend on whether the stray resistance acts like it's in series with the inductance, in parallel with the whole thing, etc. However, it seems unlikely that the perfect vanishing of Z^{-1} will be preserved when this perturbation is added. A good way to visualize this is to imagine Z^{-1} as a point in the plane, which traces out a curve (actually a line) as we vary ω. Without the stray resistance, this curve passed exactly through the origin. Putting in any stray resistances will distort this curve and move the zero-intersection point in some random direction. This is guaranteed to eliminate the intersection with the origin, unless we can fine-tune the resistance in some special way with infinite precision, which is impossible in reality.

14-6 At DC, the capacitor has infinite impedance, and so the branch acts like an open circuit, giving an equivalent impedance $R_1 =$ 500 Ω. In the opposite limit of infinite frequency, the capacitor has no impedance, so the circuit has equivalent impedance $(1/R_1 + 1/R_2)^{-1}$. If this is to equal 300 Ω, algebra shows that $R_2 =$ 750 Ω. Finally we have the finite frequency. The impedance at this frequency is

$$Z = \left[\frac{1}{R_1} + \frac{1}{R_2 - i/\omega C}\right].$$

This results in

$$C = \frac{-i}{\omega}\left[\left(\left(\frac{1}{Z} - \frac{1}{R_1}\right)^{-1}\right) - R_2\right].$$

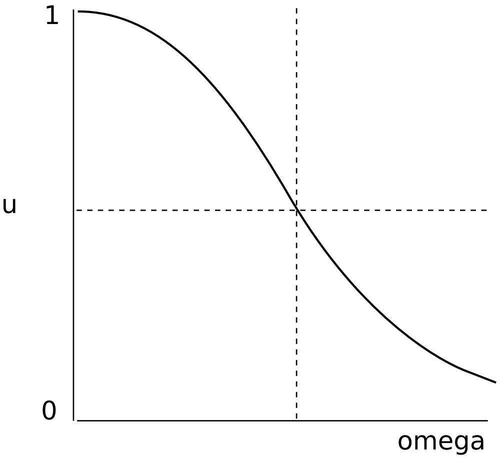
The model is stated in such a way that this has to come out real. Plugging in complex numbers on the right and doing the arithmetic gives $C =$ 8.0 nF.

14-7 As one capacitor discharges into the other, the changing electric field produces a magnetic field. This magnetic field takes up some of the energy. The system is an oscillating LC circuit, which will oscillate at its characteristic frequency. Eventually, radiation will dissipate the energy of the oscillations.

14-8 (a) A DC current can't flow through a capacitor, so the steady-state behavior of the circuit is that the capacitor is charged up, and there is no current flowing through the inductor. That means there's an electric field in the capacitor, but no magnetic field in the inductor. All of the energy is stored in the capacitor, and none in the inductor.
(b) The energy is being shuffled back and forth between the capacitor and the inductor.
(c) The fields oscillate sinusoidally, and the energy in a field is proportional to the square of the field, so the variation in each field's energy over time has the form of a square of a sinusoidal function. The square of a sine wave is a function that oscillates between 0 and 1, and its average value is 1/2, so in each case, the average energy in the field equals half the maximum energy. If we have $q = \tilde{q}\sin\omega t$ for the charge on one of the plates of the capacitor, then differentiatoin gives $I = \tilde{q}\omega\cos\omega t$. Then

$$\begin{aligned}
u &= \frac{\overline{U_C}}{\overline{U_L}+\overline{U_C}} \\
&= \frac{U_{C,max}}{U_{L,max}+U_{C,max}} \\
&= \left(\frac{U_{L,max}}{U_{C,max}}+1\right)^{-1} \\
&= \left(\frac{(1/2)L\tilde{I}^2}{(1/2C)\tilde{q}^2}+1\right)^{-1} \\
&= \left(\frac{LC\tilde{q}^2\omega^2}{\tilde{q}^2}+1\right)^{-1} \\
&= \frac{1}{LC\omega^2+1}
\end{aligned}$$

The figure shows the behavior of this expression as a function of frequency. The capacitor's share is 100% at zero frequency, in agreement with part a. It drops to 50% at resonance ($\omega = 1/\sqrt{LC}$), and at twice the resonant frequency, it equals 20%.



15-1 Ampère's law is

$$\frac{4\pi k}{c^2}I_{\text{through}} = \int \mathbf{B}\cdot d\boldsymbol{\ell}.$$

We exploit the symmetry of the situation by choosing our Ampèrian surface to be a disk perpendicular to the wire, with the wire passing through the disk's center. We've already determined that the field pattern is such that along the boundary of this disk, $\mathbf{B}$ is parallel to $d\boldsymbol{\ell}$, so that $\mathbf{B}\cdot d\boldsymbol{\ell} = |\mathbf{B}||\,d\boldsymbol{\ell}|\cos 0 = B\,d\ell$. The magnitude B of the field is constant along this curve, so we can take it outside the integral. The current passing through the disk is simply the current I in the wire, so we have

$$\frac{4\pi k}{c^2}I = B\int d\ell,$$

but this integral is just the circumference of the circle, $2\pi r$, and now straightforward algebra produces the claimed result.

15-2 (a) Ampère's law is

$$\frac{4\pi k}{c^2}I_{\text{through}} = \int \mathbf{B}\cdot d\boldsymbol{\ell}.$$

We make an Ampèrian surface in the shape of a rectangle of width w in the x direction, so that a portion of the sheet with width w passes perpendicularly through the rectangle. The top and bottom of the rectangle contribute to the right-hand side of the equation, but the sides don't. If we choose the orientation of the loop to be counterclockwise, then the current through the rectangle is $+w\eta$, and the $\mathrm{d}\boldsymbol{\ell}$ vector points in the $-x$ direction on the top and in the $+x$ direction on the bottom. Thus

$$\frac{4\pi k}{c^2}w\eta = -B_{\mathrm{x,top}}w + B_{\mathrm{x,bottom}}w,$$

or

$$\Delta B_x = -\frac{4\pi k}{c^2}\eta.$$

The sign depends on the details of our definitions: that ΔB is defined as top minus bottom, and that the current is coming out of the page.

(b) The field lines are horizontal. This means that there is tension in the x direction and pressure in the y direction.

If the current is flowing through a material conductor, then the conductor will be pressed by this pressure from both sides. If the magnitude of the magnetic field is equal on both sides (which it doesn't have to be), then the pressure on the sides is equal, and the conductor will not accelerate vertically. This is similar to the door of a house in which the air pressure is equalized on both sides.

The tension will tend to crumple the sheet in the horizontal direction. This makes sense, because parallel currents attract.

15-3 (a) We apply Faraday's law to the rectangular surface bounded by the U and the rolling wire. We take this surface to be oriented so that the flux in the figure is positive, which means that a counterclockwise current is positive. This surface changes its area over time, which is OK and does not invalidate Faraday's law. Let x be the position of the rolling wire, which depends on time, and let V be the voltage drop across the wire. We have

$$-\frac{\partial \Phi_B}{\partial t} = \int \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell},$$

and because the resistance of the U is negligible, the voltage drop is only along the rolling wire, not the U, so

$$\begin{aligned} -\frac{\partial}{\partial t}(x\ell b) &= V \\ V &= -\ell B v. \end{aligned}$$

Ohm's law then gives $I = -(\ell/R)Bv$. The minus sign means that the current is clockwise.

(b) The resistance of the wire produces heat, and by conservation of energy, this has to come from somewhere. It comes from the wire's kinetic energy, so that the wire will be slowing down. This means that the force is to the left.

(c) This also checks out against the usual right-hand rules. If the current is coming toward us, and the field is up, then the force should be to the left.

(d) The magnitude of the force per unit length is $F/\ell = IB$, which comes out to be

$$F = \frac{\ell^2}{R}B^2 v.$$

(e) The force is an even function of B and an odd function of v. Reversing the field would have led to an opposite induced current, but then the force of the field on the induced current would still have been to the left. This is consistent with conservation of energy, because there would still have to be conversion of KE into heat. Reversing the direction of motion of the wire would reverse the direction of the force, which means that the force would still be opposite to the direction of motion, causing a deceleration. This is again what we need if conservation of energy is to hold.

15-4 The relationship between the field and the voltage is

$$E = \frac{V}{d} \qquad ,$$

so for the flux we have

$$\Phi_E = \frac{\pi R^2 V}{d} \qquad .$$

Based on the symmetry of the capacitor, we clearly need to choose an Amperian surface that is a circle centered on the axis. To extract any information about the magnetic field at radius R, we're going to have to make it a circle of radius R. Using Maxwell's equation for the circulation of the magnetic field,

$$\begin{aligned} c^2\Gamma_B &= \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} \\ &= \frac{\pi R^2 s}{d} \end{aligned}$$

Now the magnetic field could have components in various directions, but only the component tangent to the circle will contribute to this circulation. That means we can't use this technique to get the components of the field in the radial direction and in the direction parallel to the axis. Let's compute the tangential component, B_t, first, and then worry about the other two.

$$\begin{aligned} c^2 \cdot 2\pi R B_t &= \frac{\pi R^2 s}{d} \\ B_t &= \frac{sR}{2c^2 d} \end{aligned}$$

The radial component must vanish because if it didn't, the magnetic field would be diverging from or converging on the capacitor, and Gauss's law for magnetism forbids this.

The axial component is a little trickier to rule out. If there was an axial component, then the field pattern would have to point in some direction along the axis, let's say up. Since the field cannot diverge or converge, the field pattern would then have to come back down in the region outside the capacitor. But then the magnetic field would have a circulation around a loop lying in an up-down plane. This would violate Maxwell's equation for the circulation of B, because there is no electric flux through such a surface (the electric field doesn't twirl around the axis), and there are no currents through such a surface (the currents are all axial and radial currents in the wires and the plates of the capacitor).

15-5 (a) The changing magnetic field induces an electric field, which causes a current to run around the loop. The resistance of the wire causes energy to be turned into heat. This energy can only be coming from the mechanical energy (KE and gravitational PE) of the loop, so it must be making the loop fall with an acceleration less than g. The force must be upward.

(b) An electron in the wire has negative charge, so the force it feels in a magnetic field is in the direction of $-\mathbf{v}\times\mathbf{B}$. This is into the page for an electron in the top, and out of the page for an electron in the bottom. Because the field is stronger at the top, that force dominates, and a clockwise current will be established. Once the current is established, an electron at the top has a nonvanishing v_y, and this new component gives rise to a new component of $-\mathbf{v}\times\mathbf{B}$, pointing up. The force at the bottom of the loop is downward, but it's weaker, so the total force is upward.

(c) As worked out in problems 9 and 10 in this chapter (whose answers are in the back of the book), the force exerted on a wire by a magnetic field is $I\ell B$. Let subscripts t and b stand for the top and bottom of the loop. Ignoring signs except where they're necessary, we have:

$$\begin{aligned} F &= F_t - F_b \\ &= I\ell k(z_t - z_b) \\ &= IAk \\ &= Ak\Gamma_E/R \\ &= \frac{Ak}{R}\frac{\partial\Phi_B}{\partial t} \\ &= \frac{Ak}{R}\frac{\partial}{\partial t}\int kz\,\mathrm{d}y\,\mathrm{d}z \\ &= \frac{Ak^2(\text{width})}{R}\frac{\partial}{\partial t}\frac{z_t^2 - z_b^2}{2} \\ &= \frac{Ak^2(\text{width})}{R}\left(z_t\frac{\mathrm{d}z_t}{\mathrm{d}t} - z_b\frac{\mathrm{d}z_b}{\mathrm{d}t}\right) \\ &= \frac{Ak^2(\text{width})(\text{height})v}{R} \\ &= \frac{A^2k^2v}{R} \end{aligned}$$

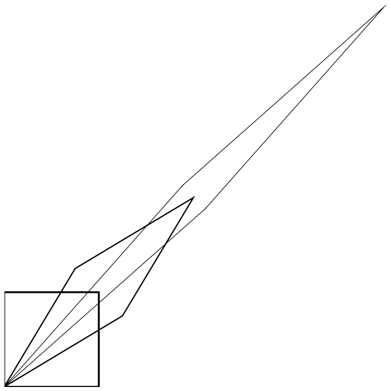
The behavior of this expression is sensible. The argument in part a didn't depend on the sign of k, so it makes sense that the final answer depends on k^2, and is therefore also independent of the sign of k. It also makes sense that the final answer has v to the first power. If we threw the loop upward, the original argument would still imply that mechanical energy was being taken out and dumped into ohmic heating, but the force required would be downward. Having R in the denominator makes sense. For instance, if we cut the wire, then it would be an open circuit, and there would be no way to establish a current; this corresponds to the case of $R = \infty$, which would give $F = 0$ in our expression.

15-6 The field of a wire is $B = 2kI/c^2R$, in the direction tangent to the circle centered on the wire and passing through the point of interest. As suggested in the problem, the field approaches zero as R approaches infinity, so we can neglect the contributions from the short, infinitely distant pieces of wire at the ends of the rectangle. Of the two infinite wires, each makes an equal contribution to the circulation, so we only need to integrate over one of them, and then double that result. Let the wire lie along the z axis, and let the wires be parallel to the x axis. Then

$$\begin{aligned}
\Gamma_B &= 2\int_{-\infty}^{+\infty} \mathbf{B}\cdot\hat{\mathbf{x}}\,\mathrm{d}x \\
&= 2\int_{-\infty}^{+\infty} |\mathbf{B}||\hat{\mathbf{x}}|\cos\theta_{Bx}\,\mathrm{d}x \\
&= 2\int_{-\infty}^{+\infty} \frac{2kI}{c^2R}\cdot 1\cdot\frac{h}{R}\,\mathrm{d}x \\
&= \frac{4kIh}{c^2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}x}{R^2} \\
&= \frac{4kIh}{c^2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}x}{h^2+x^2} \\
&= \frac{4kIh}{c^2}\cdot\frac{1}{h}\tan^{-1}\frac{x}{h}\Big]_{-\infty}^{+\infty} \\
&= \frac{4\pi kI}{c^2}
\end{aligned}$$

This is the result expected from Ampère's law.

17-1 To get good precision, you need to do this on a full-size printout, as suggested by the problem. To keep from using up a whole page on my solutions handouts, I've shown the idea below at about 25% of the actual size.



First I drew the long diagonal twice as long again, meaning about 276 mm. To keep the area the same, I had to make its "cross-bar" half as long as on the preceding parallelogram, or about 17.5 mm. The speed represented by this new parallelogram is equal to the slope of its bottom edge. To get this slope, which is the rise divided by the run, we have to draw the rise and run accurately, i.e., lined up straight with the edges of the paper. I did this by using a second piece of paper as a right angle. The result is that the slope is (rise)/(run) = 91 mm/105 mm = .86, representing 86% of the speed of light.

17-2 Its kinetic energy is equal to its mass-energy minus its resting value,

$$\begin{aligned}
K &= \mathcal{E} - mc^2 \\
&= m(\gamma-1)c^2 \\
&= (8.0\times10^7\ \text{kg})\left(\frac{1}{\sqrt{1-(1/2)^2}}-1\right)c^2 \\
&= 1.1\times10^{24}\ \text{J}
\end{aligned}$$

This is a thousand times greater than the total energy content of the world's nuclear arsenals. In other words, the Enterprise is the ultimate weapon of mass destruction. If it crashed into a planet (as it did, in one of the movies), it would destroy all life on the planet's surface.

17-3 (a) v = (slope) = (rise)/(run) = x/γ, so $x = v\gamma$. (b) Because we're using units in which

$c = 1$, the diagram has symmetry with respect to reflection across the 45-degree diagonal. Therefore the (t, x) coordinates of P and Q are simply related by a swap of t and x, and Q has coordinates $(v\gamma, \gamma)$. (c) Points P and Q are separated by $|\Delta t| = (1 - v)\gamma$, and $|\Delta x| = (1 - v)\gamma$ as well. The Pythagorean theorem results in $\text{PQ} = \sqrt{2}(1 - v)\gamma$. (d) Averaging the coordinates of points P and Q, we find that C has coordinates $x = t = (1 + v)\gamma/2$. The Pythagorean theorem then gives $\text{OC} = (1 + v)\gamma/\sqrt{2}$. (e) The area of triangle PQO is (1/2)(base)(height) = $(1/2)\text{PQ} \cdot \text{OC} = (1/2)(1 - v)(1 + v)\gamma^2$, which simplifies to $(1/2)(1 - v^2)\gamma^2$. Doubling this to find the area of the quadrilateral, we obtain $(1-v^2)\gamma^2$, and setting this equal to 1 and solving for γ gives the desired result.

17-4 (a)

$$\begin{aligned} p &= m\gamma v \\ &= \frac{mv}{\sqrt{1 - v^2}} \\ p^2 \left(1 - v^2\right) &= m^2 v^2 \\ p^2 &= \left(m^2 + p^2\right) v^2 \\ v &= \frac{p}{\sqrt{m^2 + p^2}} \end{aligned}$$

(b) At low velocities, the second term inside the square root is negligible, and we have

$$\begin{aligned} v &\approx \frac{p}{\sqrt{m^2}} \\ &= \frac{p}{m} \qquad , \end{aligned}$$

which is the nonrelativistic result.

(c) For very large momenta, the m^2 term is negligible compared to the p^2 term, so

$$\begin{aligned} v &= \frac{p}{\sqrt{p^2}} \\ &= \pm 1, \end{aligned}$$

which is the speed of light in natural units.

(d) Converting to SI units, we have

$$v = \frac{p}{\sqrt{m^2 + p^2/c^2}}.$$

17-5 (a) If v is very close to c then the denominator in $\gamma = 1/\sqrt{1 - v^2/c^2}$ gets very close to zero. Dividing by a very small number gives a very big result. Ultrarelativistic particles have very large values of γ.

(b) The energy due to its mass is mc^2, which is a big number because c is big. However, the energy due to its motion equals $m(\gamma - 1)c^2$, which is even bigger, because not only does it have the c^2 factor, it also has $\gamma - 1$, which is big. The particle has much more energy due to its motion than due to its mass.

(c) $\mathcal{E}/p = m\gamma c^2/m\gamma v = c^2/v$.

(d) For $v = c$, this becomes $\mathcal{E}/p = c^2/c = c$.

(e) The beam of light was emitted with a power of 1 watt, i.e., 1 joule per second. Since it lasted for one second, its energy is one joule.

$$\begin{aligned} p &= \mathcal{E}/c \\ &= (1\ \text{J})/(3 \times 10^8\ \text{m/s}) \\ &= (1\ \text{kg}\cdot\text{m}^2/\text{s}^2)/(3 \times 10^8\ \text{m/s}) \\ &= 3 \times 10^{-9}\ \text{kg}\cdot\text{m/s} \end{aligned}$$

It has only a miniscule amount of momentum, which fits the correspondence principle: until Einstein came along, nobody even knew light had momentum at all.

17-8 Really this boils down to expanding γ in a Taylor series, and that task can be simplified a lot by letting $x = v^2$, and thinking of γ as a function of x. Then

$$\begin{aligned} \gamma &= (1 - x)^{-1/2} \\ \gamma' &= \frac{1}{2}(1 - x)^{-3/2} \\ \gamma'' &= \frac{3}{4}(1 - x)^{-5/2} \end{aligned}$$

Evaluating these at $x = 0$, we get 1, 1/2, and 3/4, so the Taylor series is

$$\begin{aligned} \gamma &= 1 + \frac{1}{1!} \cdot \frac{1}{2}x + \frac{1}{2!} \cdot \frac{3}{4}x^2 + \ldots \\ &= 1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \ldots \\ K &= \frac{1}{2}mv^2 + \frac{3}{8}mv^4 + \ldots \qquad , \end{aligned}$$

which in ordinary units would be

$$K = \frac{1}{2}mv^2 + \frac{3}{8c^2}mv^4 + \ldots \qquad .$$

The odd terms vanish, because kinetic energy is a scalar that doesn't depend on the direction of motion. The first term is the nonrelativistic expression, as claimed.

17-9 We have $p = m\gamma v = m(1-v^2)^{-1/2}v$. A brute-force evaluation of the Taylor series for this expression can be done, but it's messy. One way to simplify the calculation is by evaluating the Taylor series for γ, and then simply multiplying by v. Now that we've reduced the problem to evaluating γ as a Taylor series, a second simplification is to let $x = v^2$, and think of γ as a function of x. Then

$$\begin{aligned}\gamma &= (1-x)^{-1/2}\\ \gamma' &= \frac{1}{2}(1-x)^{-3/2}\\ \gamma'' &= \frac{3}{4}(1-x)^{-5/2}\end{aligned}$$

Evaluating these at $x = 0$, we get 1, 1/2, and 3/4, so the Taylor series is

$$\begin{aligned}\gamma &= 1 + \frac{1}{1!}\cdot\frac{1}{2}x + \frac{1}{2!}\cdot\frac{3}{4}x^2 + \ldots\\ &= 1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \ldots\end{aligned}$$

Applying this to the expression for momentum, we have

$$\begin{aligned}p &= m\gamma v\\ &= mv\left(1 + \frac{1}{2}v^2 + \ldots\right)\\ &= mv + \frac{1}{2}mv^3 + \ldots \qquad ,\end{aligned}$$

or, in ordinary units,

$$p = mv + \frac{1}{2c^2}mv^3 + \ldots \qquad .$$

The even-order terms vanish, because when v flips its sign, p must flip its sign as well. The first nonvanishing term is the newtonian expression mv.

17-10 Among the spacelike vectors, **a** and **e** are clearly congruent, because they're the same except for a rotation in space; this is the same as the definition of congruence in ordinary Euclidean geometry, where rotation doesn't matter. Vector **b** is also congruent to these, since it represents an interval $3^2 - 5^2 = -4^2$, just like the other two.

The lightlike vectors **c** and **d** both represent intervals of zero, so they're congruent, even though **c** is a double-scale version of **d**.

The timelike vectors **f** and **g** are not congruent to each other or to any of the others; **f** represents an interval of 2^2, while **g**'s interval is 4^2.

17-11 Since $m^2 = \mathbf{p}\cdot\mathbf{p}$, the three cases correspond to the possibility that m^2 is positive, negative, or zero. If the energy-momentum vector is timelike, then m^2 is positive, which just means that the mass is real and nonzero. If the vector is lightlike, m^2 must be zero, so the particle is massless. It's not possible for the vector to be spacelike, because then the mass would have to be an imaginary number.

17-12 *Compton scattering:* Here, as in the analysis of pair production in the example in the text, it's helpful to discuss the process in the special frame of reference (called the center of mass frame) in which the total momentum is zero, $p_\gamma = -p_e = p$. In this frame, the gamma ray and the electron collide head-on, so the most natural thing to guess for the outcome of the collision is that they both emerge along the same line, i.e., the whole process is confined to a single line. If this was a collision between two cars, we would expect a large amount of energy to be converted into other forms such as heat, sound, and permanent deformation of the cars. But none of this can happen with subatomic particles, so we expect the collision to be perfectly "bouncy" (technically the term is "elastic"). In Newtonian mechanics, in a perfectly elastic collision, the result in the center of mass frame is that both objects simply reverse their momentum vectors. This also works in Compton scattering: if we reverse each particle's momentum, then its four-momentum vector changes from $(\mathcal{E}, p)$ to $(\mathcal{E}, -p)$, and four-momentum is conserved:

$$(\mathcal{E}_\gamma, p) + (\mathcal{E}_e, -p) = (\mathcal{E}_\gamma, -p) + (\mathcal{E}_e, p)$$

We also need to check that $(\mathcal{E}_\gamma, -p)$ and $(\mathcal{E}_e, p)$ are still legal four-momentum vectors for the gamma and electron; they are, because $m^2 = \mathcal{E}^2 - p^2$ is unchanged when we flip the sign of p. The conclusion is that Compton scattering *can* occur between a gamma ray and an electron, regardless of the absence of any third particle.

Photoelectric effect: If the photoelectric effect could occur without the presence of any third particle such as an atomic nucleus, then the initial state would look just like the initial state of Compton scattering: an electron and a gamma ray. We've already analyzed a collision with this initial state by going into the center of mass frame. The final state of the photoelectric effect would be different, however, because only the electron would exist, the gamma having been absorbed. What would this final state look like in the center of mass frame? The electron would have to have zero momentum. But this would imply that it has *lost* mass-energy as a result of absorbing the gamma, which would violate conservation of mass-energy.

17-13 The first derivative of the function $D(v)$ is

$$D' = \frac{1}{2}\left(\frac{1-v}{1+v}\right)^{-1/2}\left[-\frac{1}{1+v} + (1-v)(-1)(1+v)^{-2}\right]$$

Evaluating this at $v = 0$ gives $D'(0) = -1$. The first two terms of the Taylor series are

$$\begin{aligned} D(v) &\approx D(0) + D'(0)v \\ &= 1 - v \qquad . \end{aligned}$$

This is identical to the nonrelativistic Doppler shift for the wavelength detected from a source receding at velocity v relative to the medium (if v is expressed as a fraction of the wave's speed). (The nonrelativistic version is actually more complicated, because in principle both the source and the observer could be moving relative to the medium. For Doppler shifts of light, there is no medium, so clearly this can't be relevant. In the nonrelativistic case we get shifts of $1 - v$ and $1/(1 + v)$ for a moving source and a moving observer, but the latter's Taylor series is $1 - v + \ldots$, which still matches the relativistic expression in its first two terms.)

17-14 (a) As the relative velocity gets closer and closer to c, the length of the long diagonal of the parallelogram approaches infinity. But the area is supposed to stay the same, so this means the short diagonal's length must approach zero. The parallelogram becomes a line. But one of our basic rules here is that observers in different frames of reference must always agree on whether or not events are the same, and this rule is being violated here, because points that were distinct on the original square are on top of each other when the square is compressed to a line. The conclusion is that we can never have a frame of reference that moves at c relative to another frame of reference.

(a) The paradox implicitly assumes the existence of a frame of reference that moves along with beam number 1, so that we can then talk about the velocity of beam 2 in beam 1's frame of reference. But there aren't frames of reference that move at c.

17-15 (a) Use natural units, with $c = 1$. Let the massive particle have mass m and velocity v. For a massless particle, we have $E = |p|$. Because this is the center of mass frame, we have $p_\gamma + p_m = 0$, so $|p_\gamma| = |p_m| = m\gamma|v|$. Thus

$$E_\gamma = m\gamma|v|,$$

while

$$\mathcal{E}_m = m\gamma,$$

and therefore $E_\gamma < \mathcal{E}_m$, since $|v| < 1$ in natural units.

(b) Let $R = KE_m/E_\gamma$. Then $R = m(\gamma - 1)/m\gamma = 1 - 1/\gamma < 1$. Therefore $KE_m < E_\gamma$.

17-17 Unlike the example of the school bus paradox, the answer here is not frame-dependent. We are not asking whether all parts of the ball fit in the hole simultaneously (which would be a frame-dependent question because simultaneity is frame-dependent). We are simply asking whether it is possible that no part of the ball ever intersects any part of the wall. Intersections of world-lines are frame-independent events.

Length contraction will not help the ball to fit through the hole, because length contraction occurs only along the direction of motion, not along the transverse axes.

17-18 (a) In units with $c = 1$, we have

$$\begin{aligned} v &= L/t \\ K &= m(\gamma - 1) \\ \tau &= t/\gamma. \end{aligned}$$

The solution of these equations is

$$K = m\left[\sqrt{1+\left(\frac{L}{\tau}\right)^2} - 1\right],$$

or, reinserting factors of c,

$$K = mc^2\left[\sqrt{1+\left(\frac{L}{c\tau}\right)^2} - 1\right].$$

(b) For the given data, the result is something like 10^{12} megatons of TNT. Your ultrarelativistic friend's body has so much kinetic energy that if he collided with the earth, it would be the end of the world, so I think Congress should pass a law prohibiting him from doing this.

6 Modern Physics

1-1 Plugging in to $K \approx (1/2)mv^2 = (1/2)m(0.1c)^2$ gives 4×10^{22} J. This is about 40 times greater than the total energy content of the world's nuclear arsenals. In other words, the Enterprise is the ultimate weapon of mass destruction. If it crashed into a planet (as it did, in one of the movies), it would destroy all life on the planet's surface.

1-2 (a) $t = \sqrt{\mathcal{I}^2 + d^2}$

(b) Sirius: 13 years. Adhara: 570 years.

(c) All three quantities have units of time except for d, so let's make the d piece have time units. We can do this by dividing d by c. The result is $t = \sqrt{\mathcal{I}^2 + (d/c)^2}$.

2-1 $T = 1/f = 3.82$ ms, and $\lambda = v/f = (340 \text{ m/s})/f = 1.3$ m.

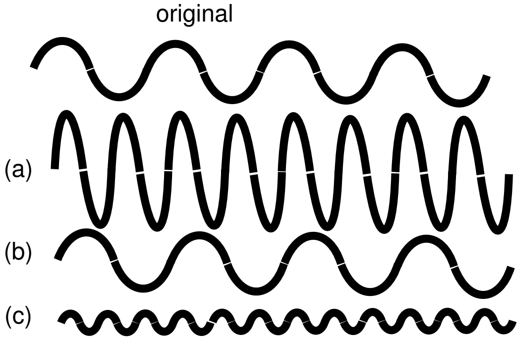
2-2 1% of the speed of sound, or about 3 m/s

2-3 Since it's a graph of the wave as a function of position, what it shows is amplitude and wavelength.

(a) The amplitude is doubled, and $\lambda = v/f$, so the wavelength is cut in half.

(b) Doubling both velocity and frequency leaves the wavelength unchanged. This is exactly the same as the orginal wave.

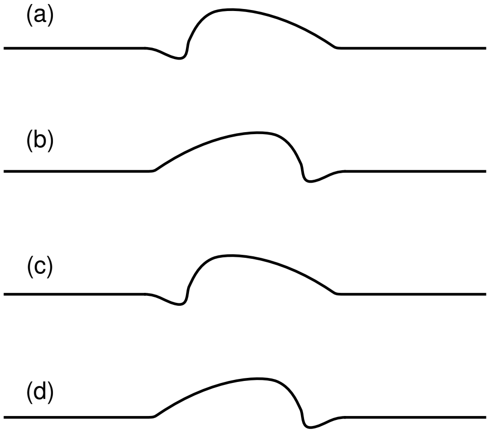
(c) The information about the velocity is irrelevant. We're given the wavelength and amplitude directly, so that's all we need to show.



2-4 (a) The scale on the time axis is not comparable to the scale of the distance axis, so there is no reason why your answer has to be the same width on the page. As the pulse passes through a certain point traveling to the right, the water will first go down a little, then up, so the graph should look similar, but reversed.

(b) As the pulse travels to the left, the water will first do the big, slow rise, then the small, short dip. The graph is not reversed compared to the original.

(c) and (d) are the same idea – the roles of question and answer are just reversed.



2-5 We know that ωt and kx both change by the same amount (2π) over one period. Since ω and k are constants, this means that $\omega\Delta t = k\Delta x$, or $\Delta x/\Delta t = \omega/k$. The units make sense, because ω/k has units of $(1/\text{s})/(1/\text{m}) = \text{m/s}$.

2-6 (a) To keep the stuff inside the cosine the same, we'd have to decrease x while t increased. The wave is moving in the negative x direction.

(b) At a particular point, the quantity $0.45\pi t$ has to change by 2π for one cycle to go by, so $T = 2\pi/0.45\pi = 4.4$, which we assume represents 4.4 seconds, since we're told that everything is set up for SI units. The frequency is $f = 1/T = 0.23$ Hz. The relationship between x and the wavelength is the same as the relationship between t and the period, so $\lambda = 2\pi/0.73\pi = 2.7$ m.

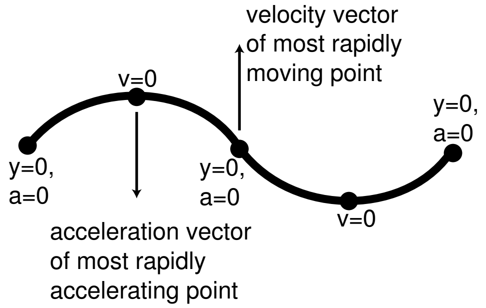
(c) We could do this by plugging in to $v = f\lambda$, but let's use a different approach. If x

changes by Δx while enough time goes by for the inside of the cosine to stay the same, then $0.73\pi\Delta x + 0.45\pi\Delta t = 0$. The velocity is $\Delta x/\Delta t = -0.45/0.73 = 0.62$ m/s.
(d) The velocity of the point on the string is

$$\begin{aligned} y' &= -3.5\sin(0.73\pi x + 0.45\pi t + 0.37\pi)(0.45\pi) \\ &= -4.9\sin(0.73\pi x + 0.45\pi t + 0.37\pi) \quad , \end{aligned}$$

which varies between -4.9 m/s and $+4.9$ m/s. This is completely different from the wave's velocity of 0.62 m/s, and is in the perpendicular direction.

2-8 The points with $v = 0$ are the ones that have gone as far out to one side as they'll go, and are now ready to come back. The points that are whipping across the center line are the ones that are going the fastest, and they also have zero acceleration because their v vs t graph is at a peak, where the tangent line has zero slope.


velocity vector of most rapidly moving point
v=0
y=0, a=0
y=0, a=0
y=0, a=0
v=0
acceleration vector of most rapidly accelerating point

2-9 (a) In the expression

$$\lambda' = \left(1 - \frac{v_s}{v}\right)\lambda \quad ,$$

we eliminate wavelength λ and replace it with v/f:

$$\frac{v}{f'} = \left(1 - \frac{v_s}{v}\right)\frac{v}{f} \quad ,$$

We simplify by dividing out the factor of v on both sides, and inverting both sides:

$$f' = \frac{f}{1 - \frac{v_s}{v}}$$

(b) The ratio v_s/v is unitless, so it makes sense to subtract it from a unitless 1. Dividing a frequency by a unitless number gives a frequency.

(c) If $v_s = 0$, then $f' = f$, which makes sense. For v_s greater than zero (but less than v), the denominator is less than one, so $f' > f$, indicating an upward frequency shift, which makes sense if positive v_s indicates motion toward the observer.

2-10

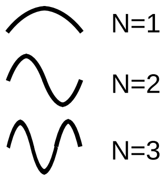
There are a couple of things that make it difficult to devise an accurate experiment: (1) it's hard to produce waves in a controlled way, and (2) it's hard to measure the height of the superposing waves in real time.

Possible solution: Find a machine that's capable of making spreading ripples of a consistent height, e.g., a wind-up toy car whose wheels go around. This takes care of problem 1. You could put two such cars in the water, and see if the peaks formed by constructive interference become twice as tall at a point equidistant from the two cars, compared to their height when only one car was turned on.

The toy cars also help to solve problem 2, because the waves are steady. You can put a ruler in the water, and watch the steady up-and-down vibration of the surface, measuring the height of the wave. This could be made easier by taking a video on a cell phone and then clicking through it frame by frame.

2-11 (a) The lowest frequency corresponds to the longest possible wavelength. Because the string is fixed at the ends, the longest-wavelength pattern will be one with a single hump in the middle, i.e., one in which half a wavelength fits on the string. We find $\lambda = 0.82$ m.
(b) $v = f\lambda = 360$ m/s.
(c) We have $v = \sqrt{T/\mu}$, so $T = \mu v^2 = 143$ N.

2-12 (a)



(b) In the first one, half a wavelength fits on the string, so $\lambda = 2$ m. Continuing in the same way, we get $\lambda = 1$ m for $N = 2$ and $\lambda = 2/3$ m

for $N = 3$.
(c) N is the number of half-wavelengths, so $N = L/(\lambda/2)$, or $\lambda = 2L/N$. This checks out when we plug in and compare with the answers from b. It also matches the result of problem 11a for $N = 1$.
(d) We have $v = f\lambda$, so $f = Nv/2L$.

2-13 (a) Partial differentiation with respect to x means treating y as a constant in addition to u and v. The result is $u\ln(vy)$. (b) Treating x as a constant, the derivative is uvx/y.

2-14

$$\begin{aligned}\nabla^2\Psi &= \frac{\partial^2}{\partial x^2}\Psi + \frac{\partial^2}{\partial y^2}\Psi \\ &= 4\cdot e^{2x+y} + 1\cdot e^{2x+y} \\ &= 5e^{2x+y}\end{aligned}$$

2-16 The derivative of a sine is a cosine, so taking the partial derivative with respect to x, and applying the chain rule, gives $k_x A\cos(\ldots)$. The second partial derivative is $-k_x^2\sin(\ldots)$, which is the same as $-k_x^2 u$. Doing a similar calculation for y, and adding, we get $-(k_x^2 + k_y^2)u$, which is the claimed result.

2-17 We have $\mathbf{k}_3 = k_0(6,2)$, so $|\mathbf{k}_3| = k_0\sqrt{40}$. We then get $\omega_3/\omega_1 = k_3/k_1 = \sqrt{20}$.

2-18 (a) A piece of rope with length ℓ has volume $A\ell$ and mass $m = \rho A\ell$, so $\mu = m/\ell = \rho A$. Setting $v = \sqrt{T/\mu}$ equal to c and solving for T, we find $T = c^2\rho A$.
(b) Plugging in $\rho = 1\ \text{g/cm}^3 = 10^3\ \text{kg/m}^3$ gives a maximum tensile strength of $T/A = 10^{20}$ Pa, which is about a thousand times greater than that hypothesized for the strongest carbon nanotubes.
(c) Most likely the rope would break. The existence of a theoretical maximum strength for the rope also suggests that even the strongest rope theoretically possible would break.

2-19 (a) Following the approach suggested in the hint in the back of the book, let the origin be at the center of the loop. The distance from the center to a point on the rope (x, y) is $\sqrt{x^2+y^2}$, which must equal the radius of the loop, so for y in terms of x we have $y = \sqrt{r^2 - x^2}$. The second derivative with respect to x, evaluated at $x = 0$ (where the rope is parallel to the x axis), comes out to be $-1/r$. The equation $\partial^2 y/\partial t^2 = (T/\mu)\partial^2 y/\partial x^2$ then gives $a = -T/\mu r$, where the minus sign indicates that the acceleration is in the negative y direction. The magnitude of the acceleration in circular motion is $\omega^2 r$, so $\omega^2 r = T/\mu r$, and $T = \mu\omega^2 r^2$.
(b) $v = \sqrt{T/\mu} = \omega r$. This means that the magnitude of the wave velocity is the same as the speed of a point on the rope due to its circular motion. A pulse traveling in the same direction as the rotation will have double this speed, but the cool thing is that a pulse traveling against the rotation stands still!

3-1 Setting $(c^2/8\pi k)B^2 = (1/8\pi k)E^2$ gives $E/B = c$.

3-2 The energy density is proportional to the square of the field, so if the field patterns are identical except for a scaling up by a factor of 2, the energy in 3He is greater by a factor of 4.

3-3 The wave propagates in the direction of the Poynting vector, which is in the direction of $\mathbf{E}\times\mathbf{B}$. Using the right-hand rule, this is in the $-x$ direction.

3-4 The momentum density needs to be in the $+x$ direction. Since it is perpendicular to both $\mathbf{E}$ and $\mathbf{B}$, the magnetic field can't be along the x axis. It also can't be along the y axis, because then it would be parallel or antiparallel to $\mathbf{E}$, and the the cross product $\mathbf{E}\times\mathbf{B}$ would be zero. The remaining possibilities are the $\pm z$ directions, and of these, the one that gives momentum in the $+x$ direction is if $\mathbf{B}$ is in the $-z$ direction.

3-5 Since it's a plane wave, the direction of the magnetic field is perpendicular to the electric field and the direction of propagation, so it must be in the $\pm y$ direction. What gives $\mathbf{E}\times\mathbf{B}$ in the positive z direction is if the magnetic field is in the $+y$ direction.

The Poynting vector is $\mathbf{S} = (c^2/4\pi k)\mathbf{E}\times\mathbf{B}$. Because $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other, the magnitude of their cross product is $|\mathbf{E}||\mathbf{B}|$. We also have $|\mathbf{E}| = c|\mathbf{B}|$ for a plane wave, so $|\mathbf{E}\times\mathbf{B}| = cB^2$. Putting this together,

$$a = \frac{c^2}{4\pi k},$$

so

$$B = \sqrt{\frac{4\pi k a}{c^3}}.$$

3-7 The given information is the Poynting vector, and we need to find the momentum. Comparing the expressions for the Poynting vector and the momentum density, we find

$$\frac{p}{v} = c^{-2}S.$$

The result is $p = 15$ kg·m/s.

3-8 (a) In units of 10^{-27} kg, the amount of mass lost is

$$1.67495 - 1.67265 - 0.00091 = 0.00139 \qquad ,$$

i.e., 0.00139×10^{-27} kg. This mass has been converted into energy:

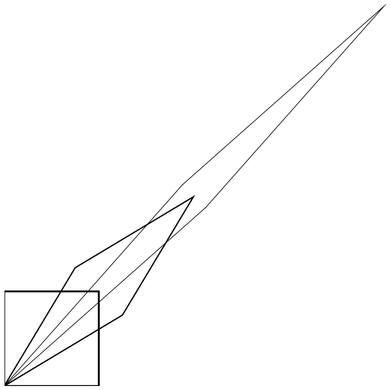
$$\begin{aligned} E &= mc^2 \\ &= (0.00139\times10^{-27})(3\times10^8)^2 \\ &= 1.25\times10^{-13} \text{ joules} \qquad . \end{aligned}$$

(In the meter-kilogram-second system of units, kilograms multiplied velocity squared give units of joules.)

(b) For this process, the total mass changes from 1.67265 to 1.67586 (again in units of 10^{-27} kg). This is an *increase* in mass, which means that we would have to have a source of energy to make the reaction happen. A free proton has no such source of energy; it's all by itself, so there's nothing for it to get energy from.

3-10 The Poynting vector of the superimposed waves is $(\mathbf{E}_1+\mathbf{E}_2)\times(\mathbf{B}_1+\mathbf{B}_2)$. If we multiply this expression out, we get four terms. Two of these are just $\mathbf{S}_1$ and $\mathbf{S}_2$, the Poynting vectors of the separate waves. Now suppose the polarization of the two waves differs by an angle θ. Because they are propagating in the same direction, this angle θ is both the angle between the electric field vectors and the angle between the magnetic field vectors. The remaining terms in the cross product equal $(E_1B_2 + E_2B_1)\sin(\theta+\pi/2)$. As a function of the randomly chosen θ, the sine factor oscillates symmetrically through positive and negative values, and therefore averages to zero.

4-1 To get good precision, you need to do this on a full-size printout, as suggested by the problem. To keep from using up a whole page on my solutions handouts, I've shown the idea below at about 25% of the actual size.



First I drew the long diagonal twice as long again, meaning about 276 mm. To keep the area the same, I had to make its "cross-bar" half as long as on the preceding parallelogram, or about 17.5 mm. The speed represented by this new parallelogram is equal to the slope of its bottom edge. To get this slope, which is the rise divided by the run, we have to draw the rise and run accurately, i.e., lined up straight with the edges of the paper. I did this by using a second piece of paper as a right angle. The result is that the slope is (rise)/(run) = 91 mm/105 mm = .86, representing 86% of the speed of light.

4-2 We have $D = \sqrt{(1+v)/(1-v)}$, so $D^2(1-v) = 1+v$, and $v = (D^2-1)/(D^2+1)$. As a simple check, if $D = 1$, we get $v = 0$, which makes sense.

4-3

$$\begin{aligned} \frac{D}{\gamma} &= \sqrt{\frac{1+v}{1-v}}\sqrt{1-v^2} \\ &= 1+v \end{aligned}$$

When v is close to 1, this is approximately 2.

4-4 (a) The computation of this Taylor seris can be simplified a lot by letting $x = v^2$, and thinking of γ as a function of x. Then

$$\gamma = (1-x)^{-1/2}$$
$$\gamma' = \frac{1}{2}(1-x)^{-3/2}$$

Evaluating these at $x = 0$, we get 1 and 1/2, so the Taylor series is

$$\begin{aligned}\gamma &= 1 + \frac{1}{1!}\cdot\frac{1}{2}x + \ldots \\ &= 1 + \frac{1}{2}v^2 + \ldots\end{aligned}$$

The odd terms vanish, because kinetic energy is a scalar that doesn't depend on the direction of motion.

(b) In SI units, we can't add 1 plus $(1/2)v^2$, which have different units. The γ factor is unitless, so we need to make the v^2 term unitless. The result is $\gamma \approx 1 + (1/2)(v/c)^2$.

(c) Let $r = 6.4 \times 10^6$ m be the radius of the earth, $R = 1.5 \times 10^{11}$ m the radius of its orbit, and $T = 1$ year $= 3.2 \times 10^7$ s the period of its orbit. Then the velocity is $v = (2\pi R)/T$, and we have $\gamma = 1/\sqrt{1 - v^2/c^2} = 1 + 4.8 \times 10^{-9}$. The resulting contraction of the diameter is about $(4.8 \times 10^{-9})(2r) = 6$ cm.

5-1 (a) Since the result only depends on the ratios of speeds, we can drop the powers of 10:

$$R = \frac{2.2 - 3.0}{2.2 + 3.0} = -0.15$$

(b) Energy is proportional to the square of amplitude, so the fraction of the energy reflected is $(0.15)^2 = 0.022$.

5-2 (a) We have $R = (\alpha - 1)/(\alpha + 1)$. Inverting α gives $R = (1/\alpha - 1)/(1/\alpha + 1) = (\alpha - 1)/(1 + \alpha)$, which means we've simply negated R. The interpretation is that the energy of the reflected pulse is unchanged, but the reflection's inverting or uninverting character is flipped.

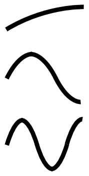
(b) Based on part a, we expect that there will be two values of α that will do the job, and that they will be inverses of each other. Solving for α in terms of R gives $\alpha = (1 + R)/(1 - R)$. Taking $R = 1/2$ gives $\alpha = 3$, and without having to do the arithmetic for the case of the inverting reflection, we can tell that it will happen for $\alpha = 1/3$.

5-3 (a) These are all different standing wave patterns that fit within the tube. Notice how all the frequencies are multiples of 150 Hz. (Since we're getting all the multiples, not just the odd multiples, the saxophone apparently acts like a symmetric air column, and we can visualize the series of wave patterns as the one in which we fit some number of half-wavelengths in the length of the tube.)

(b) Higher frequencies correspond to shorter wavelengths. You can fit more than one half-wavelength in the tube, but you can't put less than one half-wavelength in, because the wave wouldn't be doing the right things at the ends.

5-4 $L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{340\ \mathrm{m/s}}{2f} = 65$ cm

5-5 (a)



(b) In the first one, a quarter a wavelength fits on the string, so $\lambda = 4$ m. Continuing in the same way, we get $\lambda = 4/3$ m and $4/5$ m for the other two patterns. (c) The pattern is $4L/1$, $4L/3$, $4L/5$, ..., where the denominators are the successive odd integers.

(d) We have $v = f\lambda$, so $f = v/4L$, $3v/4L$, $5v/4L$, ...

(e) The harmonics are at frequencies f_o, $3f_o$, $5f_o$, ..., so given $f_o = 147$ Hz, the third harmonic has a frequency five times that, or 735 Hz.

5-6 (a)

C:	261.6	523.2	784.8	1046.4	1308.0
G:	392.0	784.0	1176.0	1568.0	1960.0

The 784.8 and 784.0 Hz frequencies differ by much less than 1%, and the 1046.4 and 1176.0

Hz frequencies differ by 12%. Neither combination is really in the 1-10% range that causes dissonance.

(b)

C:	261.6	523.2	784.8	1046.4	1308.0
Bb:	466.2	932.4	1398.6	1864.8	2331.0

The 1308.0 and 1398.6 Hz frequencies differ by about 7%, which is clearly going to make them dissonant.

5-7 (a) The wave velocity at height x is $\sqrt{T/\mu}$, where the tension, T, depends on x. The tension in the string has to be enough to support the weight of the part of the string below it, so $T = \mu g x$, and $v = \sqrt{gx}$.

(b)

$$\begin{aligned} a &= \frac{\mathrm{d}v}{\mathrm{d}t} \\ &= \frac{\mathrm{d}v}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t} \\ &= \frac{\mathrm{d}v}{\mathrm{d}x} \cdot v \\ &= \frac{1}{2}(gx)^{-1/2} g \cdot v \\ &= g/2 \end{aligned}$$

(c) It has zero velocity at $x = 0$, but that doesn't mean it stops there and just disappears — that would violate conservation of energy. If you tried the experiment, you saw that the pulse turned around and headed back up the string. In other words, it has constant acceleration before, at, and after the moment when it reaches the tip.

5-8 (a) These are both reflections back into a faster medium. I don't even remember off the top of my head whether such a reflection is inverting or not, but even if it is inverting, two inversions in a row leave the wave with the same phase it had before. Since $d = 0$, the distance traveled by the two types of waves is the same, so their phase is the same when they emerge, and the interference is constructive.

(b) To get constructive interference again, we need the extra distance $2d$ traveled by the twice-reflected wave to equal one complete wavelength. In other words, $d = \lambda/2$.

(c) Each cycle means the top plate has dropped by a distance $\lambda/2$, so the diameter of the filament is $n\lambda/2$.

5-9 Snell's law is

$$n_a \sin\theta_a = n_d \sin\theta_d \qquad ,$$

where a refers to air and d to diamond, and $n_a \approx 1$. Total internal reflection occurs when there is no solution for θ_a, which occurs when the value required for $\sin\theta_a$ is greater than 1. The critical angle is $\theta_d = \sin^{-1}(1/n_d) = 24.4$ deg. It's measured from the normal, since that's how the angles in Snell's law are always measured. It's a minimum, because making θ_d bigger would require $\sin\theta_a$ to be more than 1, which is impossible. In a substance with a smaller index of refraction, $1/n$ would be bigger, and $\sin^{-1}(1/n)$ would also get bigger, since the inverse sine increases as you make its input bigger; the minimum angle for total internal reflection would be bigger, meaning that it would be harder to get total internal reflection.

5-10 (a) For a medium with a fixed set of properties, the energy content of a wave is proportional to the square of its amplitude. The reflected wave is in the same medium as the original one, so the fraction of the energy that is reflected is R^2. The fraction transmitted is then $f = 1 - R^2 = 1 - [(\alpha-1)/(\alpha+1)]^2 = 4\alpha/(1+\alpha)^2$, which isn't the same as T^2.

(b) We now have $\alpha = v_2/v_1$ and $\beta = v_3/v_2$, and the fraction of the energy transmitted is

$$\frac{4\alpha}{(1+\alpha)^2} \cdot \frac{4\beta}{(1+\beta)^2} \qquad .$$

We have to maximize this quantity subject to the constraint $\alpha\beta = v_3/v_1 =$ constant. Since the variables α and β are treated completely symmetrically, we suspect that the result must be $\alpha = \beta$, in which case we have $v_2 = \sqrt{v_1 v_3}$.

To prove more rigorously that the solution is as claimed, it helps if we start with some simplifications. Constant factors like 4 don't affect the result, and in fact the constraint on the product $\alpha\beta$ guarantees that the entire numerator is constant. Thus maximizing the transmission amounts to simply minimizing the denominator. Since the denominator is a perfect square, all we really have to do is minimize its square root, which is $(1+\alpha)(1+\beta)$. Defining $x=\alpha\beta=v_3/v_1$ for ease of writing, and substituting x/α for β, the quantity to be minimized becomes $(1+\alpha)(1+x/\alpha)=1+x+\alpha+x/\alpha$. The derivative with respect to α is $1-x\alpha^{-2}$, and setting this equal to zero gives $\alpha=\sqrt{x}=\beta$, as expected.

5-11 (a) As suggested in section 10.5.9, let's identify sinusoidal functions with points in the complex plane. The function $4\sin\omega t$ corresponds to the real number 4, and $3\cos\omega t$ to $3i$. Their sum is the function corresponding to $4+3i$. Now we have to go backwards and find what sinusoidal function $4+3i$ represents. The magnitude of this complex number represents the amplitude, A, of the result, so $A=\sqrt{4^2+3^2}=5$. The argument represents the phase angle $\delta=\tan^{-1}(3/4)=37°$.
(b)

$$\begin{aligned}5\sin(\omega t+37°)&=5[\sin\omega t\cos 37°+sin37°\cos\omega t]\\&=3.9\sin\omega t+3.0\cos\omega t\end{aligned}$$

To within rounding errors, this verifies that the result was correct.

5-15 (a) Multiplication adds the arguments, so raising something to the third power triples its argument. The result is 3θ (or $3\theta+2\pi n$, where n is any integer).
(b) If $a^3=z$, then the argument of a should be 1/3 that of z. (There would be additional possibilities such that $\arg 3a=\arg z+2\pi n$, but we're only talking about the first quadrant, so that can't happen.) The only one that looks like it has 1/3 the argument of z is x.
(c) The argument has to be 1/3 that of $a+bi$, i.e., $(1/3)\tan^{-1}(b/a)$.
(d) The easy way to do the cube root is to use the fact that if $y=x^3$, then $|y|=|x|^3$, and $\arg y=3\arg x$. Therefore if y is the quantity inside the exponent in the denominator, we have $|x|=(\sqrt{8})^{1/3}=\sqrt{2}$, and the easiest possibility for the argument is $\arg x=(1/3)\arg y=\pi/4$. (There are other possibilities such as $11\pi/12$, because arguments that differ by a multiple of 2π represent the same thing.) Division divides the magnitudes, giving a magnitude of 1. Division subtracts the arguments, giving 0. Therefore the result of the calculation is 1.

5-16 This is impractical to carry out by the methods of freshman calculus, without using complex numbers. The trick is to use Euler's formula to rewrite $e^x\cos x$ as $(1/2)e^x(e^{ix}+e^{-ix})$. The result is -2^{50}.

5-18 When you have an exponent with a funky base, the standard method for reexpressing it in terms of base e is to apply the identity $\ln(a^b)=b\ln a$, which gives $a^b=\exp(b\ln a)$. This gives $i^i=e^{i\ln i}$. This reduces the problem to one of finding the natural log of i. Euler's formula tells us that raising e to the power $(\pi/2)i$ gives i. (Actually there is more than one solution, but we'll come back to that.) Taking $\ln i=(\pi/2)i$, we find

$$i^i=e^{-\pi/2}\qquad.$$

Euler's formula tells us that e^x is the same as $e^{x+2\pi i}$, $e^{x+4\pi i}$, etc. That means that $\ln i$ has more than one possible value. It can be $(\pi/2+2\pi n)i$, where n is any integer. This means that i^i could be any real number of the form $e^{-\pi/2-2\pi n}$. It shouldn't be too surprising that an exponent such as i^i can have more than one value. For example, $4^{1/2}$ can equal either 2 or -2.

5-19 $FWHM=f_{res}/Q=100$ Hz, so since the curve is approximately symmetric, the range will be ±50 Hz, i.e., 250-350 Hz.

5-20 (a) Since $\arg(z\cdot 1/z)=0$, and arguments add when we multiply, $\arg(1/z)=-\theta$.
(b) $|z|=\sqrt{3+1}=2$.
(c) It forms a 30-60-90 right triangle, so the arg is 30 degrees.
(d) $|1/z|=1/|z|=1/2$.
(e) $\arg(1/z)=-\arg z=-30\ °$.
(f) $\text{Im}(1/z)=|z|\sin(-30°)=-1/4$.

5-21 $\arg(1/\bar{z})=-\arg\bar{z}=\arg z$.

5-22 We have $n=\sin\phi/\sin\theta$. Doing implicit differentiation, we find $\mathrm{d}n=-\sin\phi(\cos\theta/\sin^2\theta)\,\mathrm{d}\theta$, which can be rewritten as $\mathrm{d}n=-n\cot\theta\,\mathrm{d}\theta$. This can be minimized by making θ as big as possible. To make θ as big as possible, we want ϕ to be as close as possible to

90 degrees, i.e., almost grazing the surface of the tank.

This result makes sense, because we're depending on refraction in order to get a measurement of n. At $\phi = 0$, we get $\theta = 0$, which provides no information at all about the index of refraction — the error bars become infinite. The amount of refraction increases as the angles get bigger.

6-1 **D** is the only one that's lightlike, so it's clearly out. For the other three, we have

$$\begin{aligned}|\mathbf{A}|^2 &= 5^2 - 1^2 = 24\\ |\mathbf{B}|^2 &= 5^2 - 3^2 = 16\\ |\mathbf{C}|^2 &= 7^2 - 5^2 = 24\end{aligned}$$

A and **C** both have $m = \sqrt{24}$, so they could be the same particle.

6-2 (a)

$$\begin{aligned}p &= m\gamma v\\ &= \frac{mv}{\sqrt{1-v^2}}\\ p^2\left(1-v^2\right) &= m^2v^2\\ p^2 &= \left(m^2+p^2\right)v^2\\ v &= \frac{p}{\sqrt{m^2+p^2}}\end{aligned}$$

(b) At low velocities, the second term inside the square root is negligible, and we have

$$\begin{aligned}v &\approx \frac{p}{\sqrt{m^2}}\\ &= \frac{p}{m} \qquad ,\end{aligned}$$

which is the nonrelativistic result.

(c) For very large momenta, the m^2 term is negligible compared to the p^2 term, so

$$\begin{aligned}v &= \frac{p}{\sqrt{p^2}}\\ &= \pm 1,\end{aligned}$$

which is the speed of light in natural units.

(d) Converting to SI units, we have

$$v = \frac{p}{\sqrt{m^2+p^2/c^2}}.$$

6-3 As in the example, we have as our starting point the relations $m^2 = E^2 - p^2$ and $v = p/E$. Here we want to eliminate p, so we substitude $p = vE$ into the definition of mass, which gives $m^2 = E^2 - v^2E^2$. Solving this for E gives an expression that can be written most compactly as $E = m\gamma$.

6-4 As we found in an earlier problem, the Taylor series of γ can be found most easily by thinking of v^2 as the independent variable. The result is $\gamma = 1 + v^2/2 + \ldots$ This gives

$$p = m\gamma v = mv + mv^3/2 + \ldots$$

The first term is mv, which is the Newtonian expression, as required by the correspondence principle. The v^2 term vanishes, which makes sense because the momentum should be an odd function of v.

6-5 $\{\hat{\mathbf{x}}\}$ is not a basis, because there are vectors such as $\hat{\mathbf{y}}$ that we can't form as a linear combination (i.e., scalar multiple) of $\hat{\mathbf{x}}$. $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}\}$ is the standard basis for this vector space. $\{\widehat{-\mathbf{x}}, \hat{\mathbf{x}}+\hat{\mathbf{y}}\}$ also works as a basis, because the two vectors are linearly independent, and it's easy to check that any vector in the plane can be formed as a linear superposition of them. $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{x}}+\hat{\mathbf{y}}\}$ is not a basis, because these three vectors are not linearly independent.

6-6 (a) The sketch for ℓ will be a 45-degree line through the origin, while r will be only the part of that line in the first quadrant. Of the two, only ℓ is a vector space. The set r isn't a vector space, because it doesn't have additive inverses.

(b) We have $(1/2)(\pi + \pi) = 0$, but $(1/2)\pi + (1/2)\pi = \pi$.

6-7 To do anything useful with these expressions describing units, we need to be able to talk about things like dividing meters by seconds to get meters per second. Thus "addition" needs to be multiplication, which corresponds to adding the exponents. Scalar "multiplication" actually has to be exponentiation, e.g., "multiplying" units of meters by the scalar 2 should give square meters.

7-1 The probabilities are independent, so $(0.014) \times (0.014) = 2.0 \times 10^{-4}$.

7-2 Let a, b, and c be the probabilities of having 0, 1, or 2 dogs, respectively. Normalization gives

$$a + b + c = 1.$$

The rule for calculating averages tells us that

$$0a + 1b + 2c = M.$$

The statistic N is stated only in terms of the part of the population that does have dogs. The probabilities b and c are not properly normalized if they are to represent only these people. Their normalized versions, for this population, would be $b/(b + c)$ and $c/(b + c)$. We therefore have

$$\frac{b + 2c}{b + c} = N.$$

Solving this system of equations gives

$$a = 1 - \frac{M}{N}$$
$$b = M\left(\frac{2}{N} - 1\right)$$

and

$$c = M\left(1 - \frac{1}{N}\right).$$

It makes sense that c will always be positive, because $N > 1$. We can also check that for $N = 2$, we get $b = 0$, which makes sense, and that if $M = 0$, we get the right results.

7-3 Suppose we record the results of a long string of games played one after another. One method of testing for independence would be to use the data to estimate the probability of winning, $P(W)$, and also break down the list into sequential pairs (1st and 2nd game, 3rd and 4th, etc.) and use these pairs to measure the probability of winning twice in a row, $P(W \text{ and } W)$. The definition of independence then gives $P(W \text{ and } W) = P(W)^2$. We can test whether the left-hand and right-hand sides of this equation are in fact equal, to within the statistical sampling error imposed by the number of games observed.

Another approach is to look for extremely long runs of wins. If independence holds, then the probability of n wins in a row is $P(W)^n$, which falls off exponentially. Any observation of a very large n would therefore prove that the rolls were not statistically independent. For a statistical analysis of data from players who ran up world-record winning streaks, see Ethier and Hoppe, arxiv.org/pdf/0906.1545. The results are described in terms of the number of rolls, which is greater than the number of games.

7-4 (a) According to the ideal gas law, $PV = nkT$, standard values of pressure and temperature give $V \propto n$, since k is also a constant.
(b)

$$\begin{aligned} V &= nkT/P \\ &= (6.0 \times 10^{23})(1.38 \times 10^{-23}\ \text{J/K})(273\ \text{K}) \\ &\qquad /(1.01 \times 10^5\ \text{Pa}) \\ &= 2.2 \times 10^{-2}\ \text{J/Pa} \\ &= 2.2 \times 10^{-2}\ \text{N$\cdot$m}/(\text{N/m}^2) \\ &= 2.2 \times 10^{-2}\ \text{m}^3 \\ &= 2.2 \times 10^{4}\ \text{cm}^3 \\ &= 22\ \text{liters} \end{aligned}$$

7-5 The motion of the piston through an infinitesimal distance $\mathrm{d}x$ will increase the interior volume of the cylinder by $A\,\mathrm{d}x$, where A is the cylinder's cross-sectional area. The work done is $\mathrm{d}W = F\,\mathrm{d}x = (PA)(\mathrm{d}V/A) = P\,\mathrm{d}V$.

Note that the result is not true in general for finite rather than infinitesimal changes. If we attempt to carry through the same proof for finite changes, it fails because $W \neq F\Delta x$ when F depends on x.

7-6 The ideal gas law says $PV = nkT$. The volume is the same as before, and the temperature is the same, since the gas is back in thermal equilibrium with the room. Since n is doubled, P must be doubled as well.

7-7 The heat required in order to raise the temperature of a monoatomic gas by ΔT is $(3/2)nk\Delta T$. The rate of temperature increase is therefore

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= \frac{2P}{3nk} \\ &= 1.5\times 10^{-8}\ \text{K/s}.\end{aligned}$$

I was surprised by how small this was. It would take several years to heat by one degree.

7-8

$$\begin{aligned}P &= \frac{n}{V}kT \\ &= \left(10^{-5}\ \text{cm}^{-3}\right)\left(1.38\times 10^{-23}\ \text{J/K}\right)\left(10^{3}\ \text{K}\right) \\ &= \left(10\ \text{m}^{-3}\right)\left(1.38\times 10^{-23}\ \text{J/K}\right)\left(10^{3}\ \text{K}\right) \\ &\sim 10^{-19}\ \text{J}\cdot\text{m}^{-3} \\ &= 10^{-19}\ \text{N}\cdot\text{m}\cdot\text{m}^{-3} \\ &= 10^{-19}\ \text{Pa}\end{aligned}$$

7-10 (a) By equipartition, the average kinetic energy is $(3/2)kT$.

(b) Equating this to $(1/2)mv^2$ gives $\overline{v^2} = 3kT/m$. This comes out large because m is so small for an electron compared to an atom.

8-3 (a) It lacks any symmetry under rotation about any axis, so it acts as a polyatomic molecule.

(b) We have

$$P \propto V^{-\gamma},$$

where $\gamma = 1 + 2/\alpha = 4/3$ for a polyatomic gas. Then

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^{-\gamma},$$

which results in $P_1 = 8.5$ atmospheres.

8-4 We have $P \propto V^{-\gamma}$ (adiabatic expansion) and $PV \propto T$ (idea gas law). Eliminating $V \propto T/P$ gives $P^{\gamma-1} \propto T^{\gamma}$, or

$$T \propto P^{1-1/\gamma},$$

so $b = 1 - 1/gamma$. Applying $\gamma = 1 + 2/\alpha$, we find

$$b = \frac{2}{2+\alpha}.$$

8-5 (a)

$$\begin{aligned}P_{surface} - P_{center} &= \int_{r=0}^{b} \mathrm{d}P \\ &= -\int_{r=0}^{b} \rho g\,\mathrm{d}r\end{aligned}$$

The pressure at the surface is essentially zero, so

$$\begin{aligned}P_{center} &= \int_{r=0}^{b} \rho g\,\mathrm{d}r \\ &= \int_{r=0}^{b} \rho\frac{Gmr}{b^3}\,\mathrm{d}r \\ &= \left.\frac{G\rho m r^2}{2b^3}\right]_{r=0}^{b} \\ &= \frac{G\rho m}{2b} \\ &= \frac{3Gm^2}{8\pi b^4}\end{aligned}$$

(b) The units of Gm^2/b^2 are newtons, so the units of Gm^2/b^4 are N/m^2.

(c) 1.7×10^{11} Pa.

(d) Atmospheric pressure is 10^5 Pa, which is, as expected, about six orders of magnitude less than the answer to part c.

8-6 (a) The condition for a molecule to escape is that its kinetic energy has to equal the amount of gravitational energy it would gain while escaping. The kinetic energy depends on v^2, and the gravitational energy on M/r, so the ratio of the escape velocities is

$$\begin{aligned}\frac{v_m}{v_e} &= \sqrt{\frac{M_m r_e}{M_e r_m}} \\ &= 0.22 \qquad \text{[ratio of escape velocities]}\end{aligned}$$

(b) The absolute temperature is proportional to the average kinetic energy per molecule, so

$$\begin{aligned}\frac{v_m}{v_e} &= \sqrt{\frac{403\ \text{K}}{273\ \text{K}}} \\ &= 1.21 \qquad \text{[ratio of average velocities]}\end{aligned}$$

(c) For a given temperature, the average kinetic energy is a fixed value, so the mass is proportional to the inverse square of the velocity. If the temperatures on the earth and moon were the same, then the mass of the fluorocarbon molecule would have to be greater by a factor of $(0.22)^{-2}$. The higher temperature on the moon means we need the fluorocarbon molecules to be heavier by an additional factor of $(1.21)^2$. The result is that we need a mass ratio of $(0.22)^{-2}(1.21)^2 = 30$.

The atomic masses involved are N=14.0, F=19.0, C=12.0.

$$30(2 \times 14) = (12.0)n + (19.0)(2n + 2)$$

$$n \approx 16$$

So if some super-advanced alien civilization decided to cover the moon with an atmosphere of $C_{16}F_{34}$, you'd be able to walk around on the surface with only an oxygen mask!

9-2 The energy of 1.3 J equals the amount Q gained by the spoon and minus the amount $-Q$ lost by the coffee. The change in entropy is

$$\Delta S = \frac{Q}{T_{\text{spoon}}} - \frac{Q}{T_{\text{coffee}}} = 7.2 \times 10^{-4}\ \text{J/K}.$$

9-4 (a)

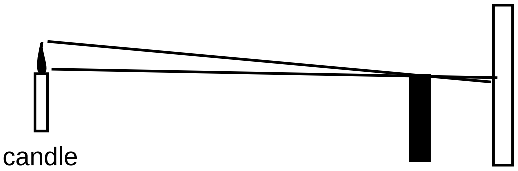
$$\begin{aligned}
\text{eff} &= \frac{Q_L}{W} \\
&= \frac{Q_L}{Q_H - Q_L} && \text{[cons. of energy]} \\
&= \left(\frac{Q_H}{Q_L} - 1\right)^{-1} \\
&= \left(\frac{T_H}{T_L} - 1\right)^{-1} && \text{[Carnot engine]} \\
&= \left(\frac{308\ \text{K}}{300\ \text{K}} - 1\right)^{-1} \\
&= 38 \\
\text{EER} &\approx (10/3)(\text{eff}) \\
&\approx 150
\end{aligned}$$

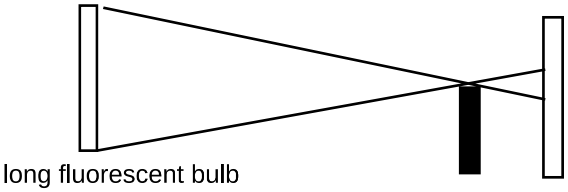
(b)

$$\begin{aligned}
\Delta S &= \frac{Q_H}{T_H} - \frac{Q_L}{T_L} \\
&= \frac{W + Q_L}{T_H} - \frac{Q_L}{T_L} && \text{[cons. of energy]} \\
&= W\left(\frac{1 + \text{eff}}{T_H} - \frac{\text{eff}}{T_L}\right) \\
&= 3 \times 10^4\ \text{J/K}
\end{aligned}$$

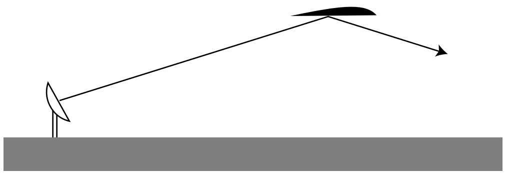
The result is positive, so it's consistent with the second law.

10-1 The fuzzy part of the shadow is the part that is only illuminated by some of the rays from the light source. This is the area between the extreme rays shown in the figures. The fuzzy area is larger in the second figure.



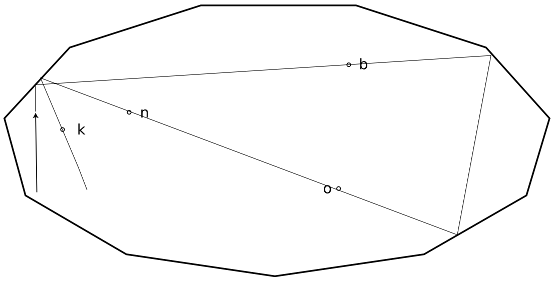


10-2 Because the surfaces are flat, you get specular reflection. In specular reflection, all the reflected rays go in one direction. Unless the plane is directly overhead, that direction won't be the right direction to make the rays come back to the radar station.



This is different from a normal plane, which has complicated, bumpy surfaces. These surfaces give diffuse reflection, which spreads the reflected rays randomly in more or less every possible direction.

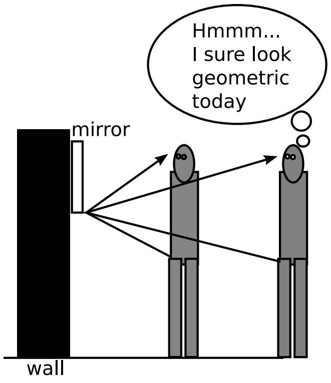
10-3 It spells “bonk.”



10-4 The image is at an equal distance behind the mirror, so the distance between the man and his image is twice his distance from the mirror, and his separation from his image is reducing at a rate of 2.0 m/s.

10-5 If you’re going to see a particular part of your body, then rays from that part of your body have to be able to go to the mirror, be reflected, and go into your eye. If we’re trying to find the lowest part you can see, then we’re interested in the case where the ray hits the very bottom of the mirror. Backing up doesn’t help, because the triangles are isoceles by the law of specular reflection, and the top of the base is fixed in height.

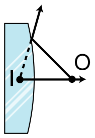
Another way of understanding the result is that if you go twice as far away, the mirror subtends half the angle, but your image also subtends half the angle, so the mirror still encompasses the same amount of your image.



10-6 An image is where *all* the rays that started from a certain point come together for one big family reunion. Only 3 represents an image. Points like 1, 2, and 4 look special, but only because of the particular rays we chose to trace, out of the infinite number of rays fanning out from the point on the object. If we had chosen a different set of rays, they would all have crossed at 3 as before, but not at 1, 2, and 4.

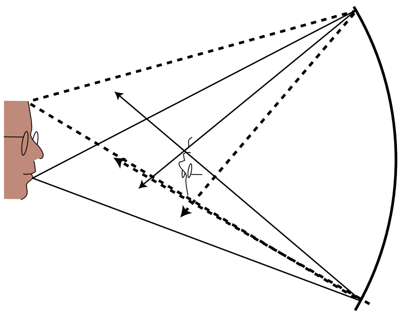
10-7 (a) The image is closer to the surface of the mirror, so the magnification is less than one.

(b) It’s virtual, because the rays only appear to have passed through point I. There is no way it could make a real image, because in order to make a real image it would have to bring the rays back together.



10-8 Usually you need to draw in a normal in order to get an accurate drawing of the reflected rays. Here, however, we already have one set of rays, and it’s pretty easy to do the new rays so that they fan in or fan out by the correct angle compared to the old ones.

As claimed, the image of the forehead (where the dashed rays cross) is below the image of the lips.



11-1 For a flat mirror, d_i and d_o are equal, so the magnification is 1, i.e., the image is the same size as the object.

11-2 If the sound gets concentrated again at your ear, it must be a real image. You're at the center of the sphere formed by extending the surface. We've already found in the text that for a real image formed by a converging mirror, the equation for the image's location is $1/f = 1/d_\text{o} + 1/d_i$. If the object and image distances are equal, then $1/f = 2/d_\text{o}$, or $f = d_\text{o}/2$. The focal length is half the distance from the surface to your head.

11-4 The diameters of the mirrors are irrelevant. The focal length simply relates to the curvature of the mirror, a more strongly curved mirror having a shorter focal length. From shortest focal length to longest, the ranking of the mirrors is C, B, A.

11-6 (a) In problem #2 we found that the equation relating the object and image distances was of the form $1/f = -1/d_i + 1/d_o$. Let's make $f = 1.00$ m. To get a virtual image we need $d_o < f$, so let $d_o = 0.50$ m. Solving for d_i, we find $d_i = 1/(1/d_o - 1/f) = 1.00$ m. The magnification is $M = d_i/d_o = 2.00$. If we change d_o to 0.55 m, the magnification becomes 2.22. The magnification changes somewhat with distance, so the store's ad must be assuming you'll use the mirror at a certain distance. It can't have a magnification of 5 at all distances.

(b) Theoretically yes, but in practical terms no. If you go through a calculation similar to the one in part a, you'll find that the images of both planets are formed at almost exactly the same d_i, $d_i = f$, since $1/d_o$ is pretty close to zero for any astronomical object. The more distant planet has an image half as big ($M = d_i/d_o$, and d_o is doubled), but we're talking about *angular* magnification here, so what we care about is the angular size of the image compared to the angular size of the object. The more distant planet has half the angular size, but its image has half the angular size as well, so the angular magnification is the same. If you think about it, it wouldn't make much sense for the angular magnification to depend on the planet's distance — if it did, then determining astronomical distances would be much easier than it actually is!

11-7 (a) This occurs when the d_i is infinite. Let's say it's a converging mirror creating a virtual image, as in problems 2 and 3. Then we'd get an infinite d_i if we put $d_o = f$, i.e., the object is at the focal point of the mirror. The image is infinitely large, but it's also infinitely far away, so its angular size isn't infinite; an angular size can never be more than about 180° since you can't see in back of your head!.

(b) It's not possible to make the magnification infinite by having $d_o = 0$. The image location and object location are related by $1/f = 1/d_o - 1/d_i$, so $1/d_i = 1/d_o - 1/f$. If d_o is zero, then $1/d_o$ is infinite, $1/d_i$ is infinite, and d_i is zero as well. In other words, as d_o approaches zero, so does d_i, and d_i/d_o doesn't blow up. Physically, the mirror's curvature becomes irrelevant from the point of view of a tiny flea sitting on its surface: the mirror seems flat to the flea. So physically the magnification would be 1, not infinity, for very small values of d_o.

11-8 See figures a-f.

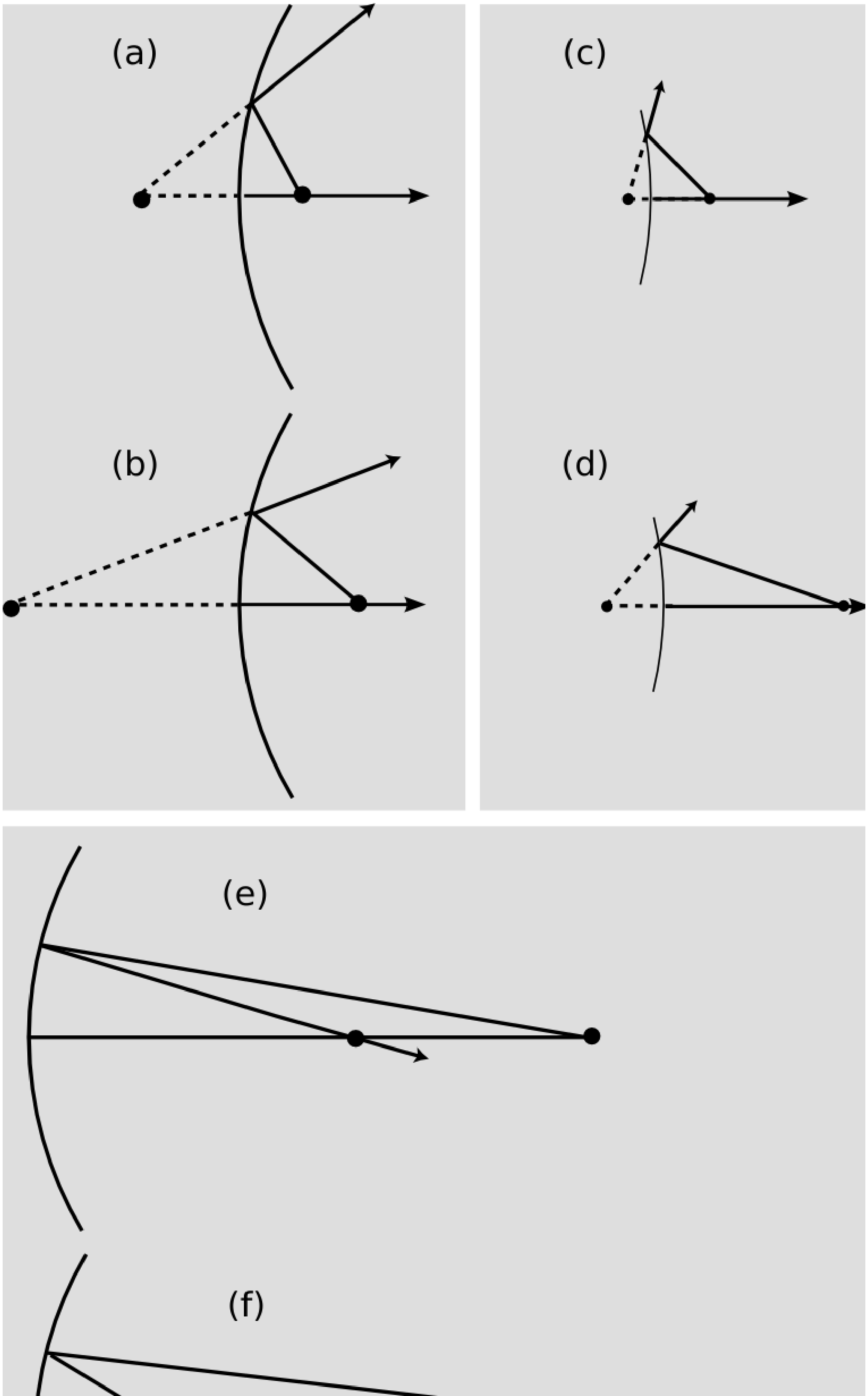
a and b: converging mirror, virtual image; increased object distance gives increased image distance

c and d: diverging mirror, virtual image: increased object distance gives increased image distance

e and f: converging mirror, real image: increased object distance gives decreased image distance

11-9 The magnification is the ratio of the image's size to the object's size. It has nothing to do with the person's location. The angular magnification, however, does depend on the person's location, because things farther away subtend smaller angles. The distance to the actual object is not changed significantly, since it's zillions of miles away in outer space, but the distance to the image does change if the observer's point of view changes. If you can get closer to the image, the angular magnification is greater.

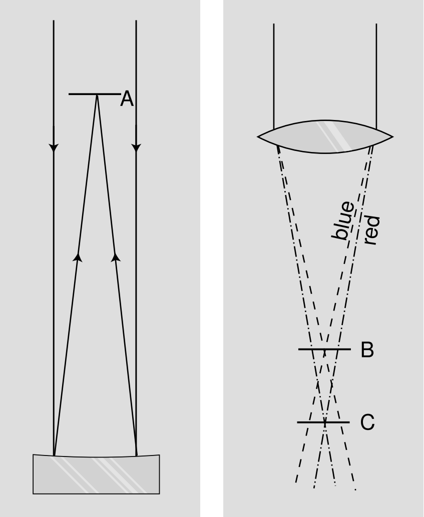
11-10 (a) For a virtual image formed by a converging mirror, the extreme possibilities are $d_o \to 0$, which gives $M \to 1$ because you can't see the curvature of the mirror from up close, and $d_o = f$, which gives parallel reflected rays,


(a)
(c)
(b)
(d)
(e)
(f)

$M = \infty$. The result is that $1 < M \leq \infty$.
(b) For a real image formed by a converging mirror, the extremes are obtained when either $d_o = \infty$ or $d_i = \infty$. We have $0 \leq M \leq \infty$.
For a diverging mirror, $d_o \to 0$ gives $M \to 1$ for the same reason as in part a, while $d_o \to \infty$ gives $M \to 0$. The result is $0 < M < 1$.

11-11 The refracted ray that was bent closer to the normal in the plastic when the plastic was in air will be bent farther from the normal in the plastic when the plastic is in water. It will become a diverging lens.

11-12 The figure shows the simplest possible reflecting and refracting telescopes, with no diagonal mirror or eyepiece.



In both cases, parallel rays enter from the star, which is infinitely far away. Each of these white rays is really a mixture of rays of all the different colors.

With the reflecting telescope, all the colors are reflected at the same angle and form a single image at one place. If we insert a piece of paper or a digital camera chip at A, only one spot is illuminated with the light from the star.

With the refracting telescope, it is never possible to get all the colors in focus at once. Each color has its own image, at a different distance from the lens. If the paper or chip is at B, where the image occurs for a specific wavelength of blue light, we get a blue dot surrounded by a fuzzy halo consisting of other colors such as red. If we move it to C, where the image occurs for a certain wavelength of red, we get a red dot surrounded by the other colors.

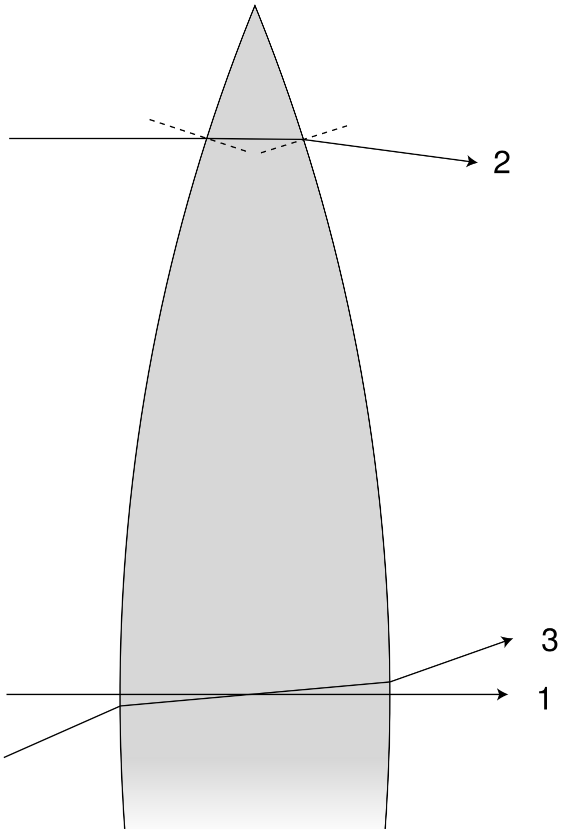
11-13 Refraction occurs only at the boundary between two substances, which in this case means the surface of the lens. Light doesn't get bent at all inside the lens, so the thickness of the lens isn't really what's important. What matters is the angles of the lens' surfaces at various points.

Ray 1 makes an angle of zero with respect to the normal as it enters the lens, so it doesn't get bent at all, and likewise at the back.

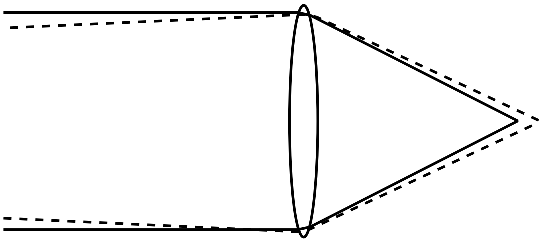
At the edge of the lens, 2, the front and back are not parallel, so a ray that traverses the lens at the edge ends up being bent quite a bit.

Although I drew both ray 1 and ray 2 coming in along the axis of the lens, it really doesn't matter. For instance, ray 3 bends on the way in, but bends an equal amount on the way out, so it still emerges from the lens moving in the same direction as the direction it originally had.

Summarizing and systematizing these observations, we can say that for a ray that enters the lens at the center, where the surfaces are parallel, the sum of the two deflection angles is zero. Since the total deflection is zero at the center, it must be larger away from the center.

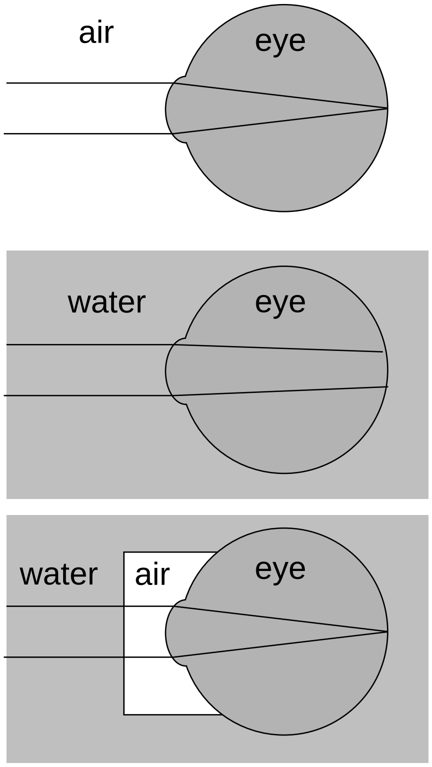


11-14 The rays from a point on a very distant object are parallel, like the solid lines in the figure. The rays from a point on a nearer object are slightly diverging, like the dashed lines. The lens cannot bend the rays from the nearer object strongly enough to make them converge as close to the lens, so the lens-to-film distance has to be greater when the object is nearer.



11-15 Normally, in air, your eyes do most of their focusing at the air-eye boundary. When you swim without goggles, there is almost no difference in speed at the water-eye interface, so light is not strongly refracted there (see figure), and the image is far behind the retina.

Goggles fix this problem for the following reason. The light rays cross a water-air boundary as they enter the goggles, but they're coming in along the normal, so they don't get bent. At the air-eye boundary, they get bent the same amount they normally would when you weren't swimming.

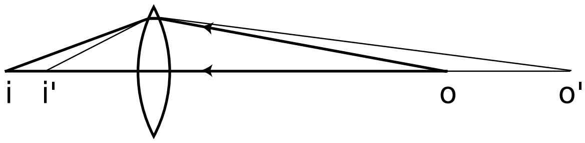


11-16 (a) See the ray diagram drawn with thick lines, showing object location o and image location i.

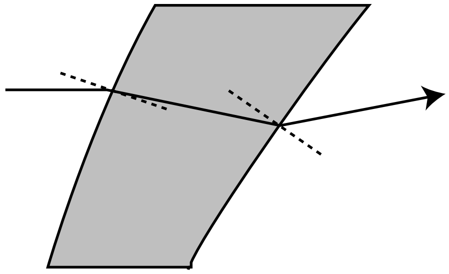
(b) If we move the object farther away, to o′ the cone of rays intercepted by the lens (thin lines) is less strongly diverging, and the lens is able to bring it to a closer focus, at i′. In the diagrams, we see that a smaller θ_o leads to a larger θ_i, so the signs in the equation $\pm\theta_o \pm \theta_i = \theta_f$ must be the same, and therefore both positive, since θ_f is positive by definition.

The equation relating the image and object locations must be $1/f = 1/d_o + 1/d_i$.

(c) Solve the equation for $d_i = (1/f - 1/d_o)^{-1} = 32$cm.



11-19 (a) See the figure below. The first refraction clearly bends it inward. However, the back surface of the lens is more slanted, so the ray makes a bigger angle with respect to the normal at the back surface. The bending at the back surface is therefore greater than the bending at the front surface, and the ray ends up being bent *outward* more than inward.



(b) Lens 2 must act the same as lens 1. It's diverging. One way of knowing this is time-reversal symmetry: if we flip the original figure over and then reverse the direction of the ray, it's still a valid diagram.

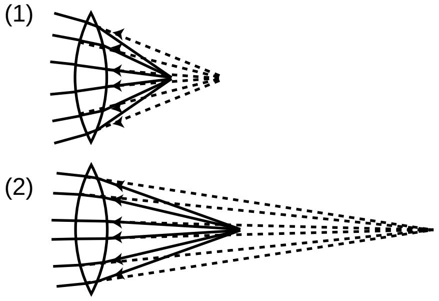
Lens 3 is diverging like lens 1 on top, and diverging like lens 2 on the bottom. It's a diverging lens.

As for lens 4, any close-up diagram we draw of a particular ray passing through it will look exactly like the corresponding close-up diagram for some part of lens 1. Lens 4 behaves the same as lens 1.

11-20 (a) See figure (1).

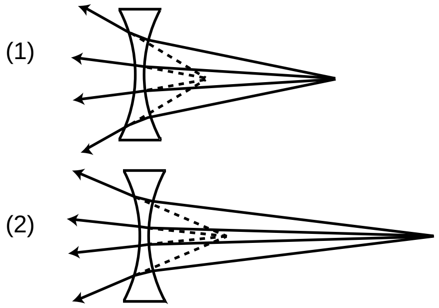
(b) If we move the object farther from the lens, as in (2), the cone of rays intercepted by the lens is less strongly diverging, so the lens is able to bring them closer together, resulting in an image that is farther from the lens. Since a smaller θ_o leads to a smaller θ_i, the signs in the equation $\pm\theta_o \pm \theta_i = \theta_f$ must be opposite. Since θ_o is greater than θ_i, we must have $\theta_o - \theta_i = \theta_f$ in order to get a positive focal angle. The equation relating the image and object locations is therefore $1/f = 1/d_o - 1/d_i$.

(c) Solving for f and plugging in, we get 18 cm.



11-21 (a) See figure (1).

(b) Decreasing θ_o, as in figure (2), makes a cone of rays that is less strongly diverging, which the lens cannot bend outward as far, so θ_i becomes smaller. Since decreasing θ_o makes θ_i smaller as well, the signs in the equation equation $\pm\theta_o \pm \theta_i = \theta_f$ must be opposite. Since θ_i is greater than θ_o, we must have $-\theta_o + \theta_i = \theta_f$ in order to get a positive θ_f. The equation relating the image and object locations must be $1/f = -1/d_o + 1/d_i$. Solving for d_i and plugging in, we get 30.8 cm.



11-26 Snell's law gives $\sin\theta_1 = (1+\epsilon)\sin\theta_2$. To get the greatest deflection, we want the limit in which θ_1 appraoches 90 °. Expanding $\sin\theta_2$ in a Taylor series around 90 °, we have $\sin\theta_2 =$

$1-\phi^2/2+\ldots$, where $\theta_2=\pi/2-\phi$. The result is $\phi=\sqrt{2\epsilon}$.

11-28 (a) Initially, the object is so close to the mirror that the mirror's curvature is irrelevant — any curve looks flat if you're close enough to it. Therefore for $d_o << f$, we have $d_i \approx d_o$, and the image's velocity equals the object's. As the object's distance approaches infinity, on the other hand, the image approaches a position that is one focal length above the mirror. Therefore,the image must accelerate from rest, move upward a distance equal to the focal length, and decelerate again, approaching zero velocity.

(b) The only two variables occurring in the statement of the problem are f and g, and there is only one way to combine these to make something with units of velocuty, which is to form the expression $\sqrt{fg}$. The answer must equal $\beta\sqrt{fg}$, where β is a unitless constant.

(c) As shown in example 5, the relatonship between the image and object locations is $1/f = 1/d_i - 1/d_o$. Since we already know the form of the solution's dependence on f and g, we can arbitrarily set $f=1$ and $g=1$ in order to simplify the writing. We then have $d_i = 1/(1+1/d_o)$, where $d_o = t^2/2$, and differentiation gives $v_i = t/(t^2/2+1)^2$. To find the maximum velocity, we differentiate again, and set the derivative equal to zero. In other words, we're looking for the time when the acceleration equals zero. The acceleration is $a_i = 1/(t^2/2+1)^2 - 2t^2(t^2/2+1)^3$, which equals zero at $t=\sqrt{2/3}$. Plugging this back into the expression for v_i, we have a maximum velocity of $2^{-7/2}3^{3/2}$. This is the mysterious unitless constant β from part b. Recovering the general solution for $f\neq 1$ and $g \neq 1$, we have a maximum velocity of $2^{-7/2}3^{3/2}\sqrt{fg}$.

11-29 (a) Let the virtual image formed by the top mirror lie at a height b above the top mirror. Then

$$\frac{1}{f}=\frac{1}{h}-\frac{1}{b} \quad .$$

Solving for b, we have

$$b=\left(\frac{1}{h}-\frac{1}{f}\right)^{-1} \quad .$$

The second image is at a distance h from the bottom mirror, so

$$\begin{aligned}\frac{1}{f}&=\frac{1}{b+h}+\frac{1}{h}\\ &=\frac{1}{h+\left(\frac{1}{h}-\frac{1}{f}\right)^{-1}}+\frac{1}{h}\end{aligned}$$

(b) Multiplying both sides by h,

$$\begin{aligned}\frac{h}{f}&=\frac{h}{h+\left(\frac{1}{h}-\frac{1}{f}\right)^{-1}}+1\\ &=\frac{1}{1+\left(1-\frac{h}{f}\right)^{-1}}+1\\ x&=\frac{1}{1+(1-x)^{-1}}+1\end{aligned}$$

This simplifies somewhat if we define a new variable $u=1-x$:

$$\begin{aligned}-u&=\frac{1}{1+1/u}\\ -u(1+1/u)&=1\\ -u-1&=1\\ u&=-2\\ x&=3\\ h&=3f\end{aligned}$$

Here's why this is a hard problem. The solution $h=3f$ is unphysical. If h was greater than f, then the first image would have been real, not virtual, which would be inconsistent with the assumptions we made in the first place. The trick is in realizing that there was a hidden assumption involved in the step where we multiplied both sides by $1+1/u$, which was that $1+1/u$ was a real number. However, if $u=0$, then $1+1/u$ is infinite, and we can't multiply both sides by it. That's the second solution referred to in the problem:

$$\begin{aligned}u&=0\\ x&=1\\ h&=f\end{aligned}$$

In other words, the first image is at infinity. Images at infinity are the crossing-over point between real and virtual images. That means it didn't actually matter that we assumed the first image would be virtual. We could have gotten the same answer by assuming it was real.

11-31 Applying small-angle approximations to Schlick's approximation gives $f \approx f(0)1 - (1/32)\theta^{10}$. From the result of the previous problem, we have $f(0) = 4n(1+n)^{-2}$. (a) Let θ_2 be the angle in air. The ratio of the widths is $g = \cos\theta/\cos\theta_2$, and using the small-angle approximation $\cos x \approx 1 - x^2/2$ and the approximation $1/(1-\epsilon) \approx 1 + \epsilon$, we find $g = 1 + P\theta^2$, where $P = (n^2-1)/2$.

(b) Multiplying the expressions for α and β, and throwing away the θ^{12} term, we have $I_2 = f(0)(1 + P\theta^2 - Q\theta^{10})$, where $Q = 1/32$. Using calculus to maximize this expression, we find

$$\theta = \left[\frac{16}{5}(n^2-1)\right]^{1/8},$$

and plugging this back in to I_2 gives

$$I_2 = f(0)(1 + k(n^2-1)^{5/4}),$$

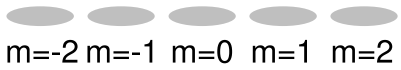
where $k = 4 \cdot 5^{-5/4} \approx 0.535$.

11-32 For simplicity, let's assume that the cylinder has unit radius, so that its focal length is 1/2, and the relationship between the object and image distances is $1/d_i - 1/d_o = 2$. To see that the standard Peaucellier linkage doesn't do the right thing, consider the case when the object point is at infinity. Then the image distance is 1/2, whereas inversion in a circle would give a point at radius zero.

The standard definition of object and image distances is relative to the surface of the mirror, whereas the geometrically relevant quantities in this problem would clearly be measured from the center of the cylinder, $u = d_o + 1$ and $v = 1 - d_i$. The relationship between these two variables is $1/u + 1/v = 2$, which is the equation of a hyperbola with perpendicular asymptotes at $u = 1/2$ and $v = 1/2$.

This gives us strong reason to hope that the a construction based on the Peaucellier can be done, since the Peaucellier linkage calculates the inverse of a number, and the graph of the function $y = 1/x$ is also a hyperbola with perpendicular asymptotes. To make the connection, let $p = u - 1/2$ and $q = v - 1/2$, so that $p = 1/4q$. This shows that we need to make a Peaucellier linkage with $k = 1/2$. Positioning it so that when the rhombus is flattened, points P and P' lie on the surface of the cylinder. The point O should not be located at the center of the cylinder; we need to modify the linkage by adding a seventh rod, with length 1/2, to tie O to the center of the cylinder. We also need to add constraints to keep the axis of the Peaucellier linkage lined up with the seventh rod.

12-1 The natural variable to use for labeling the fringes is m, as shown in the figure.



The angle θ is also different for the different fringes. The wavelength λ is a property of all the light being used, not a property of a particular fringe. For instance, if it's blue light, it might have a wavelength of 300 nm or so, and that would be the wavelength of all the light in every fringe. The variable d refers to a dimension of the physical object the light is diffracting around, e.g., the distance between two slits, so it's also not different for the different fringes. In particular, the light and dark pattern of fringes is *not* just a shadow of the slits — you could have hundreds of fringes made by only one or two slits.

12-2 The center-to-center spacing of patterns 2 and 3 are the same. The one with the small center-to-center spacing, pattern 1, corresponds to the grating with the large slit spacing, C. Grating A has more repetition, so its pattern is the one with narrower lines, 3. To summarize, 1=C, 2=B, 3=A.

12-3 Red light is at the long-wavelength end of the spectrum. Longer wavelengths give larger diffraction angles, so if the slits had been identical, the red pattern would have been the more widely spaced one. We conclude that different slits were used.

12-4 Let's write x for the distance from the slits to the wall, and y for the spacing between fringes. Three fringes cover about 5.1 cm in the photo, so the spacing is about $y = 1.70$ cm. An example in the text worked out the case where the angles are small, and we want to relate the wavelength to the angular spacing. The result (using radians, not degrees!) is

$$\begin{aligned}\lambda &\approx d\Delta\theta \\ &= d\tan^{-1}(y/x) \\ &\approx d\cdot(y/x) \\ &= 580\text{ nm}\end{aligned}$$

As a check on the precision of my result, I measured across five fringes and got 8.3 cm, which gives $y = 1.66$ cm, and $\lambda = 570$ nm. Typically, measuring across more fringes would reduce the relative error in the measurement and give a better result. However, the picture only shows five maxima, so measuring across them required me to estimate the position of the minima on the ends without being able to see the next bright fringe. Roughly, then, I'd say the two results of 580 and 570 nm are equally trustworthy, and it might make sense to average them to get about 575 nm as my best value, with error bars of five or ten nm (one or two percent).

12-5 You don't want the wave properties of light to cause all kinds of funny-looking diffraction effects. You want to see the thing you're looking at in the same way you'd see a big object. Diffraction effects are most pronounced when the wavelength of the light is relatively large compared to the size of the object the light is interacting with, so red would be the worst. Blue light is near the short-wavelength end of the visible spectrum, which would be the best.

12-6 (a) You can tell it's a single slit because of the double-width central fringe.

(b) Four fringes on the top pattern are about 23.5 mm, while five fringes on the bottom one are about 14.5 mm. The spacings are 5.88 and 2.90 mm, with a ratio of 2.03. A smaller d leads to larger diffraction angles, so the width of the slit used to make the bottom pattern was almost exactly twice as wide as the one used to make the top one.

12-7 There is not actually a sharp edge to the visible rainbow; there is a range of wavelengths at each end of the spectrum over which your eye's sensitivity drops off. However, as stated in the problem, the range is roughly 400 to 700 nm, which is slightly less than a factor of two. You get a series of rainbows, each broader than the previous one, but with a constant center-to-center angular spacing. The angle of a given color within the rainbow of order m is determined by the equation $m\lambda = d\sin\theta$, or $\theta = \sin^{-1}(m\lambda/d)$. Since the red end is very nearly twice the wavelength of the violet end, the $m = 1$ red has very nearly the same value of θ as the $m = 2$ violet, so the first and second order spectra almost overlap, but not quite. The $m = 2$ and $m = 3$ spectra will definitely overlap, because the quantity $m\lambda/d$ is greater for $m = 2$ red than for $m = 3$ violet.

12-8 For the size of the diffraction blob, we have:

$$\begin{aligned}\frac{\lambda}{d} &\sim \sin\theta \\ &\approx \theta \\ \theta &\sim \frac{700\text{ nm}}{10\text{ m}} \\ &\approx 10^{-7}\text{ radians}\end{aligned}$$

For the actual angular size of the star, the small-angle approximation gives

$$\begin{aligned}\theta &\sim \frac{10^{9}\text{ m}}{10^{17}\text{ m}} \\ &= 10^{-8}\text{ radians}\end{aligned}$$

The diffraction blob is ten times bigger than the actual disk of the star, so we can never make an image of the star itself in this way.

12-9 (a) The patterns have two structures, a coarse one and a fine one. You can look up in the book which corresponds to w and which to

d, or just use the fact that smaller features make bigger diffraction angles. The top and middle patterns have the same coarse spacing, so they have the same w. The fine structure in the top pattern has 7 fringes in 12.5 mm, for a spacing of 1.79 mm, while the middle pattern has 11 fringes in 41.5 mm, giving a spacing of 3.77 mm. The value of d for the middle pattern is therefore $(0.50\text{ mm})(1.79/3.77) = 0.23$ mm.

(b) This one has about the same d as the top one (it's difficult to measure accurately because each group has only a small number of fringes), but the coarse spacing is different, indicating a different value of w. It has two coarse groupings in 23 mm, i.e., a spacing of 12.5 mm. The coarse groupings in the original pattern were about 23 mm apart, so there is a factor of two between the $w = 0.04$ mm of the top pattern and the $w = 0.08$ mm of the bottom one.

12-10 (a) The grating's spacing is the number of centimeters per line, which is one over the number of lines per centimeter, $d = (1/2000\text{ lines/cm}) = 5\times10^{-4}$ cm/line. "Lines" are a fake unit that we can get rid of at will. Converting to meters, we have $d = 5\times10^{-6}$ m. The angles of the $m = -1$ and $m = 1$ spots are given by $\theta = \sin^{-1}(m\lambda/d) = +8.0$ degrees. The spacing of the spots on the screen is related to this by simple trig: $(2.0\text{ m})(\tan\theta) = 28$ cm.

(b) Note that the approximation $\sin\theta \approx \theta$ only works if we use units of radians. With it, we get $\theta = m\lambda/d = .139$ radians, which works out to be almost exactly the same as the answer we got without the approximation.

12-13 High frequencies have short wavelengths, which leads to small diffraction angles. This is a good thing, because we want the image to be sharply defined. Ultimately, you can't pick out any details that are smaller than one wavelength. (This only explains why they would use the highest possible frequency for imaging, not why they would use a lower one for heating. The most likely explanation is that it's just more expensive to get a speaker to vibrate really fast.)

13-1 [Note: An old version of the problem listed the parent isotope incorrectly as ^{234}Pu. It should be ^{241}Pu.]

Plutonium is atomic number 94. In alpha decay, we're removing two protons and two neutrons, so the mass number will be reduced by four, and the atomic number (number of protons) by 2. The nucleus left over after alpha decay therefore has atomic number 92 (uranium), and mass number 237; it's ^{237}U.

In electron decay, we're converting one neutron to a proton. The mass number is unchanged, but the atomic number is bumped up by one, to 95, americium. The product is ^{241}Am.

13-2 There are 82 protons in a lead nucleus. Since the lead nucleus is spherical, the electric force from the 81 other protons is the same as that of an equal amount of charge (81 times the charge of a proton) concentrated at the nucleus' center. The force is

$$k(q_{proton})\times(81q_{proton})/(6.5\times10^{-15}\text{ m})^2 = 440\text{ N} \quad ,$$

which is less than the attractive force of 8 kN.

13-3 (a) $+e - e = 0 + 0$

(b) The mass of a neutron is greater than the mass of a proton plus the mass of an electron, so this is a process that increases mass, and a hydrogen atom would therefore have to have a source of energy in order to do it.

13-4 The maximum energy of the recoiling nucleus is attained in the case where the electron gets almost 100% of the energy, because for a fixed energy, a more massive particle carries more momentum. If the electron and neutrino were to share the energy, then their momentum vectors could also partially cancel, further reducing the recoil.

Let Q be the energy released in the decay. In the approximation that the electron is ultrarelativistic, its momentum (in the case where it carries all the energy) is $p \approx Q$, and by conservation of momentum, this is also the momentum of the recoiling nucleus. Since the nucleus is nonrelativistic, its kinetic energy (reinserting factors of c) is $K = p^2/2M \approx Q^2/2Mc^2$. Plugging in numbers, we get about 24 eV, which is clearly plenty of energy to break a chemical bond.

If we don't approximate the electron as ultrarelativistic, then the momentum of the beta, in the maximum-recoil case, isn't Q but rather $\sqrt{(m+Q)^2-m^2}$, where m is the mass of the electron. The actual maximum recoil energy for the calcium is 42 eV, so although the ultrarelativistic approximation gives the right order of magnitude, it's off by almost a factor of 2.

Remark: These energies are counterintuitively low, since the nuclear energy scale of MeV is a million times bigger than the chemical energy scale of eV. In fact, for low-energy beta decays there can be a considerable probability for the parent molecule to survive without any disruption. An experiment by Snell and Pleasanton in 1958 showed that when $^{14}CO_2$ decays, emitting an electron with an energy of 0.156 MeV, an intact $^{14}NO_2^+$ ion is produced about 80% of the time.

14-1 After two years, the substance will be 75% decayed, not 100%.

14-2 (a) The activity falls off by a factor of two as you go from $t = 5$ min to $t = 32$ min, so its half-life is about 27 min. It looks like it's ^{214}Pb.
(b) If it's a single isotope with a single well-defined half-life, then the ratio of the activity at 20 min to the activity at 10 min should be the same as the ratio of the activity at 30 min to the activity at 20 min. Only the stuff produced by the second ion speed gives ratios equal to three significant figures, which is the accuracy of the data.

14-3 (a) Each rectangle has a width of 5 cm and a height of 0.005 cm^{-1}, so its area represents a probability of 0.025. The total number of rectangles under the curve is about 38, so the total area under the curve represents a probability of about $38 \times .025 = 0.95$, which is close enough to 1, considering that I had to estimate fractions of squares.
(b) The number of rectangles under the curve from 140 cm to 150 cm is about 6, so the the probability represented is about $6 \times .025 = .15$.

14-4 (a) The variable r varies continuously from 0 to b — it's not as though we have rings of finite width. However, suppose we imagine an extremely narrow ring extending from r to $r + \mathrm{d}r$. Since the person is blindfolded, the probability of hitting a given region depends only on its area. This ring has a circumference of $2\pi r$, so its area is $2\pi r\,\mathrm{d}r$. This is proportional to r, so the probability of getting a result between r and $r + \mathrm{d}r$ is proportional to r.
(b)

$$\begin{aligned}1 &= \int_0^b D(r)\,\mathrm{d}r\\ 1 &= \int_0^b kr\,\mathrm{d}r\\ 1 &= \frac{1}{2}kb^2\\ k &= \frac{2}{b^2}\\ D(r) &= \frac{2r}{b^2}\end{aligned}$$

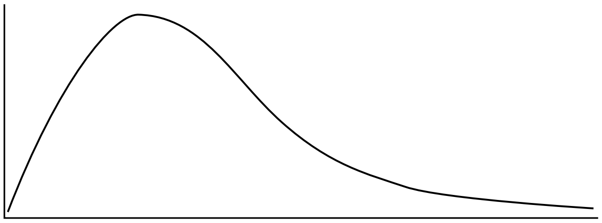
(c)

$$\begin{aligned}\bar{r} &= \int_0^b rD(r)\,\mathrm{d}r\\ &= \int_0^b \frac{2r^2}{b^2}\,\mathrm{d}r\\ &= \frac{2}{3}b\end{aligned}$$

(d) The average value of r is greater than $b/2$. This makes sense, because the area between $r = 0$ and $r = b/2$ is only one fourth of the total area.

14-5

14-6 (a) Without the factor of v in front, it would have the form of the classic bell curve. Since it's only defined for positive v, we chop off the left half of the bell. Then we multiply by v, which makes the curve drop down to zero at the origin. The result looks like this:



(b) We want to integrate D from 0 to ∞. The subsitution $u = cv^2$ gives, $\mathrm{d}u = 2cv\,\mathrm{d}v$, turning the integrand into $(b/2c)e^{-u}\,\mathrm{d}u$. The integral of $e^{-u}\,\mathrm{d}u$ from 0 to ∞ is 1, so the result is $1 = (b/2c)\cdot 1$, or $b = 2c$.

(c) I warned you not to try it by hand! (It requires a trick.) Using computer software, the result is $\int_0^\infty 2cv^2 e^{-cv^2}\,\mathrm{d}v = (1/2)\sqrt{\pi/c}$.

14-7 (a) To a good approximation, the mass of a nucleus is proportional to the number of protons plus the number of neutrons, which is the mass number. The total change in mass number from 238 to 206 is 32, so if mass is to be conserved, the number of alpha particles, with mass number four, emitted must be 32/4=8.

(b) The charge of the original nucleus was 92 (in units of the fundamental charge), and the nuclei produced have a total charge of $82 + 8 \times 2 = 98$. To conserve charge, 6 electrons must have been produced, so 6 beta decays must have occurred.

(c) The mass of a uranium atom is about 238 times the mass of a neutron or proton, so the number of surviving U atoms is

$$\frac{1230 \times 10^{-6}\ \text{kg}}{238 m_p} = 3.09 \times 10^{21} \qquad .$$

(The masses of protons and neutrons are nearly the same, so we don't have to add their masses separately.) Similarly, the number of helium atoms is

$$\frac{2.3 \times 10^{-6}\ \text{kg}}{4 m_p} = 3.44 \times 10^{20} \qquad .$$

Each of the decayed U atoms produced 8 helium atoms, so the number of decayed U atoms is $3.44 \times 10^{20}/8 = 4.30 \times 10^{19}$. The probability of decay since the rock was formed equals (number that decayed)/(number that decayed + number that survived) = 0.0137, so the probability of survival was 0.9863, and $0.9863 = 0.5^{t/t_{1/2}}$. Taking natural logarithms of both sides, $\ln 0.9863 = (t/t_{1/2}) \ln 0.5$, yielding $t = 90$ million years.

15-1 $E = hf$, and frequency corresponds to color, so if each photon was losing 20% of its energy you'd notice a pronounced change in color.

15-2 The amount of energy lost by the electrons as they leave the surface, E_s, depends on the material. With a material with a high E_s, you wouldn't be sensitive to the red end of the spectrum, which has low-energy photons.

15-3 Hertz noticed that the cutoff for producing the photoelectric effect in metals was somewhere in the visible-light spectrum. We're assuming that the energy required to ionize DNA is similar to the energy required to rip an electron out of a metal, so again, the cutoff should at a frequency that's of roughly the same order of magnitude as the frequency of visible light. The types of waves are listed in order of increasing frequency, so the only ones that can cause cancer are the ones farther down the list than the cutoff. Ultraviolet and x-rays are certainly capable of doing it. It might be possible to get cancer from visible light — our analysis is too crude to tell exactly where the cutoff is. You are not going to get cancer from power lines, FM radio, cell phones, or microwave ovens.

15-4 (a) $\lambda_1 = \lambda_4 < \lambda_2 = \lambda_3$. Since $c = f\lambda$, the frequencies go in the opposite order. The energies go in the same order as the frequencies, because $E = hf$.

(b) Conservation of energy says that the energy of both the emitted photon and the energy of the absorbed photon have to be equal to the difference in energy between the two states of the atom. Therefore only photon 2 can excite the atom in this way.

15-5 Visible light has a wavelength on the order of 10^{-6} m, so a typical photon of visible light is about $E = hf = hc/\lambda \approx 2 \times 10^{-19}$ J, so the number of photons emitted per second is about $(100\ \text{J/s})/(2 \times 10^{-19}\ \text{J/photon}) = 5 \times 10^{20}$ photons/s. The time each photon spends on its voyage is $(3\ \text{m})/c \approx 10^{-8}$ s. The number of photons in the beam equals the number of photons emitted during the time required for a photon to get across, $(5 \times 10^{20}\ \text{photons/s}) \times (10^{-8}\ \text{s}) = 5 \times 10^{12}$ photons. Given the crudeness of the initial assumptions, this is only an order-of-magnitude result, so let's call it 10^{13} photons.

15-6 (a)

$$\begin{aligned}\text{power} &= (1\ \text{mW/m}^2)(\text{cross-sectional area})\\ &= (1\ \text{mW/m}^2)(\pi r^2)\\ &\sim 10^{-32}\ \text{W}\end{aligned}$$

(rounding to a power of 10).
(b) power $= \Delta E/\Delta t$, so

$$\begin{aligned}\Delta t &= \Delta E/\text{power}\\ &\sim 10^{13}\ \text{s}\end{aligned}$$

The classical theory says it should take thousands of years for an electron to soak up enough energy to be ejected, which contradicts experiment, so the classical theory must be wrong.

15-7 The energy of the radio photon is much more than the $E = mc^2$ equivalent of the putative mass, so the photon is still ultrarelativistic. Equating $mc^2\gamma$ to hf, and solving for v/c, we get $v/c = \sqrt{1-(mc^2/hf)^2}$. This probably rounds off to one on your calculator. A way around this is to use the approximation $\sqrt{1+x} \approx 1 + x/2$, which is valid for small x and is a special case of $(1+x)^p \approx 1 + px$. Using this approximation, we have $v/c = 1-(1/2)(mc^2/hf)^2$. The relative size of the predicted discrepancy is about 10^{-34}.

15-8 In both case, we're talking about 100 joules worth of light. Consulting the chart of the electromagnetiv spectrum, we find that the frequency of visible light is greater than that of radio waves by a factor of about $(10^{14}\ \text{Hz})/(10^8\ \text{Hz}) = 10^6$. (The radio spectrum covers a lot of ground, but near the middle of it we have FM radio at about 100 MHz $= 10^8$ Hz. The visible spectrum only spans about a factor of two in frequency, so that's less of an issue.) Since a photon's energy is given by $E = hf$, that means a visible photon has a million times more energy than a radio photon. There are a million times more photons in the radio signal than in the visible light. If you assumed a different value for the frequency of the radio waves, you might have gotten an answer that differed from this one by a factor of 10 or 100.

15-9 (a) The surface area of the spherical surface is $A = 4\pi r^2$. During a time interval t, the energy from the transmitter travels a distance $w = ct$, and by the time this energy reaches the distance r, it has been spread out over a spherical shell of volume Aw. The energy emitted during this time is Pt, so the energy density is $E/V = PT/(Aw) = P/4\pi cr^2$.
(b) The number of photons n in a volume $V = \lambda^3$ is

$$\begin{aligned}n &= \frac{E}{hf}\\ &= \frac{(E/V)\lambda^3}{hc/\lambda}\\ &= \frac{P\lambda^4}{4\pi hc^2r^2}\end{aligned}$$

(c) Let's take $P = 10$ kW and $r = 10$ km. The result then comes out to be about 10^{21}. This huge number is consistent with the correspondence principle.

15-10 There's a grid superimposed on the patterns, but I just took a ruler and measured the distance between the $m = -1$ and $m = 1$ fringes. The ratio of these was $292/176 = 1.66$, so the photons in B have a wavelength that is shorter by a factor of 1.66, and an energy that is higher by a factor of 1.66. There are about 101 photons in the $m = -1$, 0, and 1 fringes in A, and 140 in B. B is going to have more energy, both because there are more photons in it and because its photons are individually more energetic. The ratio is $1.66 \times (140/101) \approx 2.3$.

15-11 The square of the wave represents probability per unit volume. In the case of this one-dimensional representation of the waves, we should interpret it as probability per unit length. The really long and laborious way to do this problem would be to draw the squared waves as four new graphs, and then count squares under the curves. But since they're all sine waves, we can dispense with some of the details about their shapes, and just compare their squared amplitudes, and the amounts of space they cover.

A	A^2	space	(space) × (A^2)
4	16	12	192
5.5	30	6	180
4	16	12	192
8	64	6	354

The final column of the table represents relative probability, and we see that it comes out about the same for the first three, so with the correct calibration of the axes on the graph, these could all come out to have a total probability of 1. The fourth one is the one that's different.

Interpretation: The first and third ones have the same amplitude, and cover the same amount of space, so naturally they have the same total probability. The fourth one has twice the amplitude and covers half the space, which you might think would compensate for each other, but probability depends on the square of the amplitude, so we get a factor of 4 due to amplitude, multiplied by the factor of 1/2 for space, which gives a total probability of twice as much as the others. That one's not properly normalized. We could say that it represented *two* photons.

15-12 Let p be the probability that it's emitted to the right. The two pieces of the wave have the same wavelength, but their amplitudes differ. Probability is proportional to amplitude squared, so $p/(1-p) = (5/1.5)^2$, giving $p = 0.92$.

16-1 Momentum is related to kinetic energy by $E = p^2/2m$, so

$$\begin{aligned}\lambda &= h/p\\ &= h/\sqrt{2mE}\\ &= h/\sqrt{2mq\Delta V}\\ &= 1\times10^{-11}\ \mathrm{m}\end{aligned}$$

16-2 As an electron speeds up, it gains momentum, which according to $\lambda = h/p$ implies a shortening of its wavelength. Number 3 is the only one that works. (Number 2 might be a tempting alternative, but it's wrong, because the momentum isn't changing. The amplitude is increasing as we go to the right, and therefore the portion of the electron's energy that we're likely to find in the right-hand region is higher than in the left-hand region. But that's not because the electron is speeding up, it's merely because the electron has a higher probability of being on the right, so that's where its energy is likely to be.)

16-3 It's conceivable that the Δx is actually *less* than the diameter of the nucleus, but it can't be *more*, since the proton stays inside the nucleus. The maximum possible value of Δx is therefore roughly the same as the diameter of the nucleus. A maximum value of Δx leads to a minimum value of Δp, since they're inversely related: $\Delta p \gtrapprox h/\Delta x$. Plugging in, we get $\Delta v = \Delta p/m \gtrapprox h/m\Delta x \gtrapprox 3\times10^7$ m/s. Note the two inequalities in a row: one because the maximum uncertainty is the diameter of the nucleus, and one because the uncertainty principle is an inequality. It's perfectly possible that they're moving much faster than this; what we've found is only a minimum.

Our result is 10% of the speed of light, so the nonrelativistic equations are good enough for this kind of estimate.

16-4 The probability distribution is the *square* of the wavefunction. Normalization requires that integrating the probability distribution give 1:

$$\begin{aligned}1 &= \int_0^L \Psi^2\,\mathrm{d}x\\ &= A^2\int_0^L \sin^2(2\pi x/L)\,\mathrm{d}x\\ &= A^2\cdot(L/2) \qquad ,\end{aligned}$$

where the final line can be performed without any techniques of integration simply because we know that the average value of $\sin^2$ over a full cycle is 1/2. The result is $A = \sqrt{2/L}$.

16-5 The wavelength equals $2L/n$, so we have

$$\begin{aligned}KE &= \frac{p^2}{2m}\\ &= \frac{(h/\lambda)^2}{2m}\\ &= \frac{h^2n^2}{8mL^2}\end{aligned}$$

16-6 (a) $\lambda = h/p = h/mv = 10^{-8}\ \mathrm{m} = 10$ nm. (b) The area of one circuit would be $10^{-8}\ \mathrm{cm}^2$,

which might be a square 10^{-4} cm on a side, i.e. 1000 nm.
(c) No. Diffraction is only an important effect when the size of the object the wave is diffracting around is comparable to the wavelength.
(d) If the linear size of a circuit was of the same order of magnitude as the wavelength, then we'd definitely have a lot of diffraction. A 10 nm × 10 nm circuit would have an area of 100 nm^2 = 10^{-12} cm^2, so you could fit 10^{12} of them on the chip.

16-7 (a) The energy of a photon is proportional to its frequency, and therefore inversely proportional to its wavelength. Of the visible-light transitions, F has the greatest energy, and thus the shortest wavelength, i.e. it is near the violet end. Transition D would be closest to the red end.
(b) A, B, and C have high energies, which correspond to short wavelengths. We're told they're not visible, so they must be in the ultraviolet. (They can't be in the x-ray or gamma regions, because the energies and wavelengths are not orders of magnitude different from those of visible light.) Similar reasoning places G and H in the infrared.
(c) A long photon wavelength corresponds to a low photon frequency, a low photon energy, and a small difference between energies of atomic levels. There is no upper limit to the wavelengths, since the states get infinitely close together near the top of the hydrogen atom's energy-level diagram. On the other hand, there is a lower limit to the photon wavelength, since the bound states of the hydrogen atom span a limited range of energies, imposing a maximum on the difference between any two levels. Physically, if you put too much energy into a hydrogen atom, you strip off the electron entirely.

16-8

$$\begin{aligned}
E_i &= E_f \\
E_{atom,i} &= E_{atom,f} + E_{photon} \\
E_{photon} &= E_{atom,i} - E_{atom,f} \\
&= \frac{mk^2e^4}{2\hbar^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)
\end{aligned}$$

But

$$\begin{aligned}
E_{photon} &= hf \\
&= hc/\lambda \qquad ,
\end{aligned}$$

so

$$\begin{aligned}
hc/\lambda &= \frac{mk^2e^4}{2\hbar^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \\
\lambda &= \frac{2\hbar^2 hc}{mk^2e^4}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)^{-1} \\
&= \frac{4\pi\hbar^3 c}{mk^2e^4}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)^{-1}
\end{aligned}$$

16-9 All of the constants cancel out, so we only need the n-dependent part of the expressions. The result is

$$\frac{\lambda_E}{\lambda_F} = \frac{E_F}{E_E} = \frac{2^{-2} - 5^{-2}}{2^{-2} - 4^{-2}} = \frac{28}{25}$$

16-10

16-11 In the approximate derivation in subsection 12.4.4, the only place charge came in was in the use of the expression $-ke^2/r$ for the electrical energy. If the charges were Ze and $-e$ rather than e and $-e$, we'd simply have Ze^2 instead of e^2. The result contains e^4, so we need to replace $e^4 = (e^2)^2$ with $(Ze^2)^2 = Z^2e^4$. The result would be

$$E_n = -\frac{mk^2Z^2e^4}{2\hbar^2} \cdot \frac{1}{n^2}.$$

16-12 (a) The relation

$$r \sim \frac{h^2n^2}{mke^2}$$

has the mass of the electron in the denominator. Increasing the mass by a factor of 207 scales the atom down by a factor of 207.
(b) The equation

$$E_n = -\frac{mk^2e^4}{2\hbar^2} \cdot \frac{1}{n^2}$$

would scale up all the energies by a factor of 207. If the atom's energies are scaled up, then so are the energies of its absorption and emission lines. They would be ultraviolet or x-ray lines.

16-13 The energies are inversely proportional to the wavelengths, so in terms of energies the equation would be $E_1 = E_2 + E_3$. This would happen for sets of transitions like G, D, and E in the previous problem, where the atom can get from one state to another either by a single hop or by two shorter hops.

16-14 We have $KE = p^2/2m \propto p^2 \propto \lambda^{-2} \sim (r/n)^{-2}$. Setting this to be $\sim |r^k|$ results in $r \sim n^{2/(2+k)}$, and taking the energy is then $E_n \sim |r^k|n^j$, where $j = 2k/(2+k)$. For the three cases discussed in the problem, we have $j = -2$, $2/3$, and 1. With $j = 2/3$, we have states that get closer together as we go up in energy. With $j = 1$, the states are evenly spaced. This is consistent with the experimental results shown in the figure.

The cases with $k > 0$ are different from the ones with $k < 0$. Positive k indicates that the energy continues to increase, no matter how far apart the particles are. Physically, it then becomes impossible to permanently and completely separate them. The constant of proportionality in $E_n \propto r^j$ is positive.

The case of $k \to \infty$ corresponds to a particle in a box. The exponent j approaches 2, which is the result for a particle in a box.

16-15 (a) The uncertainty in position is on the order of magnitude of the diameter of the nucleus, so the uncertainty principle gives $\Delta p \sim h/\Delta x \sim 10^{-18}$ kg·m/s. (b) $K = (1/2)mv^2 = p^2/(2m) \sim 10^{-7}$ J. (c) $m = E/c^2 \sim 10^{-24}$ kg, which is a thousand times greater than the actual mass of the neutron.

17-1 (a) The sizes of the realistic wavefunctions are on the order of 0.1 nm. Plugging in $n = 1$ to the approximation $r \sim h^2n^2/mke^2$, we get 2 nm. The estimate was too high by about a factor of 20, which is reasonable considering the crude assumptions we made.
(b) The proton is about 2000 times more massive than the electron, so the center of mass lies about 1/2000 of the way from the center of the proton to the electron. A typical separation between the proton and electron is 0.1 nm, so the center of the proton would typically be about (0.1 nm)/2000 $\sim 10^{-13}$ m from the center of mass. The size of a proton is about 10^{-15} m, so the center of mass is normally outside the proton.

17-2 (a) The size of the diagram is comparable to a, but b is so small that you wouldn't be able to see it on that scale.
(b) The stuff inside the exponential is $-r/a$, which is essentially zero for all values of r between 0 and b. In other words, the proton is super-tiny compared to the size of the electron cloud, and the electron's probability density therefore won't change appreciably from one part of the proton to the other. We don't need to do an integral at all. We can just multiply:

$$\begin{aligned}
P &= \Psi^2 V \\
&= \left(\pi^{-1/2}a^{-3/2}e^{-r/a}\right)^2 \left(\frac{4\pi}{3}b^3\right) \\
&\approx \left(\pi^{-1/2}a^{-3/2}\right)^2 \left(\frac{4\pi}{3}b^3\right) \\
&= \frac{4b^3}{3a^3}
\end{aligned}$$

(c) 1×10^{-15}
(d) The probability density never dies down to zero, but it gets very very close to zero. The atom is fuzzy. It doesn't have a sharply defined boundary. There is theoretically a finite probability for the electron to be a centimeter away from the proton, but it's extremely small.

17-3 The requirement of normalization is

$$\int \Psi^2 \, dv = 1 \qquad ,$$

where $\mathrm{d}v$ is an infinitesimal volume element, and the integral is over all space. The method used for carrying out this integral is similar to the one used in problem 13-8. The problem has spherical symmetry, so we can break space down into a series of concentric spherical shells, each one stretching from r to $r + \mathrm{d}r$. The volume of such a shell equals its area multiplied by its thickness, i.e., $\mathrm{d}v = 4\pi r^2\,\mathrm{d}r$.

$$\int_0^\infty \Psi^2 \cdot 4\pi r^2\,\mathrm{d}r = 4a^{-3}\int_0^\infty e^{-2r/a} r^2\,\mathrm{d}r \qquad .$$

The change of variable $u = 2r/a$ transforms this into

$$\frac{1}{2}\int_0^\infty e^{-u} u^2\,\mathrm{d}u \qquad .$$

The definite integral can be done by integration by parts, but I just used computer software (the open-source program Maxima) to find out that it equals 2. The result of the entire calculation is 1, as required by normalization.

17-4 (a)

$$\begin{aligned}
\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} &= \frac{\mathrm{d}^2}{\mathrm{d}x^2} e^{-bx^2} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(-2bxe^{-bx^2}\right) \\
&= -2be^{-bx^2} + (-2bx)(-2bx)e^{-bx^2} \\
&= 2b\left(2bx^2 - 1\right)e^{-bx^2}
\end{aligned}$$

$$\begin{aligned}
Ee^{-bx^2} &= -\frac{\hbar^2}{2m}\left(2b\left(2bx^2 - 1\right)e^{-bx^2}\right) + \frac{1}{2}kx^2e^{-bx^2} \\
E &= -\frac{\hbar^2 b}{m}\left(2bx^2 - 1\right) + \frac{1}{2}kx^2 \\
0 &= \left(\frac{1}{2}k - \frac{2\hbar^2 b^2}{m}\right)x^2 + \left(\frac{\hbar^2 b}{m} - E\right)
\end{aligned}$$

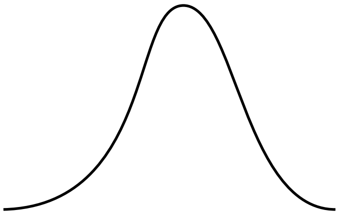
If this equation is going to be true for all values of x, then both coefficients in the polynomial have to be zero,

$$k = \frac{4\hbar^2 b^2}{m}$$

and

$$E = \frac{\hbar^2 b}{m}$$

If the problem is specified by giving the potential U, then we can find the unknowns E and b that complete the specification of the wave that is the solution. The result for b is $\sqrt{km}/2\hbar$. (b) It's a bell curve, like this:



(c) If b was bigger, then we'd get the same value of the wavefunction for smaller values of x. In other words, increasing b causes the bell to get skinnier.

(d) Making k greater makes b greater, which makes it skinnier. That makes sense, because a skinnier wavefunction means that the atom is unlikely to be found very far from the equilibrium point.

17-5 If this is to be a solution to the Schrodinger equation with a constant potential, then $\nabla^2\Psi$ has to be equal to a constant multiplied by Ψ. Calculating the partial derivatives, we have:

$$\begin{aligned}
\frac{\partial^2\Psi}{\partial x^2} &= -a^2 \sin ax e^{by} \\
\frac{\partial^2\Psi}{\partial y^2} &= b^2 \sin ax e^{by} \\
\nabla^2\Psi &= (b^2 - a^2)\Psi
\end{aligned}$$

This does equal a constant multiplied by Ψ. An easy way to tell whether this is a classically allowed region or a classically forbidden one is to test whether the kinetic energy is positive. The kinetic energy is equal to $-\nabla^2\Psi$ multiplied by positive constants. For $|b| < |a|$, the kinetic energy is positive, and the solution represents a classically allowed region.

17-6 The result of problem 13-21, for a one-dimensional box of length L, was $K = \frac{h^2n^2}{8mL^2}$. In three three dimensions, we have some number n_x of half-wavelengths that fit in the x direction, along length a, and similarly for b and c. The three contributions to the energies add, since $K \propto p^2 = p_x^2 + p_y^2 + p_z^2$ is additive. The result is

$$K = \frac{h^2}{8m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right)$$

17-7 The tritium has the same charge as the helium, so the electrical repulsion is unchanged, and therefore the height, width, and shape of the barrier are essentially unchanged. The only thing that's really changed is the mass of the particle being emitted, which has been reduced by a factor of 3/4. The tunneling probability is an exponential, and the exponent is proportional to the square root of the mass, so we're reducing it by a factor of $\sqrt{3/4}$. Since we're comparing two exponentials, it doesn't really matter what base we use. In base 10, the original tunneling probability was 1×10^{-29}, so we just need to cut the exponent from -29 to $-29\sqrt{3/4}$, which is about -25, so the result is 10^{-25}.

17-9 (a) Because Ψ_0 is an even function and Ψ_1 is odd, the integral $\int_{-\infty}^{\infty} \Psi_0\Psi_1\,\mathrm{d}x$ is zero.

(b) Either energy or parity would distinguish them.

17-10 (a) The norm of Ψ (inner product of Ψ with itself) is

$$\langle\Psi|\Psi\rangle = (2A\langle 1| + A\langle 2|)(2A|1\rangle + A|2\rangle).$$

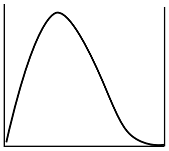
Using the linearity of the inner product, this becomes

$$\begin{aligned}
&A^2(2\langle 1| + \langle 2|)(2|1\rangle + |2\rangle) \\
&= A^2(2^2\langle 1|1\rangle + \ldots\langle 1|2\rangle \\
&\quad + \ldots\langle 2|1\rangle + 1^2\langle 2|2\rangle).
\end{aligned}$$

The second and third terms vanish because states 1 and 2 are distinguishable (or, alternatively, because one function is odd and the other is even). The result is $|A|^2(2^1 + 1^2) = 5|A|^2$, so we need $|A| = 1/\sqrt{5}$.

(b) On the left side of the box, the two functions are both positive, so the wavefunction is big and positive. On the right side of the box, there is a lot of cancellation. The resulting shape looks like this:

(c) Let x run from 0 to π. Then the two functions are basically $\sin x$ and $\sin 2x$, but we need to normalize them properly. The normalization factors come out to be $\sqrt{2/\pi}$. For the probability of being on the left side of the box, we have



$$P = \int_0^{\pi} (1/5)(2/\pi)(2\sin x + \sin 2x)^2 \, \mathrm{d}x$$

$$= (2/5\pi)\int_0^{\pi/2} (4\sin^2 x + 4\sin x \sin 2x + \sin^2 2x) \, \mathrm{d}x$$

The average values of the sine waves in the first and third terms are both 1/2. Replacing them with their average values gives

$$P = (2/5\pi)\int_0^{\pi/2} (2 + 4\sin x \sin 2x + 1/2) \, \mathrm{d}x$$

The term in $\sin x \sin 2x$ is the kind of thing that you would just do using software these days, or you can use the double-angle identity for the sine and then a change of variables to $u = \sin x$. The result is

$$\frac{1}{2} + \frac{16}{15\pi}.$$

Based on part b, it makes sense that this is greater than 1/2. (d) Probability is proportional to the square of the amplitude, so the probability of getting the ground state is four times greater than the probability of getting state 2. Therefore the probability of getting the ground state is 4/5.

(e) The shape of the wavefunction would flip, and the probability of being in the left half would now be $1 - P = 1/2 - (16/15)\pi$. Everything else would stay the same.

17-11 (a) Plugging in to the Schrödinger equation for Ψ_1 gives

$$-\Psi_1 = -\frac{1}{2m}\Psi_1 + U\Psi_1,$$

which could be a solution in a classically forbidden region. Doing the same with Ψ_2, we find

$$i\Psi_2 = \frac{1}{2m}\Psi_2 + U\Psi_2,$$

which is impossible if the mass and potential are both real.

(b) Ψ_2 shows exponential growth over time, which violates unitarity.

17-12 (a) Ψ_2 can't be a solution, because it doesn't satisfy the boundary conditions: it isn't zero at $y = 0$ and $y = b$.
(b) Ψ_1 has the form of a traveling wave, which is moving to the left. By decreasing x and increasing t by the correct amount, we can get the same quantity inside the exponential, which shows that we're surfing on one point on the wave.
(c) Ψ_3 could represent motion in a classically forbidden region, since the wavefunction falls off exponentially to the left, as expected for tunneling.
(d) Ψ_4 is the only non-separable solution. The others are clearly separable because they are expressed in the form $f(x)g(y)$.

18-1 Say the basketball has a mass of 1 kg, and in general its mass is about 0.1 m from the axis of rotation. If it's spinning at a speed of 1 m/s, then its angular momentum is on the order of 0.1 J$\cdot$s. This is roughly $10^{32}\hbar$. The fact that this number is so huge is an example of the correspondence principle. Angular momentum is quantized in units of $\hbar$, so the basketball can't continuously speed up or slow down its rotation; it has to do it in quantum leaps from one value of L to another. But because L is so huge when expressed in units of $\hbar$, the process appears smooth and continuous. Therefore quantum mechanics doesn't contradict the results of the centuries of previous experiments in which rotation appeared to be smoothly variable.

18-2 (a) The possible s_z states are -3, -2, -1, 0, 1, 2, and 3. These are 7 states, which checks against $2 \times 3 + 1$.
(b) Its spin is an integer, so it's a boson.
(c) It's made out of N neutrons and Z protons. Since neutrons and protons have half-integer spins, the fact that its ground state has a bosonic spin tells us that $N + Z$ is even. The excited states are still states of a system composed of an even number of fermions, so the excited states all have to be bosonic.

18-3 The two neutrons are identical particles, so they can't be in the same state. That means they can only be put in the spin-0 state, not the spin-1 state.

18-4 (a) As a warm-up let's do the one-dimensional version of the problem, for the length being L. Then $\lambda = 2L/n$, and the energy is $p^2/2m = \epsilon n^2$. The three-dimensional version is separable. The x and y dimensions contribute terms like the one we already calculated, while halving the wavelength for the z dimension gives four times the energy. Therefore the total energy, in units of ϵ, is $n_x^2 + n_y^2 + 4n_z^2$. The smallest energies we can make are the following.

1	1	1	6
2	1	1	9
1	2	1	9
3	1	1	14
1	3	1	14

(If we try bumping n_z up to 2, we get a minimum energy of 17, which is higher.) Counting spin, the lowest energy state has a degeneracy of 2, the second energy state is 4-fold degenerate, and the third is also 4-fold degenerate.
(b) Subject to the exclusion principle, the lowest total energy is obtained by putting 2 electrons in the lowest energy state and 3 in the second, for a total energy of 39 (still in units of ϵ).
(c) The "cheapest" way of forming an excited state is to pop one electron out of the lowest-energy state and into the second energy state, and this gives a total energy of 42. We can't boost the other one from the first to the second, because that would be all five electrons in the second state, which would violate the exclusion principle. The second-cheapest method is to boost one from the second energy level to the third, which produces an energy of 44. To excite one of the five electrons from the ground state to one of these two states, we would need a photon with an energy of either $42 - 39 = 3$ or $44 - 39 = 5$.

Photo credits